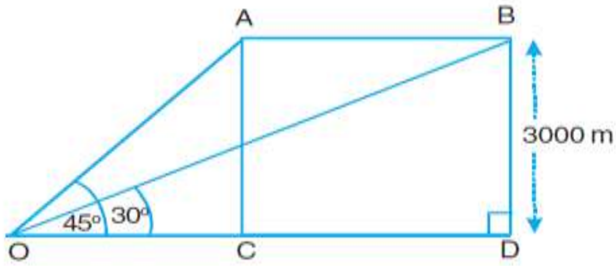
Class 10 Application of Trigonometry [Height and Distance] Solved Problems

Question 01: The angle of elevation of an areoplane from a point on the ground is 45 °. After a flight of 15 seconds, the elevation changes to 30 °. If the areoplane is flying at a constant height of 3000 meters, find the speed of the plane.

Solution: Let A and B be two positions of the plane and let O be the point of observation. Let OCD be the horizontal line.

Then ∠AOC = 45 ° and ∠BOD = 30°



By question, AC = BD = 3000 m

In rt ΔACO,
tan 45 ° = AC /OC

or 1 = AC /OC

OC = AC = 3000m ……………………..(i)

In rt ΔBDO, Tan30⁰ = BD/OD = or 1/√3 =3000 /(OC+ CD)

or, 3000 3 = 3000 + CD ……………………[By (i)]

or CD = 3000 (√3 – 1) = 3000 × 0.732 = 2196

Distance covered by the areoplane in 15 seconds = AB = CD = 2196 m

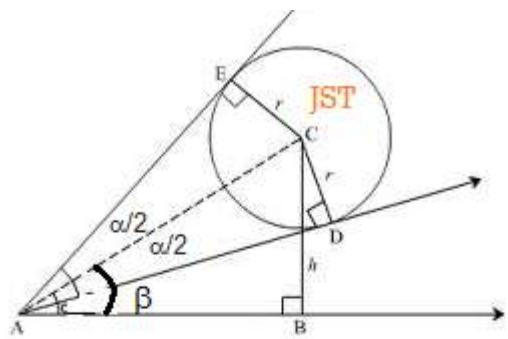
Speed of the plane = $\frac{2196}{15}\ x\ \frac{18}{5} = =\ 527.04\ \text{km/h}$

Questions 02: A around balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β , prove that the height of the centre of the balloon is r sin β cosec α/2

Solution: Let the height of centre of the balloon above the ground be h m.

Given, balloon subtends α angle at the observes eye. ∴ ∠ EAD = α

In ΔACE and ΔACD,



AE = AD (Length of tangents drawn from an external point to the circle are equal)

AC = AC (Common)

CE = CD (Radius of the circle)

∴ ΔACE ≅ ΔACD (SSS congruence criterion)

⇒ ∠EAC = ∠DAC (CPCT) = $\frac{\alpha}{2}$

In right ΔACD, $\sin\frac{\alpha}{2} = \frac{CD}{AC} \Rightarrow \sin\frac{\alpha}{2} = \frac{r}{AC} \Rightarrow \text{cosec}\frac{\alpha}{2} = \frac{AC}{r} \Rightarrow AC = r\,\text{cosec}\frac{\alpha}{2}$ ---------(i)

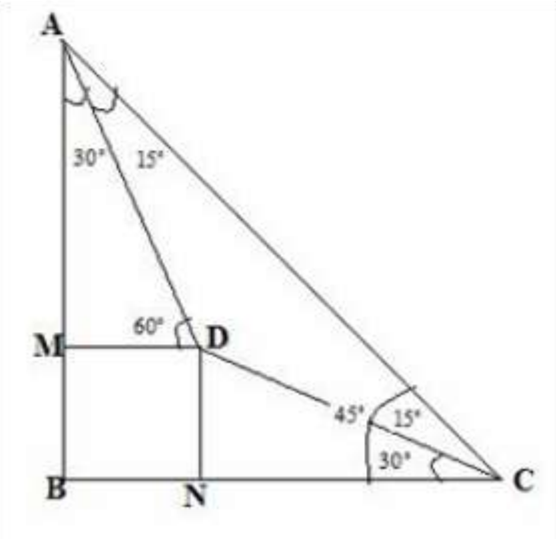
In right ΔACB, $\sin\beta = \frac{BC}{AC} \Rightarrow \sin\beta = \frac{h}{AC}$ -----------(ii)

From (i) and (ii) $\sin\beta = \frac{h}{r\,\text{cosec}\frac{\alpha}{2}} \Rightarrow h = r\sin\beta\,\text{cosec}\frac{\alpha}{2}$

Thus, the height of the centre of the balloon is ($r\sin\beta\,\text{cosec}\frac{\alpha}{2}$)

Question 03: At the foot of a mountain the elevation of its summit is 45°. After ascending 1000 m towards the mountain up a slope of 30° inclination, the elevation is found to be 60°. Find the height of the mountain.[height = 500(1+√3) m.]

Solution:



Let point A be the position of summit of the mountain and B being its foot.

Let C be the original position of observer and D, the final position after ascending 1000 metres.

Let DN and DM be perpendiculars to BC and AB respectively.

Thus CD = 1000m

<DCN = 30^0 and <ADM = 60^0

<ACB = <CAB = 45^0 and <DAM = 30^0

<DCA = <DAC = 15^0 AD = CD = 1000m

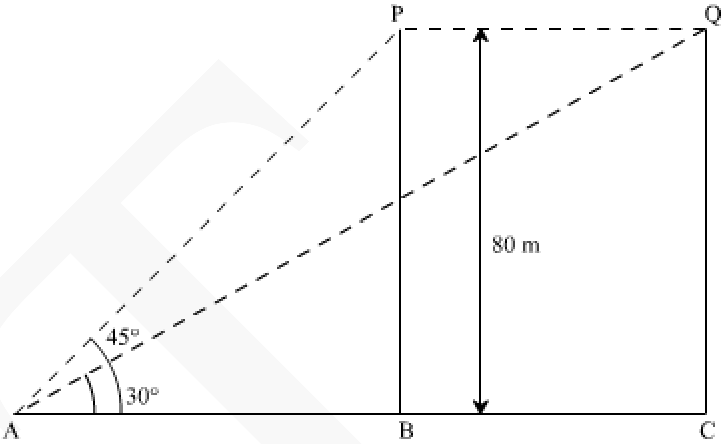
In, Δ DCN , $\sin 30^0 = \frac{DN}{DC} \Rightarrow \frac{1}{2} = \frac{DN}{1000} \Rightarrow$ DN = 500m

In, Δ ADM , $\sin 60^0 = \frac{AM}{AD} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AM}{1000} \Rightarrow$ AM = 500√3 m

Total height = BM + AM = DN + AM = 500 + 500√3 m = 500(1+√3)

Question 04: A bird is sitting on the top of a tree, which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.

Solution: . It is given that the angle of elevations of the bird in two positions P and Q from point A are 45° and 30° respectively.



∴ ∠PAB = 45°, ∠QAB = 30°. It is also given that

PB = 80 meters

In ΔABP, $\tan 45^0 = \frac{BP}{AB} \Rightarrow 1 = \frac{80}{AB} \Rightarrow AB = 80m$

In ΔACQ we have $\tan 30^0 = \frac{CQ}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{80}{AC} \Rightarrow$

$AC = 80\sqrt{3}$ m

∴ $PQ = BC = AC - AB = \{80\sqrt{3} - 80\}$ m =

$80(\sqrt{3} - 1)$ m

Thus the bird touch $80(\sqrt{3} - 1)$ m in 2 seconds

Hence speed of the bird $40(\sqrt{3} - 1)$ m

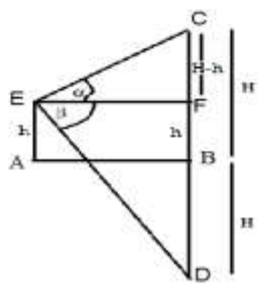
Question 05: If the angle of elevation of the cloud from a point h m above a lake is α and the angle of depression of its reflection in the lake is B, prove that the height of the cloud is

$$\boldsymbol{H = \frac{h\,(Tan\alpha + Tan\beta)}{(Tan\beta - Tan\alpha)}}$$

Solution: Let AB be the surface of the lake and let E be the point of observation. Let AE = h m

Let C be the position of the cloud and D be its reflection in the lake.

The angle of elevation of a cloud be α and the angle of depression of its reflection be β .



In, Δ CEF, $\tan\alpha = \frac{H-h}{EF}$ In Δ DEF, $\tan\beta = \frac{H+h}{EF}$

$\Rightarrow EF = \frac{H-h}{Tan\,\alpha}$ ----------(i) $\Rightarrow EF = \frac{H+h}{Tan\,\beta}$ ---(ii)

From (i) and (ii)

$$\frac{H-h}{Tan\,\alpha} = \frac{H+h}{Tan\,\beta}$$

$$\Rightarrow H\,Tan\beta - h Tan\beta = H\,Tan\alpha + h\,Tan\alpha$$

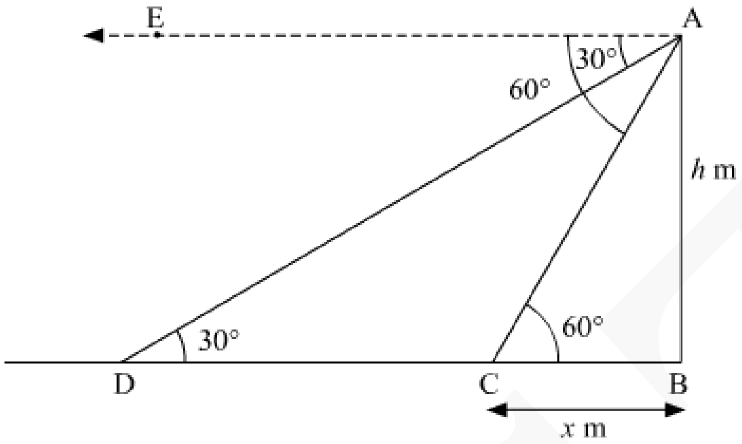
$$\Rightarrow H\,Tan\beta - H\,Tan\alpha = h\,Tan\alpha + h\,Tan\beta$$

$$\Rightarrow H\,(Tan\beta - Tan\alpha) = h\,(Tan\alpha + Tan\beta)$$

$$H = \frac{h\,(Tan\alpha + Tan\beta)}{(Tan\beta - Tan\alpha)}$$

Question 06: A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of tower with a uniform speed Six minutes later, the angle of depression of the car is found to be 60° . Find the time taken by the car to reach the foot of the tower. [3 sec]

Solution: Let AB be the vertical tower. Suppose D and C be the positions of the car when the angle of depression from the top of the tower is 30° and 60° respectively.



Time taken for the angle of depression to change from 30° to 45° = 6 sec

(Given) ∠EAD = ∠ADB = 30° (Alternate angles)

∠EAC = ∠ACB = 60° (Alternate angles)

Suppose AB = h m and BC = x m.

In ΔADB, tan $30^0 = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{(DC+x)} \Rightarrow \sqrt{3}h = DC + x --(i)$

In ΔACB, tan $60^0 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x} x \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow \sqrt{3}h = 3x ------- (ii)$

From (i) and (ii) $DC + x = 3x \Rightarrow DC = 2x$ ---(iii)

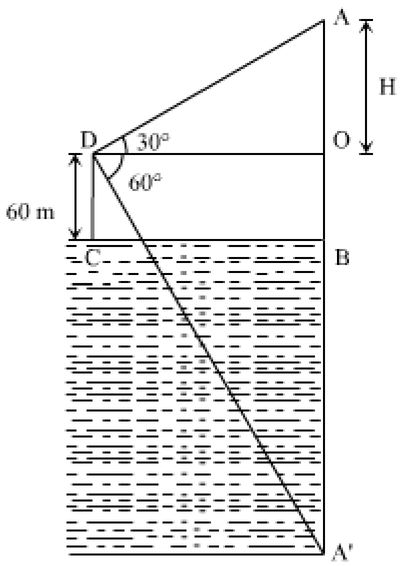
Speed = Distance/time = DC/6

Time taken to cover distance BC = x = Dist /speed = $\frac{x}{\frac{DC}{6}} = \frac{6x}{DC}$

Using value from (iii) Time = $\frac{6x}{2x}$ = 3m/s

Question 07: The angle of elevation of a cloud from a point 60m above a lake is 30^0 and the angle of depression of the reflection of the cloud in the lake is 60^0. Find the height of the cloud from the surface of the lake.

Solution:



Let AO = H CD = OB = 60 m A' B = AB = 60 + H

In Δ AOD,

Tan $30^0 = \frac{OA}{OD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{H}{OD} \Rightarrow$ OD = H$\sqrt{3}$

Now, In Δ A'OD,

Tan $60^0 = \frac{OA}{OD} \Rightarrow \sqrt{3} = \frac{OA'}{OD} \Rightarrow \sqrt{3} = \frac{120+H}{H\sqrt{3}}$

⇒ 120 + H = 3H ⇒ 2H = 120 ⇒ H = 60 m

Thus, height of the cloud above the lake = AB + A'B = 60 + 60 = 120 m

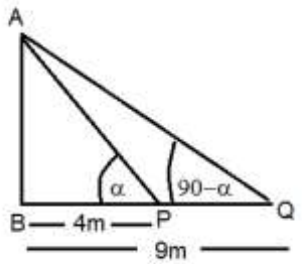
Question 08: The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution: Let, The angles of elevation of the top of a tower A from two points P and Q at a distance of 4 m and 9 m from the base B of the tower and $< APB = \alpha$ then $< AQB = (90 - \alpha)$

In Δ ABP, $Tan\ \alpha = \frac{AB}{4}$

and In Δ AQB, $Tan\ (90 - \alpha) = \frac{AB}{9} \Rightarrow \cot \alpha = \frac{AB}{9}$

Now. $\tan \alpha \times \cot \alpha = \frac{AB}{9}\ x\ \frac{AB}{4} \Rightarrow 1 = \frac{AB^2}{36} \Rightarrow AB^2 = 36 \Rightarrow AB = 6m$

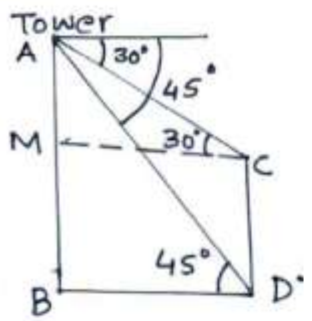


Best of Luck

Question 09: From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pole are 30° and 45° respectively. Find (i) how far the pole is from the bottom of a tower, (ii) the height of the pole. (Use √3 = 1·732) [CBSE T-II 2015]

$in\ \Delta ABD, tan\ 45 = \frac{AB}{BD} \Rightarrow 1 = \frac{AB}{BD} \Rightarrow AB = BD = 50$ m

$in\ \Delta AMC, tan\ 30 = \frac{AM}{MC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{AM}{50} \Rightarrow AM = \frac{50}{\sqrt{3}}$ m = 28.87 m.



Height of pole = $CD = BM$

$= AB - AM = 50 - 28.87 = 21.13m$

Question 10: Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole is 60° and the angle of depression from the top of another pole at point P is 30°. Find the heights of the poles and the distances of the point P from the poles. [CBSE T-II 2015]

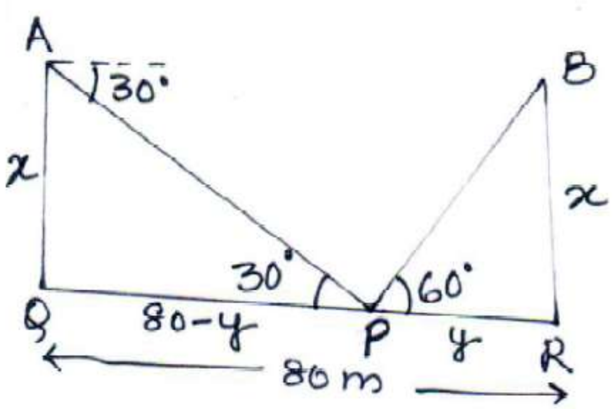


Figure 1 m

$\tan 60^\circ = \frac{x}{y}$

$\Rightarrow \quad x = y\sqrt{3}$ .... (i) ½ m

$\tan 30^\circ = \frac{x}{80 - y}$

$\Rightarrow \quad \sqrt{3}\,x = 80 - y$ .... (ii) 1 m

Solving (i) and (ii)

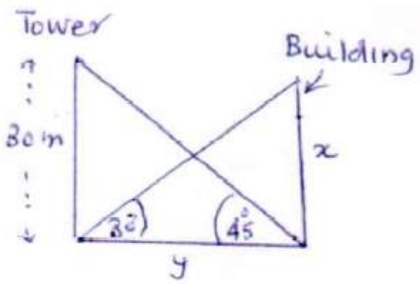
$y = 20, \quad x = 20\sqrt{3}$ m. ½ m

Height of pole $= 20\sqrt{3}$ m.

PR = 20 m.

OP = 80 – 20 = 60 m. 1 m

Question 11 : The angle of elevation of the top of a building from the foot of the tower is 30^0 and the angle of elevation of the top of the tower from the foot of the building is 45^0. if the tower is 30m high, find the height of the building[CBSE T-II 2015]



Fiqure ½ m

(i) $\frac{30}{y} = \tan 45^\circ = 1 \Rightarrow y = 30$ 1 m

(ii) $\frac{x}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow x = \frac{y}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ 1 m

$\therefore$ Height of building is $10\sqrt{3}$ m ½ m

Question 12 : From a point P on the ground the angle of elevation of the top of a tower is 30^0 and that of the top of a flag staff on the top of the tower, is 60^0. if the length of flag staff is 5 m , find the height of the tower. [CBSE T-II 2015]

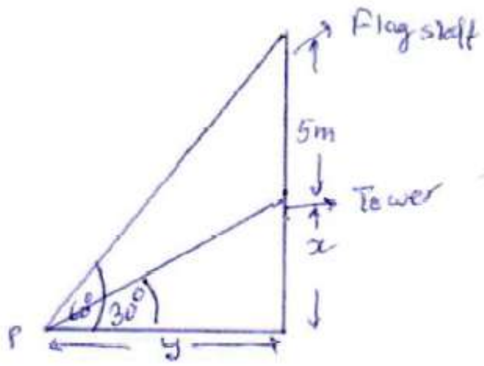


figure 1 m

Writing the trigonometric equations

(i) $\frac{x}{y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow y = \sqrt{3}\,x$ 1 m

(ii) $\frac{x+5}{y} = \tan 60^\circ = \sqrt{3}$ *or* $\frac{x+5}{\sqrt{3}x} = \sqrt{3}$ 1½ m

$\Rightarrow 3x = x + 5$

or $x = 2.5$

$\therefore$ Height of Tower = 2.5 m ½ m

Question 13: **At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30^0. The angle of depression of the reflection of the cloud in the lake, at A is 60^0. Find the distance of the cloud from A.**

correct figure 1 m

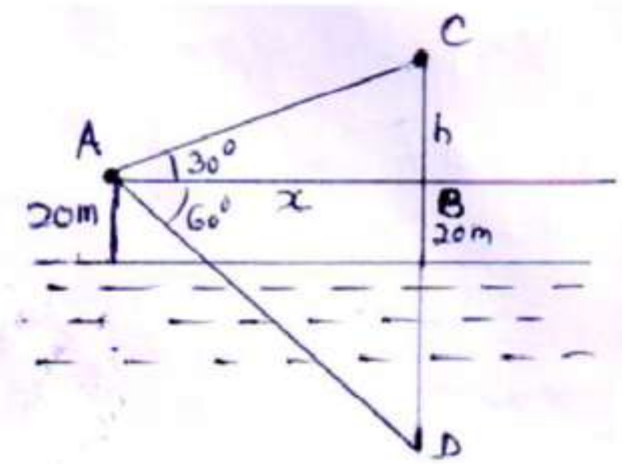
$\frac{h}{x} = \tan 30^0 = \frac{1}{\sqrt{3}} \Rightarrow x = \sqrt{3}\,h.$ ½ m

$\frac{40+h}{x} = \tan 60^0 = \sqrt{3} \Rightarrow x = \frac{40+h}{\sqrt{3}}$ ½ m

$\therefore \sqrt{3}\,h = \frac{40+b}{\sqrt{3}} \Rightarrow h = 20\,m.$ ½ m

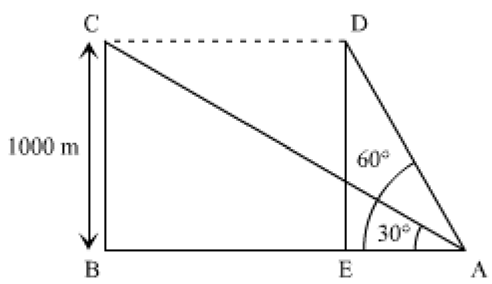
$\therefore x = 20\sqrt{3}\,m$ ½ m

$\therefore AC = \sqrt{(20)^2 + (20\sqrt{3})^2} = 40\,m.$ 1 m



[CBSE T-II 2015]

Question 14: An areoplane flying horizontal 1 km above the ground is observed at an elevation of 60^o. After 10 seconds its elevation is observed to be 30^o. Find the speed of the areoplane. [$240\sqrt{3}$ km/h]



A is the point on the ground whereas C and D are position of the aeroplane.

$$In\ \Delta CBA, Tan\ 30 = \frac{1000}{AB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{1000}{AB} = AB = 1000\sqrt{3}\ m$$

$$In\ \Delta DEA\ , Tan\ 60 = \frac{1000}{AE} \Rightarrow \sqrt{3} = \frac{1000}{AE} = AE = \frac{1000}{\sqrt{3}} m$$

BE is the distance travelled by the plane in 10 sec

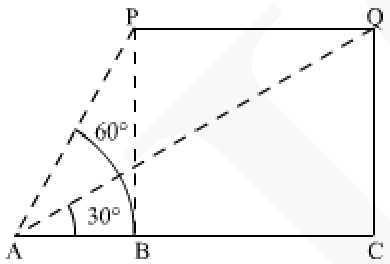
$$BE = AB - AE = 1000\sqrt{3} - \frac{1000}{\sqrt{3}} = \frac{3000 - 1000}{\sqrt{3}} = \frac{2000\sqrt{3}}{3}$$

Speed of the areoplane = Distance ÷ Time = $\frac{2000\sqrt{3}}{3} \div 10\frac{m}{s} = \frac{2000\sqrt{3}}{30}\ x\ \frac{18}{5}\ km/h = 240\sqrt{3}$ km/h

Question 15: The angle of elevation of a jet fighter from point A on the ground is 60 degree .After 15 seconds the angle of elevation changes from 60 degree to 30 degree .If the jet is flying at a speed of 720 km/hr, find the height at which the jet fighter is flying.

Solution: Let P and Q be the two position of jet and the height of the jet be h metres.

Let ABC be the horizontal line through A. It is given that A is the point of observation and angle of elevation of jet in two positions P and Q from point A are 60° and 30° respectively. ∴ ∠PAB = 60°, ∠QAB = 30°



In ΔABP ,

Tan $60^0 = \frac{PB}{AE} \Rightarrow \sqrt{3} = \frac{h}{AB} \Rightarrow$ AB $= \frac{h}{\sqrt{3}}$

In ΔACQ,

Tan $30^0 = \frac{QC}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AC} \Rightarrow \sqrt{3}\,h = AC$

Also speed of jet = 720 km/hr = 720x $\frac{5}{18}$ m/s = 200m/s

PQ = 200 × 15 = 3000 m ⇒ BC = PQ = 3000 m

Then, $AC - AB = 3000 \Rightarrow \sqrt{3}\,h - \frac{h}{\sqrt{3}}$

$\Rightarrow h = 1500\ \sqrt{3}\ m$