## 7th Triangle and its properties

Q. Define parallel lines?

Sol: [Two lines are said to be in parallel if they dont intersect each other and maintain a equal distance through out.

Q. Find the measure of an angle which is three times its supplement

Sol: Let the measure of the angle be $x$.
Supplement of the angle $x=180^{\circ}-x$.
Given, Measure of the angle $=3 \times$ Supplement of the angle
$\Rightarrow x=3\left(180^{\circ}-x\right)$
$\Rightarrow x=540^{\circ}-3 x$
$\Rightarrow x+3 x=540^{\circ}$
$\Longrightarrow 4 x=540^{\circ}$
$\Rightarrow x=\frac{540^{\circ}}{4}=135^{\circ}$
Thus, the measure of the angle is $135^{\circ}$.
Q. Define Line Segment \& Ray angle?

Ans:

A line segment is a part of a line that has two end points. It has a definite length.
A line segment corresponds to the shortest distance between two points. The line segment joining the points P and Q is denoted as $\overline{\mathrm{PQ}} \quad \stackrel{\bullet}{\mathrm{P}}$

A ray has a beginning point but no end point. The beginning point is called the vertex of the ra;


An angle is formed from two rays that have the same beginning point. The common point is called the vertex and the two rays are called the sides of the angle.
Here, the $\angle P A Q$ formed by the rays $\overrightarrow{\mathrm{AP}}$ and $\overrightarrow{\mathrm{AQ}}$.
Q. Find the supplement of the each of the following angle: $35^{\circ}$

Ans: Two angles are said to be supplementary if the sum of the measures of the angles is $180^{\circ}$.
So, the supplement of the angle of measure $35^{\circ}=180^{\circ}-35^{\circ}=145^{\circ}$
Q. the difference between the measures two complementary angles is 18. what are the measures of the two angles

Sol: Let measure of one angle be $x^{0}$ Thus, the complementary angle of this angle is $(90-x)^{0}$
Given; $x^{0}-(90-x)^{0}=18^{0} \quad-x-90+x=18 \quad \Rightarrow 2 x-90=18$
$\Rightarrow 2 x=18+90=108$
Thus, measure of one angle is $54^{\circ}$ and the measure of remaining angle is $(90-54)^{\circ}=36^{\circ}$
Q. A man 160 cm tall is at a distance of 600 cm from foot of a light source situated at top of pole of height 4.1 m high. Find distance between top of a man and source of light?

Sol: Let the height of man be $A B=160 \mathrm{~cm}$
Let the height of pole be $D C=4.1 \mathrm{~m}=410 \mathrm{~cm}$

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Let the distance between the man and pole be DC $=600$

$$
\begin{aligned}
& D C=D E+E C \\
& \therefore 410 \mathrm{~cm}=D E+160 \mathrm{~cm} \\
& \therefore D E=410 \mathrm{~cm}-160 \mathrm{~cm} \\
& =250 \mathrm{~cm}
\end{aligned}
$$

In $\triangle A E D$, right angled at E, applying Pythagoras theorem,
$A D^{2}=D E^{2}+A E^{2}$
$\therefore \mathrm{AD}^{2}=(250)^{2}+(600)^{2}$
$\Rightarrow A D^{2}=62500+360000$
$\Rightarrow A D^{2}=422500$
$\Rightarrow A D=650 \mathrm{~cm}$

Thus, the distance between the top of man and source of light is 650 cm .
Q. Each of the two angles of a triangle is $3 / 4$ th of the third angle. Find all the angles.

Ans: Let the third angle be $x$. Then other angles will be $3 x / 4$
By angle sum property we have, $3 x / 4+3 x / 4+x=180$
$5 x / 2=180 \Rightarrow 5 x=360 \Rightarrow x=72$
Other angles $3 \times 72 / 4=54$

So the angles of the triangle are 72,54 and 54
Q. To verify that the interior angle of a triangle is equal to the sum of the two interior opposite angles.

Sol: In $\triangle A B C$
$\angle A+\angle B+\angle C=180^{\circ}$ [Angle sum property ]

Also
$\angle C+\angle B C D=180^{\circ}$ [Linear Pair ]

Equating (1) and (2)
$\angle \mathrm{C}+\angle \mathrm{BCD}=\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}$
$\angle \mathrm{BCD}=\angle \mathrm{A}+\angle \mathrm{B}$

Hence, exterior angle of a triangle is equal to sum of two interior opposite angles.
Q. Prove that the sum of the interior angles of a triangle is $180^{\circ}$.

Sol: XPY is a line.
$\therefore \angle 4+\angle 1+\angle 5=180^{\circ}$
But XPY || QR and PQ, PR are transversals.
So, $\angle 4=\angle 2$ and $\angle 5=\angle 3$ (Pairs of alternate angles)

Substituting $\angle 4$ and $\angle 5$ in (1), we get
$\angle 2+\angle 1+\angle 3=180^{\circ} \quad \therefore \angle 1+\angle 2+\angle 3=180^{\circ}$
Q. In Given gig. Prove that $A B+B C+A C>2 A M$


In a triangle, the sum of the lengths of either two sides is always greater than the third side.

In $\triangle \mathrm{ABM}$,
$\mathrm{AB}+\mathrm{BM}>\mathrm{AM}$

Similarly, in $\triangle \mathrm{ACM}$,
$A C+C M>A M$
Adding equation (i) and (ii),
$A B+B M+M C+A C>A M+A M$
$A B+B C+A C>2 A M$
Q. The hypotenuse of a right triangle is 2 cm more than the longer side of the triangle. The shorter side of the triangle is 7 cm less that the longer side. Find the length of the hypotenuse.

Sol: Hypotenuse $=$ Longer side +2 cm Shorter side $=$ Longer side -7 cm

Let the longer side be $x \mathrm{~cm} . \therefore$ Hypotenuse $=(x+2) \mathrm{cm}$ and Shorter side $=(x-7) \mathrm{cm}$

In a right triangle,
$(\text { Hypotenuse })^{2}=(\text { Longer side })^{2}+(\text { Shorter side })^{2}$ [from Pythagoras theorem]
$\therefore(\mathrm{x}+2)^{2}=(\mathrm{x})^{2}+(\mathrm{x}-7)^{2}$
$\Rightarrow x^{2}+4 x+4=x^{2}+x 2-14 x+49$
$\Rightarrow x^{2}+4 x+4=2 x^{2}-14 x+49$
$\Rightarrow 2 x^{2}-x^{2}-14 x-4 x+49-4=0$
$\Rightarrow x^{2}-18 x+45=0$
$\Rightarrow x 2-15 x-3 x+45=0$
$\Rightarrow x(x-15)-3(x-15)=0$
$\Rightarrow(x-15)(x-3)=0$
$\Rightarrow \mathrm{x}-15=0$ or $\mathrm{x}-3=0$
$\Rightarrow x=15$ or $x=3$
$\therefore \mathrm{x}=15$ (When $\mathrm{x}=3$, length of shorter side is negative which is not possible)

Length of hypotenuse of the triangle $=(x+2) \mathrm{cm}=(15+2) \mathrm{cm}=17 \mathrm{~cm}$
Q. Prove that An exterior angle of a triangle is equal to the sum of its opposite interior angles?

Sol:


In $\triangle A B C$
$\angle A B D$ is formed at the point $B$. This angle lies in the exterior of the $\triangle A B C$.
$\angle A B C$ is an adjacent angle to $\angle A B D$.
So, $\angle A B C+\angle A B D=180$
Also, $\angle A B C+<A C B+<B A C=180$
So, $\angle A B C+\angle A C B+\angle B A C=\angle A B C+\angle A B D$ $\angle A C B+\angle B A C=\angle A B D$
The remaining two angles $\angle B A C$ and $\angle B C A$ are the two interior opposite angles of $\angle A B D$. So, $\angle A B D$ is the exterior angle and $\angle B A C$ and $\angle B C A$ are the two interior opposite angles of $\angle A B D$.
Q. if the angles of a triangle are in the ratio of 3:4:5 determine the three angles

Given, $\angle A: \angle B: \angle C=3: 4: 5$
: $\angle A=3 x, \angle B=4 x$ and $\angle C=5 x$, where $x$ is some constant.
In $\triangle \mathrm{ABC}, \angle A+\angle B+\angle C=180^{\circ} \quad$ (Angle Sum Property)
$\Rightarrow 3 x+4 x+5 x=180^{\circ} \Rightarrow 12 x=180^{\circ} \quad \Rightarrow x=\frac{180^{\circ}}{12}=15^{\circ}$
$\therefore \angle A=3 \times 15^{\circ}=45^{\circ} \quad \angle B=4 \times 15^{\circ}=60^{\circ} \quad \angle C=5 \times 15^{\circ}=75^{\circ}$

Q. $\triangle A B C$ is a right triangle right angled at $B$. If $A C=25 \mathrm{~cm}$ and $A B=15 \mathrm{~cm}$, then find the side $B C$.

## 7th Triangle and its properties

$\triangle A B C$ is a right-angled triangle, right-angled at $B$.
$\therefore A C^{2}=A B^{2}+B C^{2}$ (Pythagoras theorem)
$\Rightarrow 15^{2}+B C^{2}=25^{2} \quad \Rightarrow B C^{2}=252-152$
$\Rightarrow B C^{2}=625-225 \quad \Rightarrow B C^{2}=400$
$\Rightarrow B C=\sqrt{ } 400$
$\therefore B C=20 \mathrm{~cm}$

Q. A tree of height 36 m broke at a point $P$, but it did not separate. The top of the tree touched the ground at a distance of 12 m from the base. find the distance of the point $P$ from the base of the tree.

Ans:


Height of the tree $=36 \mathrm{~m}$
$A P+P B=36 m$
$P B=36-A P$
DAPB is a right-angled triangle, so by Pythagoras
theorem
$\mathrm{PB} 2=\mathrm{AP} 2+\mathrm{AB} 2$
$(36-A P) 2=A P 2+(12) 2$
$1296+$ AP2 - 72AP $=$ AP2 +144
$1296+$ AP2 - 72AP - AP2 $-144=0$
$1152-72 A P=0$
$\mathrm{AP}=\frac{-1152}{-72} \mathrm{AP}=16$
$\begin{array}{ll}-72 A P=-1152 & A P \\ \text { Distance of point } P \text { from the base }=16 \mathrm{~m}\end{array}$
Q. In a right angled isosceles triangle, find the ratio of their sides.

Sol: We have $A B=B C$
By Pythagorean Theorem
$A C 2=A B^{2}+B C^{2} \quad A C 2=A B 2+A B^{2} \quad A C 2=2 A B^{2} \quad A C 2=2 A B^{2}$
$A C=\sqrt{ } 2: A B \quad A C=A B \sqrt{ } 2$
$A B: B C: A C=A B: A B: A B \sqrt{ } 2=1: 1: \sqrt{ } 2$

