Derivation or Proof-of-Mirror formula(X) physics:

Mirror formula is the relationship between object distance (u), image distance (v) and focal length.

\[
\frac{1}{v} + \frac{1}{u} = \frac{1}{f}
\]

In \(\triangle ABC\) and \(A'B'C\)

\(<A = <A' = 90^0\)

\(<C = <C\) (vert. opp. <s)

\(\triangle ABC \sim \triangle A'B'C\) [AA similarity]

\(\frac{AB}{A'B'} = \frac{AC/A'C} ----(I)\)

Similarly,

In \(\triangle ABC\) and \(A'B'C\)

\(<A = <A' = 90^0\)

\(<C = <C\) (vert. opp. <s)

\(\triangle ABC \sim \triangle A'B'C\) [AA similarity]

\(\frac{AB}{A'B'} = \frac{AC/A'C} ----(1)\)

Similarly, In \(\triangle FPE \sim A'B'F\)

\(\frac{EP}{A'B'} = \frac{PF/A'F} \)

\(\frac{AB}{A'B'} = \frac{PF/A'F} [ AB=EP] ----(II)\)

From (i) & (ii)

\(\frac{AC/A'C} = \frac{PF/A'F} \)

\(\Rightarrow \frac{A'C}{AC} = \frac{A'F}{PF} \)

\(\Rightarrow \frac{(CP-A'P)/(AP- CP)} = \frac{(A'P- PF)/PF} \)

Now, \(PF = -f\); \(CP = 2PF = -2f\);

\(AP = -u\); and \(A'P = -v\)

Put these value in above relation:
Derivation or Proof-of Lens formula(X) physics

Let AB is an object placed between f1 and f2 of the convex lens. The image A1B1 is formed beyond 2F2 and is real and inverted.

OA = Object distance = u ; OA1 = Image distance = v ; OF2 = Focal length = f

In \( \triangle OAB \) and \( \triangle OA_1B_1 \) are similar

\[ \angle BAO = \angle B_1A_1O = 90^\circ \]
\[ \angle AOB = \angle A_1OB_1 \] [vertically opp. <s]

\( \triangle OAB \sim \triangle OA_1B_1 \)

\[ \frac{A_1B_1}{AB} = \frac{OA_1}{OA} \] ------------------(i)

Similarly, \( \triangle OCF_2 \sim \triangle F_2A_1B_1 \)

\[ \frac{A_1B_1}{OC} = \frac{F_2A_1}{OF_2} \]

But we know that OC = AB

\[ \Rightarrow \frac{A_1B_1}{AB} = \frac{F_2A_1}{OF_2} \] ------------------(ii)

From equation (i) and (ii), we get

\[ \frac{OA_1}{OA} = \frac{F_2A_1}{OF_2} \]
\[ \frac{OA_1}{OA} = \frac{(OA_1 - OF_2)}{OF_2} \]

\[ \frac{v}{-u} = \frac{(v-u)}{f} \]
\[ vf = -u(v-f) \]
\[ vf = -uv + uf \]

Dividing equation (3) throughout by uvf

\[ \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \]