Exercise 8.1

Question 1:

The angles of quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

Answer:

Let the common ratio between the angles be x. Therefore, the angles will be 3x, 5x,

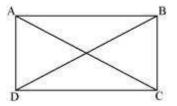
9*x*, and 13*x* respectively.

As the sum of all interior angles of a quadrilateral is 360°,

 $\therefore 3x + 5x + 9x + 13x = 360^{\circ}$ $30x = 360^{\circ}$ $x = 12^{\circ}$ Hence, the angles are $3x = 3 \times 12 = 36^{\circ}$ $5x = 5 \times 12 = 60^{\circ}$ $9x = 9 \times 12 = 108^{\circ}$ $13x = 13 \times 12 = 156^{\circ}$ Question 2:

If the diagonals of a parallelogram are equal, then show that it is a rectangle.





Let ABCD be a parallelogram. To show that ABCD is a rectangle, we have to prove that one of its interior angles is 90°.

In $\triangle ABC$ and $\triangle DCB$,

AB = DC (Opposite sides of a parallelogram are equal)

BC = BC (Common)

AC = DB (Given)

 $\therefore \Delta ABC \cong \Delta DCB$ (By SSS Congruence rule)

 $\Rightarrow \angle ABC = \angle DCB$

It is known that the sum of the measures of angles on the same side of transversal is 180°.

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\angle ABC + \angle DCB = 180^{\circ} (AB \parallel CD)
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\Rightarrow \angle ABC + \angle ABC = 180^{\circ}
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\Rightarrow 2 \angle ABC = 180^{\circ}
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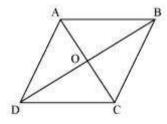
 $\Rightarrow \angle ABC = 90^{\circ}$

Since ABCD is a parallelogram and one of its interior angles is 90°, ABCD is a rectangle.

Question 3:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Answer:



Let ABCD be a quadrilateral, whose diagonals AC and BD bisect each other at right angle i.e., OA = OC, OB = OD, and $\angle AOB = \angle BOC = \angle COD = \angle AOD = 90^{\circ}$. To prove ABCD a rhombus, we have to prove ABCD is a parallelogram and all the sides of ABCD are equal.

In $\triangle AOD$ and $\triangle COD$,

OA = OC (Diagonals bisect each other)

 $\angle AOD = \angle COD$ (Given)

OD = OD (Common)

 $\therefore \Delta AOD \cong \Delta COD$ (By SAS congruence rule)

∴ AD = CD (1)

Similarly, it can be proved that

AD = AB and CD = BC (2)

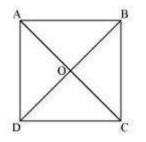
From equations (1) and (2),

$$AB = BC = CD = AD$$

Since opposite sides of quadrilateral ABCD are equal, it can be said that ABCD is a parallelogram. Since all sides of a parallelogram ABCD are equal, it can be said that ABCD is a rhombus.

Question 4:

Show that the diagonals of a square are equal and bisect each other at right angles. Answer:



Let ABCD be a square. Let the diagonals AC and BD intersect each other at a point O. To prove that the diagonals of a square are equal and bisect each other at right angles, we have to prove AC = BD, OA = OC, OB = OD, and $\angle AOB = 90^{\circ}$.

In $\triangle ABC$ and $\triangle DCB$,

AB = DC (Sides of a square are equal to each other)

 $\angle ABC = \angle DCB$ (All interior angles are of 90°)

BC = CB (Common side)

 $\therefore \Delta ABC \cong \Delta DCB$ (By SAS congruency)

 \therefore AC = DB (By CPCT)

Hence, the diagonals of a square are equal in length.

In $\triangle AOB$ and $\triangle COD$,

 $\angle AOB = \angle COD$ (Vertically opposite angles)

 $\Box ABO = \Box CDO$ (Alternate interior angles)

AB = CD (Sides of a square are always equal)

 $\Box \Delta AOB \Box \Delta COD$ (By AAS congruence rule)

 \Box AO = CO and OB = OD (By CPCT)

Hence, the diagonals of a square bisect each other.

In $\triangle AOB$ and $\triangle COB$,

As we had proved that diagonals bisect each other, therefore,

AO = CO

AB = CB (Sides of a square are equal)

BO = BO (Common)

 $\Box \Delta AOB \Box \Delta COB$ (By SSS congruency)

 $\Box \Box AOB = \Box COB (By CPCT)$

However, $\Box AOB + \Box COB = 180^{\circ}$ (Linear pair)

2□AOB = 180°

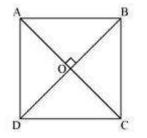
 $\Box AOB = 90^{\circ}$

Hence, the diagonals of a square bisect each other at right angles.

Question 5:

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer:



Let us consider a quadrilateral ABCD in which the diagonals AC and BD intersect each other at O. It is given that the diagonals of ABCD are equal and bisect each other at right angles. Therefore, AC = BD, OA = OC, OB = OD, and $\Box AOB = \Box BOC = \Box COD$ = $\Box AOD = 90^{\circ}$. To prove ABCD is a square, we have to prove that ABCD is a parallelogram, AB = BC = CD = AD, and one of its interior angles is 90°. In ΔAOB and ΔCOD ,

AO = CO (Diagonals bisect each other)

OB = OD (Diagonals bisect each other)

 $\Box AOB = \Box COD$ (Vertically opposite angles)

 $\Box \Delta AOB \Box \Delta COD$ (SAS congruence rule)

 \Box AB = CD (By CPCT) ... (1)

And, $\Box OAB = \Box OCD$ (By CPCT)

However, these are alternate interior angles for line AB and CD and alternate interior

angles are equal to each other only when the two lines are parallel.

□ AB || CD ... (2)

From equations (1) and (2), we obtain

ABCD is a parallelogram.

In $\triangle AOD$ and $\triangle COD$,

AO = CO (Diagonals bisect each other)

 $\Box AOD = \Box COD$ (Given that each is 90°)

OD = OD (Common)

 $\Box \Delta AOD \Box \Delta COD$ (SAS congruence rule)

 \Box AD = DC ... (3)

However, AD = BC and AB = CD (Opposite sides of parallelogram ABCD)

 $\Box AB = BC = CD = DA$

Therefore, all the sides of quadrilateral ABCD are equal to each other.

In \triangle ADC and \triangle BCD,

AD = BC (Already proved)

AC = BD (Given)

DC = CD (Common)

 $\Box \Delta ADC \Box \Delta BCD$ (SSS Congruence rule)

 $\Box \Box ADC = \Box BCD (By CPCT)$

However, $\Box ADC + \Box BCD = 180^{\circ}$ (Co-interior angles)

 $\Box \Box ADC + \Box ADC = 180^{\circ}$

 \Box 2 \Box ADC = 180°

 $\Box \Box ADC = 90^{\circ}$

One of the interior angles of quadrilateral ABCD is a right angle.

Thus, we have obtained that ABCD is a parallelogram, AB = BC = CD = AD and one

of its interior angles is 90°. Therefore, ABCD is a square.

Question 6:

Diagonal AC of a parallelogram ABCD bisects $\Box A$ (see the given figure). Show that

- (i) It bisects $\Box C$ also,
- (ii) ABCD is a rhombus.

Answer:

(i) ABCD is a parallelogram.

 $\Box \Box DAC = \Box BCA$ (Alternate interior angles) ... (1)

And, $\Box BAC = \Box DCA$ (Alternate interior angles) ... (2)

However, it is given that AC bisects $\Box A$.

 $\Box \Box DAC = \Box BAC \dots (3)$

From equations (1), (2), and (3), we obtain

 $\Box DAC = \Box BCA = \Box BAC = \Box DCA \dots (4)$

 $\Box \Box DCA = \Box BCA$

Hence, AC bisects \Box C.

(ii)From equation (4), we obtain

 $\Box DAC = \Box DCA$

 \Box DA = DC (Side opposite to equal angles are equal)

However, DA = BC and AB = CD (Opposite sides of a parallelogram)

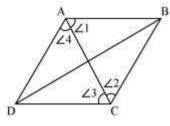
 $\Box AB = BC = CD = DA$

Hence, ABCD is a rhombus.

Question 7:

ABCD is a rhombus. Show that diagonal AC bisects $\Box A$ as well as $\Box C$ and diagonal BD bisects $\Box B$ as well as $\Box D$.

Answer:



Let us join AC.

In ∆ABC,

BC = AB (Sides of a rhombus are equal to each other)

 $\Box \Box 1 = \Box 2$ (Angles opposite to equal sides of a triangle are equal)

However, $\Box 1 = \Box 3$ (Alternate interior angles for parallel lines AB and CD)

 $\Box \Box 2 = \Box 3$

Therefore, AC bisects \Box C.

Also, $\Box 2 = \Box 4$ (Alternate interior angles for || lines BC and DA)

 $\Box \Box 1 = \Box 4$

Therefore, AC bisects $\Box A$.

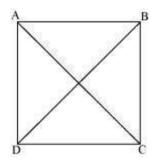
Similarly, it can be proved that BD bisects \Box B and \Box D as well.

Question 8:

ABCD is a rectangle in which diagonal AC bisects $\Box A$ as well as $\Box C$. Show that:

(i) ABCD is a square (ii) diagonal BD bisects \Box B as well as \Box D.

Answer:



(i) It is given that ABCD is a rectangle.

 $\Box \Box A = \Box C$

$$\Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle C$$

$$\Rightarrow \angle DAC = \angle DCA \qquad (AC \text{ bisects } \angle A \text{ and } \angle C)$$

CD = DA (Sides opposite to equal angles are also equal)

However, DA = BC and AB = CD (Opposite sides of a rectangle are equal)

 $\Box AB = BC = CD = DA$

ABCD is a rectangle and all of its sides are equal.

Hence, ABCD is a square.

(ii) Let us join BD.

In ΔBCD ,

BC = CD (Sides of a square are equal to each other)

 \Box CDB = \Box CBD (Angles opposite to equal sides are equal)

However, \Box CDB = \Box ABD (Alternate interior angles for AB || CD)

 $\Box \Box CBD = \Box ABD$

 \Box BD bisects \angle B.

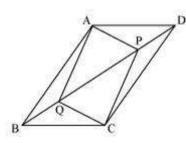
Also, \Box CBD = \Box ADB (Alternate interior angles for BC || AD)

 $\Box \Box CDB = \Box ABD$

 \Box BD bisects \Box D.

Question 9:

In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see the given figure). Show that:



(i) ΔAPD □ ΔCQB

(ii) AP = CQ

(iii) ΔAQB □ ΔCPD

(iv) AQ = CP

(v) APCQ is a parallelogram

Answer:

(i) In $\triangle APD$ and $\triangle CQB$,

 \Box ADP = \Box CBQ (Alternate interior angles for BC || AD)

AD = CB (Opposite sides of parallelogram ABCD)

DP = BQ (Given)

 $\Box \Delta APD \Box \Delta CQB$ (Using SAS congruence rule)

(ii) As we had observed that $\triangle APD \Box \triangle CQB$,

 \Box AP = CQ (CPCT)

(iii) In $\triangle AQB$ and $\triangle CPD$,

 $\Box ABQ = \Box CDP$ (Alternate interior angles for AB || CD)

AB = CD (Opposite sides of parallelogram ABCD)

BQ = DP (Given)

 \Box Δ AQB \Box Δ CPD (Using SAS congruence rule)

(iv) As we had observed that $\triangle AQB \Box \triangle CPD$,

 \Box AQ = CP (CPCT)

(v) From the result obtained in (ii) and (iv),

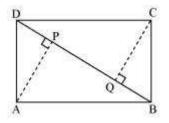
AQ = CP and

AP = CQ

Since opposite sides in quadrilateral APCQ are equal to each other, APCQ is a parallelogram.

Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (See the given figure). Show that



(i) $\triangle APB \Box \triangle CQD$

(ii) AP = CQ

Answer:

(i) In $\triangle APB$ and $\triangle CQD$,

 $\Box APB = \Box CQD$ (Each 90°)

AB = CD (Opposite sides of parallelogram ABCD)

 $\Box ABP = \Box CDQ$ (Alternate interior angles for AB || CD)

 $\Box \Delta APB \Box \Delta CQD$ (By AAS congruency)

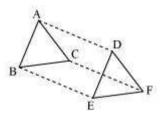
(ii) By using the above result

 $\triangle APB \Box \triangle CQD$, we obtain

AP = CQ (By CPCT)

Question 11:

In \triangle ABC and \triangle DEF, AB = DE, AB || DE, BC = EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F respectively (see the given figure). Show that



(i) Quadrilateral ABED is a parallelogram

- (ii) Quadrilateral BEFC is a parallelogram
- (iii) AD || CF and AD = CF
- (iv) Quadrilateral ACFD is a parallelogram
- (v) AC = DF
- (vi) $\triangle ABC \Box \triangle DEF$.

Answer:

(i) It is given that AB = DE and $AB \parallel DE$.

If two opposite sides of a quadrilateral are equal and parallel to each other, then it will be a parallelogram.

Therefore, quadrilateral ABED is a parallelogram.

(ii) Again, BC = EF and BC || EF

Therefore, quadrilateral BCEF is a parallelogram.

(iii) As we had observed that ABED and BEFC are parallelograms, therefore

AD = BE and AD || BE

(Opposite sides of a parallelogram are equal and parallel)

And, BE = CF and BE || CF

(Opposite sides of a parallelogram are equal and parallel)

 \Box AD = CF and AD || CF

(iv) As we had observed that one pair of opposite sides (AD and CF) of quadrilateral

ACFD are equal and parallel to each other, therefore, it is a parallelogram.

(v) As ACFD is a parallelogram, therefore, the pair of opposite sides will be equal and parallel to each other.

 \Box AC || DF and AC = DF

(vi) $\triangle ABC$ and $\triangle DEF$,

AB = DE (Given)

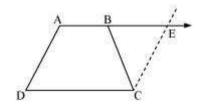
BC = EF (Given)

AC = DF (ACFD is a parallelogram)

 $\Box \Delta ABC \Box \Delta DEF$ (By SSS congruence rule)

Question 12:

ABCD is a trapezium in which AB || CD and AD = BC (see the given figure). Show that



(i) $\Box A = \Box B$

(ii) $\Box C = \Box D$

(iii) ΔABC □ ΔBAD

(iv) diagonal AC = diagonal BD

[*Hint*: Extend AB and draw a line through C parallel to DA intersecting AB produced at E.]

Answer:

Let us extend AB. Then, draw a line through C, which is parallel to AD, intersecting

AE at point E. It is clear that AECD is a parallelogram.

(i) AD = CE (Opposite sides of parallelogram AECD)

However, AD = BC (Given)

Therefore, BC = CE

 \Box CEB = \Box CBE (Angle opposite to equal sides are also equal)

Consider parallel lines AD and CE. AE is the transversal line for them.

 $\Box A + \Box CEB = 180^{\circ}$ (Angles on the same side of transversal)

 $\Box A + \Box CBE = 180^{\circ}$ (Using the relation $\Box CEB = \Box CBE$) ... (1)

However, $\Box B + \Box CBE = 180^{\circ}$ (Linear pair angles) ... (2)

From equations (1) and (2), we obtain

 $\Box A = \Box B$

(ii) AB || CD

 $\Box A + \Box D = 180^{\circ}$ (Angles on the same side of the transversal)

Also, $\Box C + \Box B = 180^{\circ}$ (Angles on the same side of the transversal)

 $\Box \Box A + \Box D = \Box C + \Box B$

However, $\Box A = \Box B$ [Using the result obtained in (i)]

 $\Box \Box C = \Box D$

(iii) In $\triangle ABC$ and $\triangle BAD$,

AB = BA (Common side)

BC = AD (Given)

 $\Box B = \Box A$ (Proved before)

 $\Box \Delta ABC \Box \Delta BAD$ (SAS congruence rule)

(iv) We had observed that,

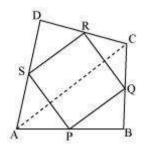
 $\triangle ABC \Box \triangle BAD$

 \Box AC = BD (By CPCT)

Exercise 8.2

Question 1:

ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see the given figure). AC is a diagonal. Show that:



(i) SR || AC and SR =
$$2 \text{ AC}$$

1

(ii)
$$PQ = SR$$

(iii) PQRS is a parallelogram.

Answer:

(i) In ΔADC , S and R are the mid-points of sides AD and CD respectively.

In a triangle, the line segment joining the mid-points of any two sides of the triangle is parallel to the third side and is half of it.

 $\Box \text{ SR } || \text{ AC and SR} = \frac{1}{2} \text{ AC } \dots (1)$

(ii) In \triangle ABC, P and Q are mid-points of sides AB and BC respectively. Therefore, by using mid-point theorem,

PQ || AC and PQ = $\frac{1}{2}$ AC ... (2) Using equations (1) and (2), we obtain PQ || SR and PQ = SR ... (3) \Box PQ = SR (iii) From equation (3), we obtained PQ || SR and PQ = SR Clearly, one pair of opposite sides of quar

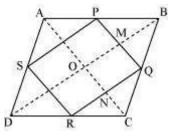
Clearly, one pair of opposite sides of quadrilateral PQRS is parallel and equal.

Hence, PQRS is a parallelogram.

Question 2:

ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.

Answer:



In \triangle ABC, P and Q are the mid-points of sides AB and BC respectively.

$$\square$$
 PQ || AC and PQ = $\frac{1}{2}$ AC (Using mid-point theorem) ... (1)
In ΔADC.

R and S are the mid-points of CD and AD respectively.

1

$$\square$$
 RS || AC and RS = $\frac{1}{2}$ AC (Using mid-point theorem) ... (2)

From equations (1) and (2), we obtain

 $PQ \parallel RS$ and PQ = RS

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Let the diagonals of rhombus ABCD intersect each other at point O.

In quadrilateral OMQN,

MQ || ON (* PQ || AC)

QN || OM (** QR || BD)

Therefore, OMQN is a parallelogram.

 $\Box \Box MQN = \Box NOM$

 $\Box \Box PQR = \Box NOM$

However, \Box NOM = 90° (Diagonals of a rhombus are perpendicular to each other)

 $\Box \Box PQR = 90^{\circ}$

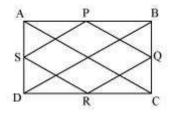
Clearly, PQRS is a parallelogram having one of its interior angles as 90°.

Hence, PQRS is a rectangle.

Question 3:

ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Answer:



Let us join AC and BD.

In ∆ABC,

P and Q are the mid-points of AB and BC respectively.

 \square PQ || AC and PQ = $\frac{1}{2}$ AC (Mid-point theorem) ... (1) Similarly in ΔADC,

SR || AC and SR = $\frac{1}{2}$ AC (Mid-point theorem) ... (2)

Clearly, PQ || SR and PQ = SR

Since in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

 \square PS || QR and PS = QR (Opposite sides of parallelogram)... (3)

In Δ BCD, Q and R are the mid-points of side BC and CD respectively.

 $\frac{1}{2}$ BD (Mid-point theorem) ... (4) However, the diagonals of a rectangle are equal.

$$\Box$$
 AC = BD ...(5)

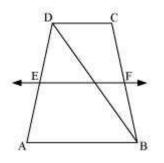
By using equation (1), (2), (3), (4), and (5), we obtain

PQ = QR = SR = PS

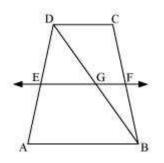
Therefore, PQRS is a rhombus.

Question 4:

ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid - point of AD. A line is drawn through E parallel to AB intersecting BC at F (see the given figure). Show that F is the mid-point of BC.



Answer: Let EF intersect DB at G.



By converse of mid-point theorem, we know that a line drawn through the mid-point of any side of a triangle and parallel to another side, bisects the third side.

In ∆ABD,

EF || AB and E is the mid-point of AD.

Therefore, G will be the mid-point of DB.

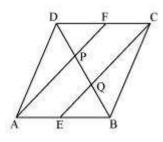
As EF || AB and AB || CD,

 \Box EF || CD (Two lines parallel to the same line are parallel to each other)

In Δ BCD, GF || CD and G is the mid-point of line BD. Therefore, by using converse of mid-point theorem, F is the mid-point of BC.

Question 5:

In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see the given figure). Show that the line segments AF and EC trisect the diagonal BD.



Answer: ABCD is a parallelogram.

□AB || CD

And hence, AE || FC

Again, AB = CD (Opposite sides of parallelogram ABCD)

$$\frac{1}{2}_{AB} = \frac{1}{2}_{CD}$$

AE = FC (E and F are mid-points of side AB and CD)

In quadrilateral AECF, one pair of opposite sides (AE and CF) is parallel and equal to each other. Therefore, AECF is a parallelogram.

□ AF || EC (Opposite sides of a parallelogram)

In Δ DQC, F is the mid-point of side DC and FP || CQ (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that P is the mid-point of DQ.

 $\Box DP = PQ \dots (1)$

Similarly, in $\triangle APB$, E is the mid-point of side AB and EQ || AP (as AF || EC). Therefore, by using the converse of mid-point theorem, it can be said that

Q is the mid-point of PB.

 \Box PQ = QB ... (2)

From equations (1) and (2),

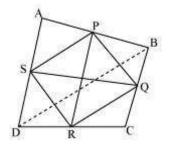
DP = PQ = BQ

Hence, the line segments AF and EC trisect the diagonal BD.

Question 6:

Show that the line segments joining the mid-points of the opposite sides of a guadrilateral bisect each other.

Answer:



Let ABCD is a quadrilateral in which P, Q, R, and S are the mid-points of sides AB, BC, CD, and DA respectively. Join PQ, QR, RS, SP, and BD.

In \triangle ABD, S and P are the mid-points of AD and AB respectively. Therefore, by using mid-point theorem, it can be said that

SP || BD and SP =
$$\frac{1}{2}$$
 BD ... (1)
Similarly in Δ BCD,
QR || BD and QR = $\frac{1}{2}$ BD ... (2)
From equations (1) and (2), we obtain
SP || QR and SP = QR
In quadrilateral SPQR, one pair of opposite sides is equal and parallel to
each other. Therefore, SPQR is a parallelogram.
We know that diagonals of a parallelogram bisect each other.
Hence, PR and QS bisect each other.

Question 7:

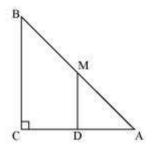
ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that

(i) D is the mid-point of AC

(ii) MD □ AC

(iii)
$$CM = MA = \frac{1}{2}AB$$

Answer:



(i) In ∆ABC,

It is given that M is the mid-point of AB and MD || BC.

Therefore, D is the mid-point of AC. (Converse of mid-point theorem)

(ii) As DM || CB and AC is a transversal line for them, therefore,

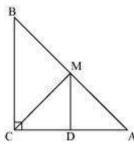
 \square MDC + \square DCB = 180° (Co-interior angles)

 $\Box MDC + 90^{\circ} = 180^{\circ}$

 \Box MDC = 90°

 \Box MD \Box AC

(iii) Join MC.



In $\triangle AMD$ and $\triangle CMD$, AD = CD (D is the mid-point of side AC) $\Box ADM = \Box CDM$ (Each 90°)

DM = DM (Common)

 $\Box \Delta AMD \Box \Delta CMD$ (By SAS congruence rule)

Therefore, AM = CM (By CPCT)

 $\frac{1}{2}$ However, AM = $\frac{1}{2}$ AB (M is the mid-point of AB) Therefore, it can be said that

$$CM = AM = \frac{1}{2}AB$$