रोल नं.

Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित (मानक)
MATHEMATICS (STANDARD)
*

निर्धारित समय : *3 घण्टे*
Time allowed : 3 hours

अधिकतम अंक : *80*
Maximum Marks : 80

नोट / NOTE :

(i) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **27** हैं ।

Please check that this question paper contains **27** printed pages.

(ii) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।

Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(iii) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।

Please check that this question paper contains **38** questions.

(iv) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।**

Please write down the serial number of the question in the answer-book before attempting it.

(v) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

General Instructions :

Read the following instructions carefully and follow them :

(i)   *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii)   *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii)   *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv)   *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v)   *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi)   *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii)   *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each. Internal choice is provided in **2** marks questions in each case-study.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix)   *Draw neat diagrams wherever required. Take $\pi = \dfrac{22}{7}$ wherever required, if not stated.*

(x)   *Use of calculators is **not** allowed.*

SECTION A

*This section comprises multiple choice questions (MCQs) of 1 mark each.*

1. The number of polynomials having zeroes -3 and 5 is :

(a)   only one   (b)   infinite

(c)   exactly two   (d)   at most two

2. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if :

(a) $a = b$ (b) $3a = 2b$

(c) $2a = 3b$ (d) $ab = 6$

3. The common difference of the A.P. whose n^{th} term is given by $a_n = 3n + 7$, is :

(a)   7   (b)   3

(c)   3n   (d)   1

4. In the given figure, DE ∥ BC. The value of x is :
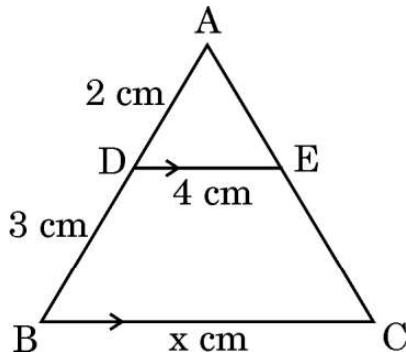
(a) 6	(b) 12·5
(c) 8	(d) 10

5. A quadratic equation whose roots are $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ is :

(a) $x^2 - 4x + 1 = 0$	(b) $x^2 + 4x + 1 = 0$
(c) $4x^2 - 3 = 0$	(d) $x^2 - 1 = 0$

6. If $\tan \theta = \dfrac{5}{12}$, then the value of $\dfrac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$ is :

(a) $-\dfrac{17}{7}$	(b) $\dfrac{17}{7}$
(c) $\dfrac{17}{13}$	(d) $-\dfrac{7}{13}$

7. The distance between the points $P\left(-\dfrac{11}{3}, 5\right)$ and $Q\left(-\dfrac{2}{3}, 5\right)$ is :

(a) 6 units	(b) 4 units
(c) 2 units	(d) 3 units

8. In the given figure, AB = BC = 10 cm. If AC = 7 cm, then the length of BP is :
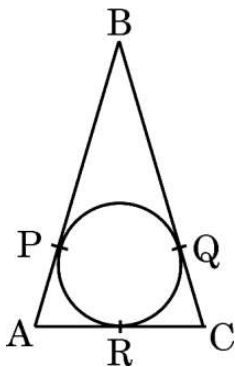
(a) 3·5 cm	(b) 7 cm
(c) 6·5 cm	(d) 5 cm

9. Water in a river which is 3 m deep and 40 m wide is flowing at the rate of 2 km/h. How much water will fall into the sea in 2 minutes ?

(a) 800 m³

(b) 4000 m³

(c) 8000 m³

(d) 2000 m³

10. If the mean and the median of a data are 12 and 15 respectively, then its mode is :

(a) 13·5

(b) 21

(c) 6

(d) 14

11. In the given figure, AB is a tangent to the circle centered at O. If OA = 6 cm and $\angle$ OAB = 30°, then the radius of the circle is :
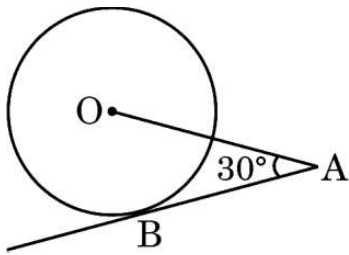
(a) 3 cm

(b) $3\sqrt{3}$ cm

(c) 2 cm

(d) $\sqrt{3}$ cm

12. $\left(\dfrac{2\tan 30°}{1+\tan^2 30°}\right)$ is equal to :

(a) sin 60°

(b) cos 60°

(c) tan 60°

(d) sin 30°

13. In Δ ABC and Δ DEF, $\dfrac{AB}{DE} = \dfrac{BC}{FD}$. Which of the following makes the two triangles similar ?

(a) $\angle$ A = $\angle$ D

(b) $\angle$ B = $\angle$ D

(c) $\angle$ B = $\angle$ E

(d) $\angle$ A = $\angle$ F

14. The 11th term from the end of the A.P. : 10, 7, 4, ..., – 62 is :

(a) 25 (b) 16

(c) – 32 (d) 0

15. Two coins are tossed together. The probability of getting at least one tail is :

(a) $\dfrac{1}{4}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{3}{4}$ (d) 1

16. In the given figure, AC and AB are tangents to a circle centered at O. If $\angle COD = 120°$, then $\angle BAO$ is equal to :
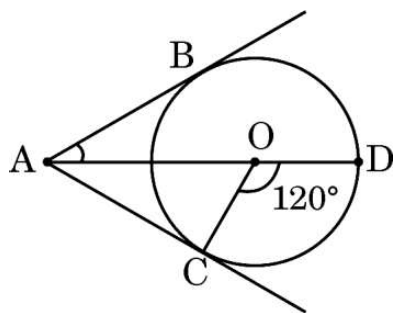
(a) 30° (b) 60°

(c) 45° (d) 90°

17. Which of the following numbers *cannot* be the probability of happening of an event ?

(a) 0 (b) $\dfrac{7}{0{\cdot}01}$

(c) 0·07 (d) $\dfrac{0{\cdot}07}{3}$

18. If every term of the statistical data consisting of n terms is decreased by 2, then the mean of the data :

(a) decreases by 2

(b) remains unchanged

(c) decreases by 2n

(d) decreases by 1

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If the points A(4, 3) and B(x, 5) lie on a circle with centre O(2, 3), then the value of x is 2.

Reason (R) : Centre of a circle is the mid-point of each chord of the circle.

20. *Assertion (A) :* The number 5^n cannot end with the digit 0, where n is a natural number.

Reason (R): Prime factorisation of 5 has only two factors, 1 and 5.

SECTION B

*This section comprises very short answer (VSA) type questions of 2 marks each.*

21. (a) The line segment joining the points A(4, – 5) and B(4, 5) is divided by the point P such that AP : AB = 2 : 5. Find the coordinates of P.

OR

(b) Point P(x, y) is equidistant from points A(5, 1) and B(1, 5). Prove that x = y.

22. In the given figure, PT is a tangent to the circle centered at O. OC is perpendicular to chord AB. Prove that $PA \cdot PB = PC^2 - AC^2$.
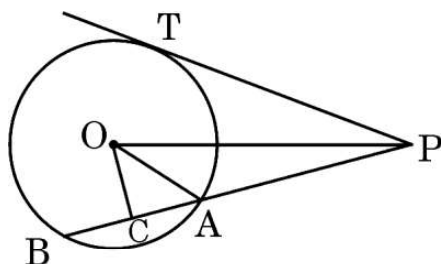
23. Using prime factorisation, find HCF and LCM of 96 and 120.

24. Find the ratio in which y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$.

25. (a) If $a \cos \theta + b \sin \theta = m$ and $a \sin \theta - b \cos \theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.

OR

(b) Prove that :
$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \operatorname{cosec} A$$

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

**26.** (a) Prove that $\sqrt{3}$ is an irrational number.

OR

(b) The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change together next ?
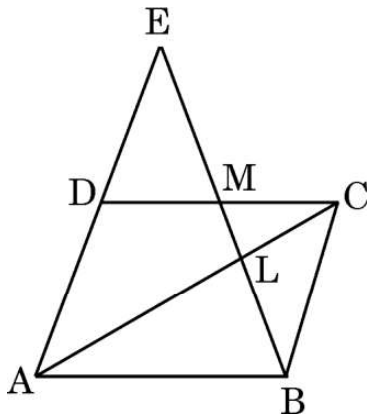
27. If p^{th} term of an A.P. is q and q^{th} term is p, then prove that its nth term is $(p + q - n)$.

28. (a) In the given figure, CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove that $\dfrac{CF}{CD} = \dfrac{FG}{DG}$.
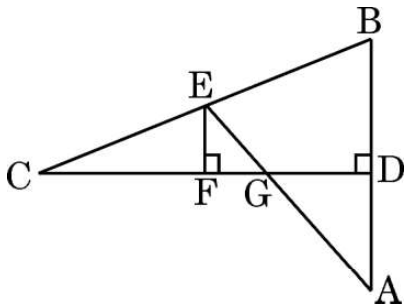
OR

(b) In the given figure, ABCD is a parallelogram. BE bisects CD at M and intersects AC at L. Prove that EL = 2BL.

29. Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 hours. Had they walked in the same direction with same speeds as before, they would have met in 8 hours. Find their walking speeds.

30. Prove that :

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \, \text{cosec} \, \theta$$

31. Find the mean of the following frequency distribution :

Classes	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60
Frequency	14	22	16	6	5	3	4

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find :

(a)   The height of the basket from the ground.

(b)   The distance of the basket from the first observer's eye.

(c)   The horizontal distance of the second observer from the basket.

33.   (a)   A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of the sides AB and AC, if it is given that area $\triangle$ ABC = 90 cm^2.
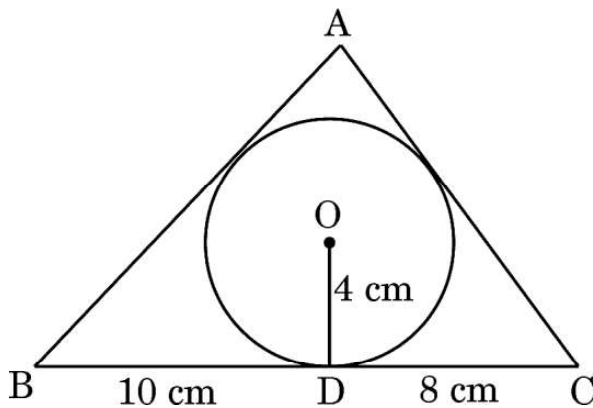
OR

(b) Two circles with centres O and O′ of radii 6 cm and 8 cm, respectively intersect at two points P and Q such that OP and O′P are tangents to the two circles. Find the length of the common chord PQ.
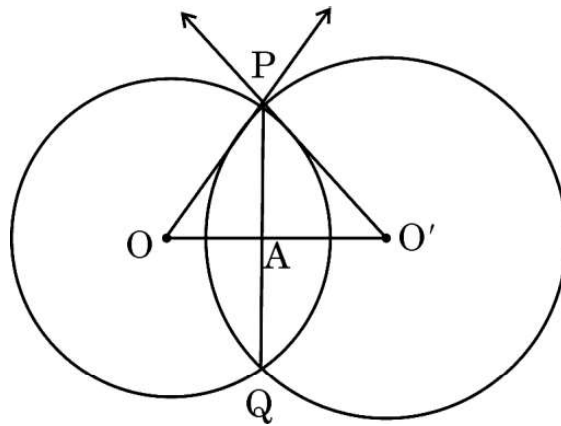
34. (a) A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the journey, what was its first average speed ?

OR

(b) Two pipes together can fill a tank in $\frac{15}{8}$ hours. The pipe with larger diameter takes 2 hours less than the pipe with smaller diameter to fill the tank separately. Find the time in which each pipe can fill the tank separately.

35. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. (Use π = 3·14)

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A golf ball is spherical with about 300 − 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4·2 cm and the surface has 315 dimples (hemi-spherical) of radius 2 mm.
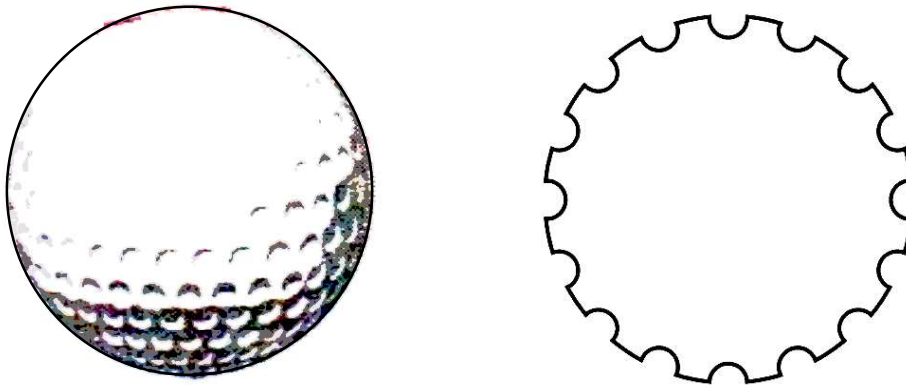
Based on the above, answer the following questions :

(i)   Find the surface area of one such dimple.   *1*

(ii) Find the volume of the material dug out to make one dimple.   *1*

(iii) (a)   Find the total surface area exposed to the surroundings.   *2*
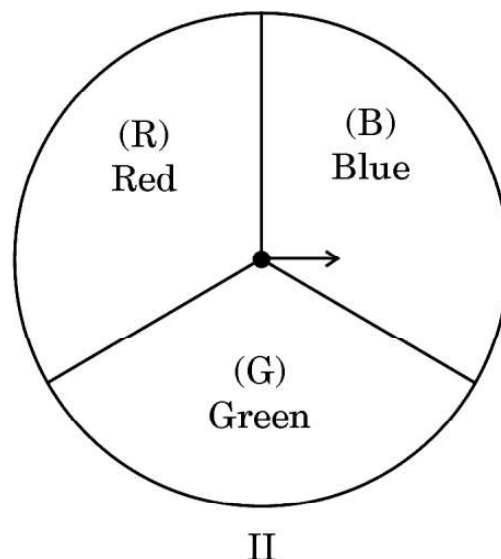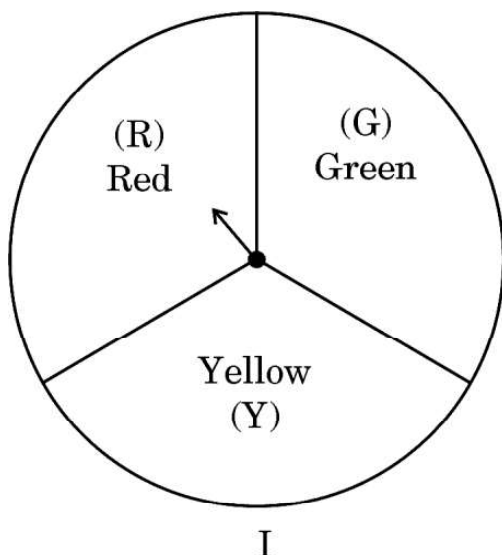
                                    OR

(iii) (b)   Find the volume of the golf ball.   *2*

37. A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival.

I

II

Making Purple : Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'.

Based on the above, answer the following questions :

(i) List all possible outcomes of the game. *1*

(ii) Find the probability of 'Making Purple'. *1*

(iii) (a) For each win, a participant gets ₹ 10, but if he/she loses, he/she has to pay ₹ 5 to the school.
If 99 participants played, calculate how much fund could the school have collected. *2*

OR

(iii) (b) If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school ? (Number of participants = 99) *2*
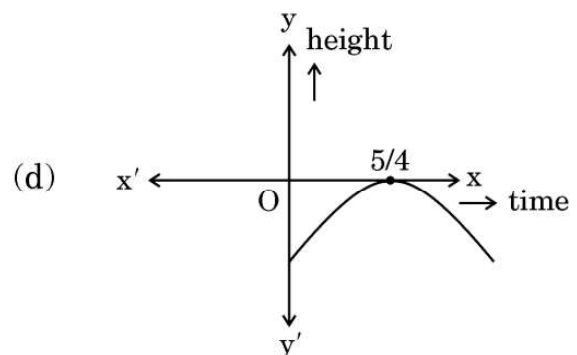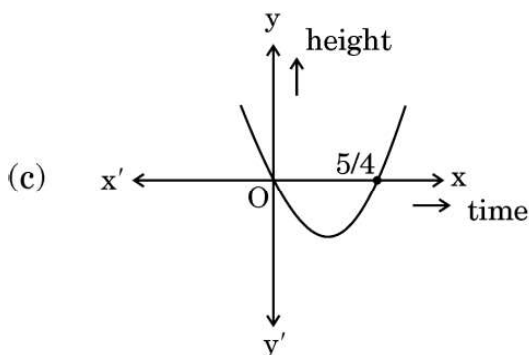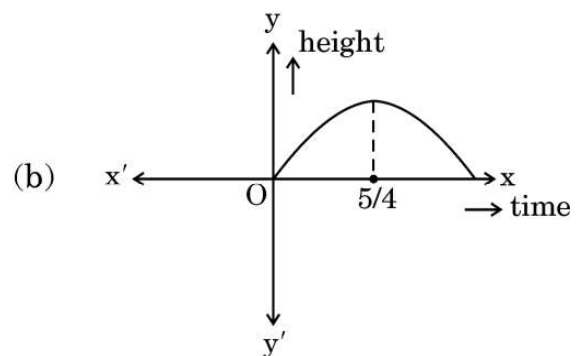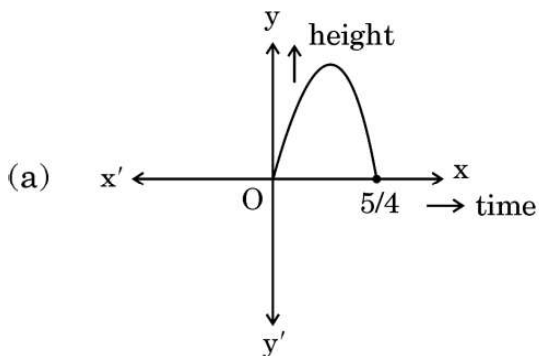
Case Study – 3

38. In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.
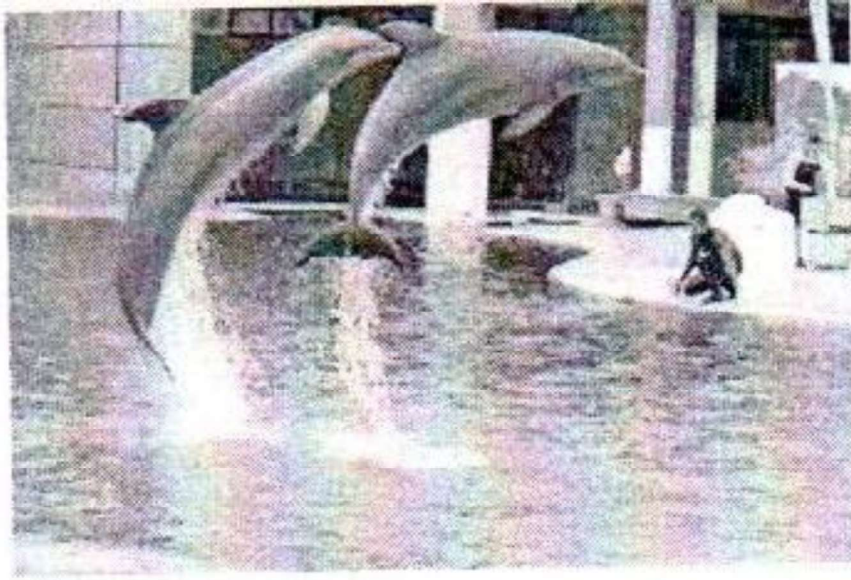
Based on the above, answer the following questions :

(i) Find zeroes of polynomial $p(t) = 20t - 16t^2$. *1*

(ii) Which of the following types of graph represents p(t) ? *1*

(a)

(b)

(c)

(d)

(iii)   (a)   What would be the value of h at $t = \dfrac{3}{2}$? Interpret the result.   *2*

   OR

(iii)   (b)   How much distance has the dolphin covered before hitting the water level again ?   *2*