Q. Prove that the circle drawn with any side of a rhombus as a diameter, passes through the point of its diagonals.

Let ABCD be a rhombus whose diagonals AC and BD intersect at O.
We know that, **diagonals of a rhombus intersect each other at right angle.**
∴ ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°
Now, circles with AB, BC, CD and DA as diameter passes through O.
(***Angle in a semi-circle is 90°**)
Thus, the circles described on the four sides of a rhombus as diameter, pass through the point of intersection of its diagonals.
Q. If the circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side

Let the side AB and AC are the diameters and AD be the common chord

Prove that D lies on BC

Proof: ∠ADB = 90° (Angle in a semicircle) and ∠ADC = 90° (Angle in a semicircle)
So, ∠ADB + ∠ADC = 90° + 90° = 180°
Therefore, BDC is a line. Hence the point of intersection of two circles lie on the third side.

Q. Prove that angles subtended by an arc at the centre is double the angle subtended by it at any other point on the circle

In the given condition, clearly we have three cases:
Case (1) \(\widehat{AB}\) is a minor arc
Case (2) \(\widehat{AB}\) is a semicircle
Case (3) \(\widehat{AB}\) is a major arc
Construction: Join PO and extend it to a point Q.

(i)
(ii)
(iii)

To prove: \(\angle AOB = 2\angle APB\)

Proof:
We know that, an exterior angle of a triangle is equal to the sum of the interior opposite angles.
In \(\triangle OPB\),
\(\angle QOB = \angle OPB + \angle OBP \ldots (1)\)
\(OB = OP\) (Radius of the circle)
⇒ \(\angle OPB = \angle OBP\) (In a triangle, equal sides have equal angle opposite to them)
∴ \( \angle QOB = \angle OPB + \angle OPB \)
⇒ \( \angle QOB = 2\angle OPB \) ...(2)

In \( \triangle OPA \)
\[ \angle QOA = \angle OPA + \angle OAP \] ...(3)
OA = OP (Radius of the circle)
⇒ \( \angle QOA = \angle OPA \) (In a triangle, equal sides have equal angle opposite to them)
∴ \( \angle QOA = 2\angle OPA \) ...(4)

Adding (2) and (4), we have
\[ \angle QOA + \angle QOB = 2\angle OPA + \angle OPB \]
⇒ \( \angle AOB = 2\angle APB \)

For the case 3, where AB is the major arc, \( \angle AOB \) is replaced by reflex \( \angle AOB \).
∴ reflex \( \angle AOB = 2\angle APB \)

Q. Prove that there is one and only one circle passing through three given non-collinear points.

Given: Three non collinear points P, Q and R
To prove: There is one and only one circle passing through the points P, Q and R.

Construction: Join PQ and QR.

Draw perpendicular bisectors AB of PQ and CD of QR. Let the perpendicular bisectors intersect at the point O.

Now join OP, OQ and OR.
A circle is obtained passing through the points P, Q and R.

Proof: We know that, each and every point on the perpendicular bisector of a line segment is equidistant from its ends points.
Thus, OP = OQ [Since, O lies on the perpendicular bisector of PQ]
and OQ = OR. [Since, O lies on the perpendicular bisector of QR]
So, OP = OQ = OR.
Let OP = OQ = OR = r.
Now, draw a circle \( C(O, r) \) with O as centre and r as radius.
Then, circle \( C(O, r) \) passes through the points P, Q and R.
Next, we show: this circle is the only circle passing through the points P, Q and R.

If possible, suppose there is another circle \( C(O', t) \) which passes through the points P, Q, R.
Then, \( O' \) will lie on the perpendicular bisectors AB and CD.
But O was the intersection point of the perpendicular bisectors AB and CD.
So, \( O' \) must coincide with the point O. [Since, two lines can not intersect at more than one point]
As, \( O'P = t \) and \( OP = r \); and \( O' \) coincides with O, we get \( t = r \)
Therefore, \( C(O, r) \) and \( C(O, t) \) are congruent.
Thus, there is one and only one circle passing through three the given non-collinear points.

We can draw circles from more than 3 non-collinear points as a circle consists of infinite number of points.

Q. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in their hand to talk each other. Find the length of the string of each phone.

It is given that \( AS = SD = DA \)
Therefore, \( \triangle ASD \) is an equilateral triangle. \( OA = r = 20 \) m
Medians of equilateral triangle pass through the circumcentre (O) of the equilateral triangle ASD. We also know that medians intersect each other in the ratio 2: 1. As AB is the median of equilateral triangle ASD, we can write
\[
\Rightarrow OA = \frac{2}{1}
\]
\[
\Rightarrow OB = \frac{20}{1}
\]
\[
\Rightarrow OB = \left(\frac{20}{2}\right)m = 10\ m
\]
\[\therefore AB = OA + OB = (20 + 10)\ m = 30\ m\]

In \(\triangle ABD\),
\[AD^2 = AB^2 + BD^2\]
\[\Rightarrow AD^2 = (30)^2 + \left(\frac{AD}{2}\right)^2\]
\[\Rightarrow AD^2 = 900 + \frac{1}{4}AD^2\]
\[\Rightarrow \frac{3}{4}AD^2 = 900\]
\[\Rightarrow AD^2 = 1200\]
\[\Rightarrow AD = 20\sqrt{3}\]

Therefore, the length of the string of each phone will be \(20\sqrt{3}\) m.

Q. If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection; prove that the chords are equal.

Given that: \(AB\) is the diameter of the circle with centre \(O\). \(AP\) and \(AQ\) are two intersecting chords of the circle such that \(\angle PAB = \angle QAB\).

To prove: \(AP = AQ\)

Construction: Draw \(OL \perp AB\) and \(OM \perp AC\).

Proof: In \(\triangle AOL\) and \(\triangle AOM\)
\[\angle OLA = \angle OMB\ (each\ 90^\circ),\ OA = OA\ \ (Common\ line)\]
\[\angle OAL = \angle OAM\ (\angle PAB = \angle QAB)\]
\[\therefore \triangle AOL \cong \triangle AOM\ \ (AAS\ congruence\ criterion)\]
\[\Rightarrow OL = OM\ (C.P.C.T)\]
\[\Rightarrow Chords\ AP\ and\ AQ\ are\ equidistant\ from\ centre\ O\]
\[\Rightarrow AP = AQ\ (Chords\ which\ are\ equidistant\ from\ the\ centre\ are\ equal)\]

Q. If two sides of a cyclic quadrilateral are parallel, prove that the remaining two sides are equal and the diagonals are also equal.

ABCD is the cyclic quadrilateral.
It is given that \(AB\) is parallel to \(CD\).
Prove that: \(AD\) and \(BC\) are equal and that \(AC\) and \(BD\) are equal.

Since ABCD is a cyclic quadrilateral, \(<DAB + <DCB = <CDA + <CBA = 180\>
Since \(AB\) is parallel to \(CD\), \(<DAB + <CDA = <DCB + <CBA = 180\>

Comparing the above two equations, it can be said that \(<CDA = <DCB\>

This is a property of an isosceles trapezium. Thus, \(AD = BC\).

In \(\triangle DAC\) and \(\triangle CBD\); \(AD = BC\), \(<CDA = <DCB\), \(CD = DC\)
Thus, \(\triangle DAC \equiv\ \triangle CBD\)

Thus, \(AC = BD\). Thus, it has been proven that \(AD = BC\) and \(AC = BD\)