DEVELOPMENT OF SUPPORT MATERIAL IN MATHEMATICS FOR CLASS XI

GROUP LEADER

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REVIEW OF SUPPORT MATERIAL : 2012

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CONTENTS

COURSE STRUCTURE

CLASS XI

One Paper **Three Hours** **Max. Marks. 100**

Units		**Marks**
I.	Sets and Functions	29
II.	Algebra	37
III.	Coordinate Geometry	13
IV.	Calculus	06
V.	Mathematical Reasoning	03
VI.	Statistics and Probability	12
		100

Unit-I : Sets and Functions

1. **Sets :** **(12) Periods**

   Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement Sets.

2. **Relations and Functions :** **(14) Periods**

   Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of reals with itself (upto $R \times R \times R$). Definition of relation, pictorial diagrams, domain, codomain and range of a relation. Function as a special kind of relation from one set to another. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

3. **Trigonometric Functions : (18) Periods**

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trignometric functions and their graphs. Expressing sin ($x \pm y$) and cos ($x \pm y$) in terms of sin x, sin y, cos$x$ and cos y. Deducing the identities like the following:

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \ \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x},$$

$$\sin x + \sin y = 2 \sin\frac{x+y}{2}\cos\frac{x-y}{2},$$
$$\cos x + \cos y = 2 \cos\frac{x+y}{2}\cos\frac{x-y}{2},$$

$$\sin x - \sin y = 2 \cos\frac{x+y}{2}\sin\frac{x-y}{2},$$
$$\cos x - \cos y = -2 \sin\frac{x+y}{2}\sin\frac{x-y}{2}.$$

Identities related to sin2x, cos2x, tan 2x, sin3x, cos3x and tan3x. General solution of trigonometric equations of the type $\sin\theta = \sin\alpha$, $\cos\theta = \cos\alpha$ and $\tan\theta = \tan\alpha$. Proof and simple applications of sine and cosine formulae.

Unit-II : Algebra

1. **Principle of Mathematical Induction : (06) Periods**

Process of the proof by induction, motivating the applications of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.

2. **Complex Numbers and Quadratic Equations : (10) Periods**

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quardratic equations. Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system. Square root of a complex number.

3. **Linear Inequalities : (10) Periods**

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical solution of system of linear inequalities in two variables.

4. **Permutations and Combinations : (12) Periods**

Fundamental principle of counting. Factorial n (n!) Permutations and combinations, derivation of formulae and their connections, simple applications.

5. **Binomial Theorem : (08) Periods**

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.

6. **Sequence and Series : (10) Periods**

Sequence and Series. Arithmetic progression (A.P.) arithmetic mean (A.M.) Geometric progression (G.P.), general term of a G.P., sum of n terms of a G.P., Arithmetic and Geometric series, Infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Sum to n terms of the special series $\sum_{k=1}^{n} k$, $\sum_{k=1}^{n} k^2$ and $\sum_{k=1}^{n} k^3$.

Unit-III : Coordinate Geometry

1. **Straight Lines : (09) Periods**

Brief recall of two dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line : parallel to axes, point-slope form, slope-intercept form, two-point form, intercept form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.

2. **Conic Sections : (12) Periods**

Sections of a cone : circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. **Introduction to Three-Dimensional Geometry (08) Periods**

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Unit-IV : Calculus

1. Limits and Derivatives : (18) Periods

Limit of function introduced as rate of change of distance function and its geometric meaning. $\lim\limits_{x \to o} \frac{\log_e (1 + x)}{x}$, $\lim\limits_{x \to o} \frac{e^x \; 1}{x}$. Definition of derivative, relate it to slope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V : Mathematical Reasoning

1. Mathematical Reasoning : (08) Periods

Mathematically acceptable statements. Connecting words/phrases-consolidating the understanding of “if and only if (necessary and sufficient) condition”, “implies”, “and/or”, “implied by”, “and”, “or”, “there exists” and their use through variety of examples related to real life and Mathematics. Validating the statements involving the connecting words, difference between contradiction, converse and contrapositive.

Unit-VI : Statistics and Probability

1. **Statistics :** **(10) Periods**

   Measures of dispersion, mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.

2. **Probability :** **(10 Periods)**

   Random experiments; outcomes, sample spaces (set representation). Events; occurrence of events, “not”, "and" and “or” events, exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability, connections with the theories of earlier classes. Probability of an event, probability of “not”, “and” and “or” events.

CHAPTER - 1

SETS

KEY POINTS

- A set is a well-defined collection of objects.
- There are two methods of representing a set :–
  - (a) Roster or Tabular form.
  - (b) Set-builder form or Rule method.
- Types of sets :–
  - (i) Empty set or Null set or void set
  - (ii) Finite set
  - (iii) Infinite set
  - (iv) Singleton set
- Subset :– A set A is said to be a subset of set B if $a \in A \Rightarrow a \in B$, $\forall\ a \in A$
- Equal sets :– Two sets A and B are equal if they have exactly the same elements i.e $A = B$ if $A \subset B$ and $B \subset A$
- Power set : The collection of all subsets of a set A is called power set of A, denoted by P(A) i.e. $P(A) = \{ B : B \subset A \}$
- If A is a set with $n(A) = m$ then $n[P(A)] = 2^m$.

Types of Intervals

Open Interval $(a, b) = \{ x \in R : a < x < b \}$

Closed Interval $[a, b] = \{ x \in R : a \leq x \leq b \}$

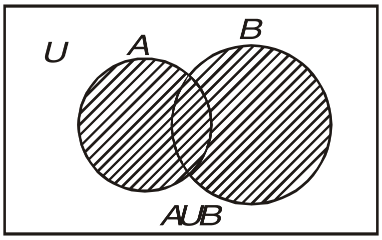
Semi open or Semi closed Interval,

$(a,b] = \{ x \in R : a < x \leq b\}$

$[a,b) = \{ x \in R : a \leq x < b\}$

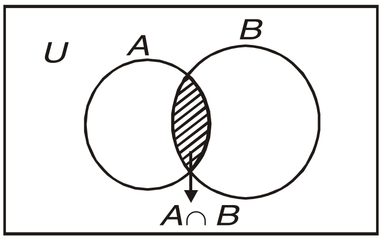
- Union of two sets A and B is,

  $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

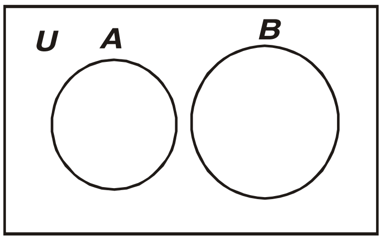


- Intersection of two sets A and B is,

  $A \cap B = \{ x : x \in A \text{ and } x \in B\}$

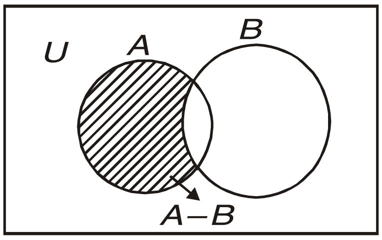


- Disjoint sets : Two sets A and B are said to be disjoint if $A \cap B = \phi$



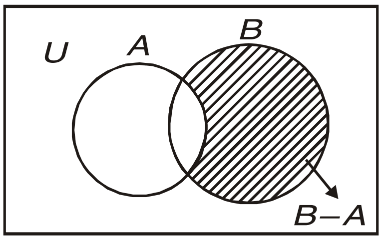
- Difference of sets A and B is,

  $A - B = \{ x : x \in A \text{ and } x \notin B\}$



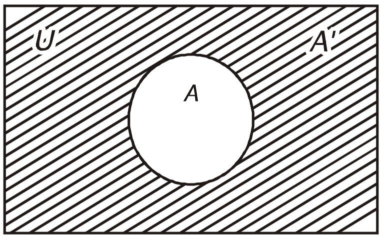
- Difference of sets B and A is,

  $B - A = \{ x : x \in B \text{ and } x \notin A \}$



- Complement of a set A, denoted by A' or A^c is

  $A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A\}$



- Properties of complement sets :

  1. Complement laws

     (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

2. De Morgan's Laws

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Note : This law can be extended to any number of sets.

3. $\phi' = \bigcup$ and $\bigcup' = \phi$

- $A - B = A \cap B'$
- Commutative Laws :–

  (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$
- Associative Laws :–

  (i) $(A \cup B) \cup C = A \cup (B \cup C)$ (ii) $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributive Laws :–

  (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Which of the following are sets? Justify your answer.

1. The collection of all the months of a year beginning with letter M
2. The collection of difficult topics in Mathematics.

   Let A = {1,3,5,7,9}. Insert the appropriate symbol $\in$ or $\notin$ in blank spaces :– (Question- 3,4)
3. 2 —— A
4. 5 —— A
5. Write the set $A = \{ x : x$ is an integer, $-1 \le x < 4\}$ in roster form
6. List all the elements of the set,

   $$A = \left\{x : x \in Z, -\frac{1}{2} < x < \frac{11}{2}\right\}$$

7. Write the set B = {3,9,27,81} in set-builder form.

Which of the following are empty sets? Justify. (Question- 8,9)

8. A = { x : x $\in$ N and 3 <x <4}

9. B = { x : x $\in$ N and x^2 = x}

Which of the following sets are finite or Infinite? Justify. (Question-10,11)

10. The set of all the points on the circumference of a circle.

11. B = { x : x $\in$ N and x is an even prime number}

12. Are sets A = { –2,2}, B = { x : x $\in$ Z, x^2 –4 = 0} equal? Why?

13. Write (–5,9] in set-builder form

14. Write { x : –3 $\leq$ x <7} as interval.

15. If A = { 1,3,5}, how many elements has P(A)?

16. Write all the possible subsets of A = {5,6}.

If A = {2,3,4,5}, B = { 3,5,6,7} find (Question- 17,18)

17. A $\cup$ B

18. A $\cap$ B

19. If A = {1,2,3,6} , B = {1, 2, 4, 8} find B – A

20. If A = {p, q}, B = {p, q, r}, is B a superset of A? Why?

21. Are sets A = {1,2,3,4}, B = { x : x $\in$ N and 5 $\leq$ x $\leq$ 7} disjoint? Why?

22. If X and Y are two sets such that n(X) = 19, n(Y) = 37 and n(X $\cap$ Y) = 12, find n(X $\cup$ Y).

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

23. If $\bigcup$ = {1,2,3,4,5,6,7,8,9}, A = {2,3,5,7,9}, B = { 1,2,4,6}, verify

   (i) (A $\cup$ B)' = A' $\cap$ B'

   (ii) B – A = B $\cap$ A' = B – (A $\cap$ B)

24. Let A, B be any two sets. Using properties of sets prove that,

(i) $(A - B) \cup B = A \cup B$

(ii) $(A \cup B) - A = B - A$

[**Hint :** $A - B = A \cap B'$ and use distributive law.]

25. In a group of 800 people, 500 can speak Hindi and 320 can speak English. Find

(i) How many can speak both Hindi and English?

(ii) How many can speak Hindi only?

26. A survey shows that 84% of the Indians like grapes, whereas 45% like pineapple. What percentage of Indians like both grapes and pineapple?

27. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

28. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

29. If $A = [-3, 5)$, $B = (0, 6]$ then find (i) $A - B$, (ii) $A \cup B$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

30. In a survey it is found that 21 people like product A, 26 people like product B and 29 like product C. If 14 people like product A and B, 15 people like product B and C, 12 people like product C and A, and 8 people like all the three products. Find

(i) How many people are surveyed in all?

(ii) How many like product C only?

31. A college awarded 38 medals in football, 15 in basket ball and 20 in cricket. If these medals went to a total of 50 men and only five men got medals in all the three sports, how many received medals in exactly two of the three sports?

ANSWERS

1. Set
2. Not a set
3. $\notin$
4. $\in$
5. $A = \{-1, 0, 1, 2, 3\}$
6. $A = \{0,1,2,3,4,5\}$
7. $B = \{x : x = 3^n, n \in N \text{ and } 1 \le n \le 4\}$
8. Empty set
9. Non-empty set
10. Infinite set
11. Finite set
12. Yes
13. $\{x : x \in R, -5 < x \le 9\}$
14. $[-3,7)$
15. $2^3 = 8$
16. ϕ, $\{5\}$, $\{6\}$, $\{5,6\}$
17. $A \cup B = \{2,3,4,5,6,7\}$
18. $A \cap B = \{3, 5\}$
19. $B - A = \{4,8\}$
20. Yes, because A is a subset of B
21. Yes, because $A \cap B = \phi$
22. $n(X \cup Y) = 44$

25. (i) 20 people can speak both Hindi and English

    (ii) 480 people can speak Hindi only

26. 29% of the Indians like both grapes and pineapple.

27. **Hint :** $\cup$ – set of people surveyed

    A – set of people who play cricket

    B – set of people who play tennis

    Number of people who play neither cricket nor tennis

    $= n[(A \cup B)'] = n(U) - n(A \cup B)$

    $= 450 - 200$

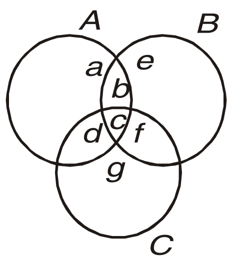
    $= 250$

28. There are 280 students in the group.

29. (i) [–3, 0]; (ii) [–3, 6]

30. **Hint :** Let A, B, C denote respectively the set of people who like product A, B, C.

a, b, c, d, e, f, g – Number of elements in bounded region

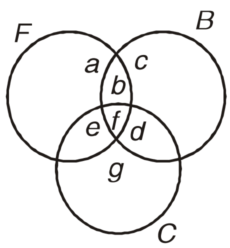


(i) Total number of Surveyed people = a + b + c + d + e + f + g = 43

(ii) Number of people who like product C only = g = 10

31. 13 people got medals in exactly two of the three sports.

Hint :



f = 5

a + b + f + e = 38

b + c + d + f = 15

e + d + f + g = 20

a + b + c + d + e + f + g = 50

we have to find *b* + *d* + *e*

CHAPTER - 2

RELATIONS AND FUNCTIONS

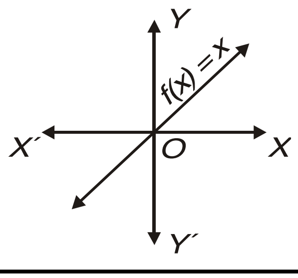
KEY POINTS

- Cartesian Product of two non-empty sets A and B is given by,

  $A \times B = \{ (a,b) : a \in A, b \in B\}$

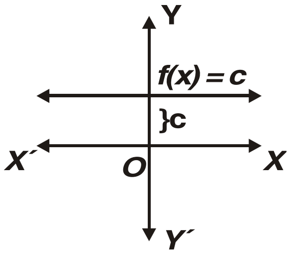
- If $(a,b) = (x, y)$, then $a = x$ and $b = y$
- Relation R from a non-empty set A to a non-empty set B is a subset of $A \times B$.
- Domain of $R = \{a : (a,b) \in R\}$
- Range of $R = \{ b : (a,b) \in R\}$
- Co-domain of R = Set B
- Range $\subseteq$ Co-domain
- If $n(A) = p$, $n(B) = q$ then $n(A \times B) = pq$ and number of relations $= 2^{pq}$
- A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.
- $D_f = \{x : f(x) \text{ is defined}\}$ $\qquad$ $R_f = \{f(x) : x \in D_f\}$
- Identity function, $f : R \to R$; $f(x) = x \ \forall x \in R$ where R is the set of real numbers.

  $D_f = R$ $\qquad$ $R_f = R$



- Constant function, f : R → R; f(x) = c $\forall x \in R$ where c is a constant

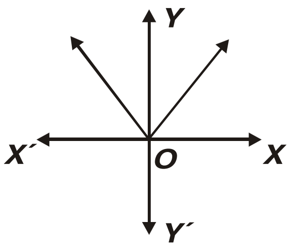
  $D_f = R$ $\qquad$ $R_f = \{c\}$



- Modulus function, f : R → R; f(x) = |x| $\forall x \in R$

  $D_f = R$

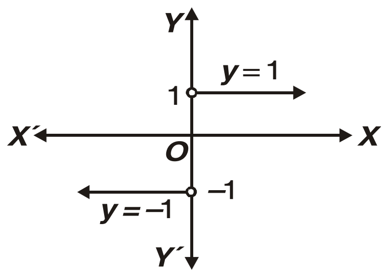
  $R_f = R^+ = \{ x \in R: x \geq 0\}$



- Signum function, f : R → R ; $f(x) = \begin{cases} 1, \text{If } x > 0 \\ 0, \text{ if } x = 0 \\ -1, \text{if } x < 0 \end{cases}$

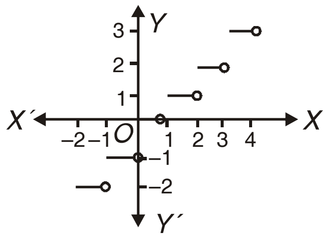
  $D_f = R$

  $R_f = \{-1,0,1\}$



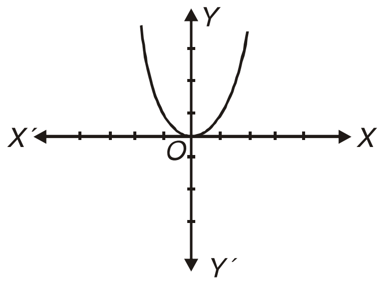
- Greatest Integer function, $f : R \to R$; $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x

  $D_f = R$ $\qquad$ $R_f = Z$



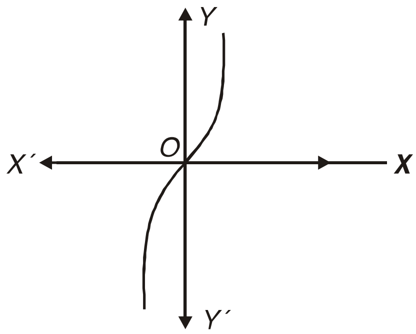
- $f : R \to R$, $f(x) = x^2$

  $D_f = R$ $\qquad$ $R_f = [0, \infty)$



- $f : R \to R$, $f(x) = x^3$

  $D_f = R$ $\qquad$ $R_f = R$



- Let $f : X \to R$ and $g : X \to R$ be any two real functions where $x \subset R$ then

  $(f \pm g)(x) = f(x) \pm g(x) \ \forall x \in X$

  $(fg)(x) = f(x)\, g(x) \ \forall x \in X$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad \forall\ x \in X \text{ provided } g(x) \neq 0$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find a and b if $(a - 1, b + 5) = (2, 3)$

   If $A = \{1,3,5\}$, $B = \{2,3\}$ find : (Question-2, 3)

2. $A \times B$

3. $B \times A$

   Let $A = \{1,2\}$, $B = \{2,3,4\}$, $C = \{4,5\}$, find (Question- 4,5)

4. $A \times (B \cap C)$

5. $A \times (B \cup C)$

6. If $P = \{1,3\}$, $Q = \{2,3,5\}$, find the number of relations from A to B

7. If $A = \{1,2,3,5\}$ and $B = \{4,6,9\}$,

   $R = \{(x, y) : |x - y| \text{ is odd}, x \in A, y \in B\}$

   Write R in roster form

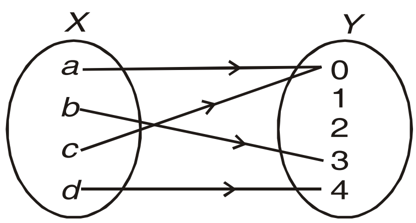
   Which of the following relations are functions. Give reason. (Questions 8 to 10)

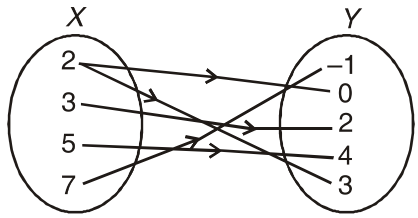
8. $R = \{ (1,1), (2,2), (3,3), (4,4), (4,5)\}$

9. $R = \{ (2,1), (2,2), (2,3), (2,4)\}$

10. $R = \{ (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)\}$

    Which of the following arrow diagrams represent a function? Why? (Question- 11,12)

11. 

12. 

Let f and g be two real valued functions, defined by, $f(x) = x^2$, $g(x) = 3x + 2$, find : (Question 13 to 16)

13. $(f + g)(-2)$

14. $(f - g)(1)$

15. $(fg)(-1)$

16. $\left(\frac{f}{g}\right)(0)$

17. If $f(x) = x^3$, find the value of,

$$\frac{f(5) - f(1)}{5 - 1}$$

18. Find the domain of the real function,

$$f(x) = \sqrt{x^2 - 4}$$

19. Find the domain of the function, $f(x) = \frac{x^2 + 2x + 3}{x^2 - 5x + 6}$

Find the range of the following functions, (Question- 20,21)

20. $f(x) = \frac{1}{1 - x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,

$R = \{ (x, y) : x, y \in Z, xy = 4\}$

Find the range of the following relations : (Question-23, 24)

23. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

24. $R = \left\{\left(x, \frac{1}{x}\right) : x \in z, 0 < x < 6\right\}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

$R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2\}$

(a) Depict this relation using arrow diagram.

(b) Find domain of R.

(c) Find range of R.

(d) Write co-domain of R.

26. Let $R = \{ (x, y) : x, y \in N \text{ and } y = 2x\}$ be a relation on N. Find :

(i) Domain

(ii) Codomain

(iii) Range

Is this relation a function from N to N?

27. Let $f(x) = \begin{cases} x^2, \text{ when } 0 \le x \le 2. \\ 2x, \text{ when } 2 \le x \le 5 \end{cases}$

$g(x) = \begin{cases} x^2, \text{ when } 0 \le x \le 3. \\ 2x, \text{ when } 3 \le x \le 5 \end{cases}$

Show that f is a function while g is not a function.

28. Find the domain and range of,

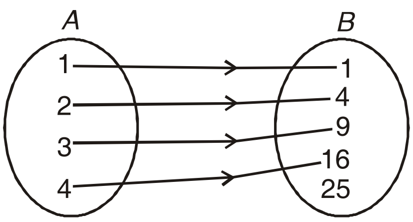
$f(x) = |2x - 3| - 3$

29. Draw the graph of the Greatest Integer function

30. Draw the graph of the Constant function, $f : R \to R$; $f(x) = 2 \ \forall x \in R$. Also find its domain and range.

ANSWERS

1. $a = 3, b = -2$
2. $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$
3. $B \times A = \{ (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$
4. $\{(1,4), (2,4)\}$
5. $\{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)\}$
6. $2^6 = 64$
7. $R = \{ (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6)\}$
8. Not a function because 4 has two images.
9. Not a function because 2 does not have a unique image.
10. Function
11. Function
12. Not a function
13. 0
14. -4
15. -1
16. 0
17. 31
18. $(-\infty, -2] \cup [2, \infty)$
19. $R - \{2,3\}$
20. $(-\infty, 0) \cup [1, \infty)$
21. $[2,\infty)$
22. $\{-4, -2, -1,1,2,4\}$
23. $\{2,4,6,8\}$
24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$
25. (a)



(b) {1,2,3,4}

(c) {1,4,9,16}

(d) {1,4,9,16,25}

26. (i) N

(ii) N

(iii) Set of even natural numbers

yes, R is a function from N to N.

28. Domain is R

Range is $[-3, \infty)$

30. Domain = R

Range = {2}

CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- A radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. We denote 1 radian by 1^c.
- π radian = 180 degree

  1 radian = $\frac{180}{\pi}$ degree

  1 degree = $\frac{\pi}{180}$ radian
- If an arc of length l makes an angle θ radian at the centre of a circle of radius r, we have

  $$\theta = \frac{l}{r}$$

-

Quadrant →	I	II	III	IV
t- functions which are positive	All	sin x cosec x	tan x cot x	cos x sec x

-

Function	$-x$	$\frac{\pi}{2} - x$	$\frac{\pi}{2} + x$	$\pi - x$	$\pi + x$	$2\pi - x$	$2\pi + x$
sin	–sin x	cos x	cos x	sin x	–sin x	–sin x	sin x
cos	cos x	sin x	–sin x	–cos x	–cos x	cos x	cos x
tan	–tan x	cot x	–cot x	–tan x	tan x	–tan x	tan x
cosec	–cosec x	sec x	sec x	cosec x	–cosec x	–cosec x	cosec x
sec	sec x	cosec x	–cosec x	–sec x	–sec x	sec x	sec x
cot	–cot x	tan x	–tan x	–cot x	cot x	–cot x	cot x

Function	Domain	Range
sin x	R	[–1,1]
cos x	R	[–1,1]
tan x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R
Cosec x	$R - \{n\pi;\ n \in z\}$	R – (–1,1)
Sec x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R – (–1,1)
cot x	$R - \{n\pi,\ n \in z\}$	R

Some Standard Results

- $\sin(x + y) = \sin x \cos y + \cos x \sin y$

  $\cos(x + y) = \cos x \cos y - \sin x \sin y$

  $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$$

  $$\cot(x+y) = \frac{\cot x . \cot y - 1}{\cot y + \cot x}$$

- $\sin(x - y) = \sin x \cos y - \cos x \sin y$

  $\cos(x - y) = \cos x \cos y + \sin x \sin y$

  $$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x . \tan y}$$

  $$\cot(x-y) = \frac{\cot x . \cot y + 1}{\cot y - \cot x}$$

- $$\tan(x+y+z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y . \tan z - \tan z \tan x}$$

- $2\sin x \cos y = \sin(x + y) + \sin(x - y)$

  $2\cos x \sin y = \sin(x + y) - \sin(x - y)$

  $2\cos x \cos y = \cos(x + y) + \cos(x - y)$

  $2\sin x \sin y = \cos(x - y) - \cos(x + y)$

- $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

  $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

  $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$

  $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

- $\text{Sin } 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$

- $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$

- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

- $\sin 3x = 3 \sin x - 4 \sin^3 x$

- $\cos 3x = 4 \cos^3 x - 3 \cos x$

- $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

- $\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$

  $= \cos^2 y - \cos^2 x$

- $\cos(x + y) \cos(x - y) = \cos^2 x - \sin^2 y$

  $= \cos^2 y - \sin^2 x$

- Principal solutions – The solutions of a trigonometric equation for which $0 \leq x < 2\pi$ are called its principal solutions.

- General solution – A solution of a trigonometric equation, generalised by means of periodicity, is known as the general solution.

General solutions of trigonometric equations :

$\sin\theta = 0 \Rightarrow \theta = n\pi, n \in z$

$\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in z$

$\tan\theta = 0 \Rightarrow \theta = n\pi, n \in z$

$\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in z$

$\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in z$

$\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, n \in z$

- Law of sines or sine formula

  The lengths of sides of a triangle are proportional to the sines of the angles opposite to them i.e..

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- Law of cosines or cosine formula

  In any Δ ABC

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the radian measure corresponding to 5° 37' 30"
2. Find the degree measure corresponding to $\left(\frac{11}{16}\right)^c$
3. Find the length of an arc of a circle of radius 5 cm subtending a central angle measuring 15°

4. Find the value of $\tan \frac{19\pi}{3}$

5. Find the value of sin(–1125°)

6. Find the value of tan 15°

7. If $\sin A = \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$, find cos A

8. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of A + B.

9. Express $\sin 12\theta + \sin 4\theta$ as the product of sines and cosines.

10. Express 2 cos4x sin2x as an algebraic sum of sines or cosines.

11. Write the range of $\cos\theta$

12. What is domain of $\sec\theta$?

13. Find the principal solutions of $\cot x = -\sqrt{3}$

14. Write the general solution of $\cos \theta = 0$

15. If $\sin x = \frac{\sqrt{5}}{3}$ and $0 < x < \frac{\pi}{2}$ find the value of cos 2x

16. If $\cos x = \frac{-1}{3}$ and x lies in quadrant III, find the value of $\sin \frac{x}{2}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

18. It the angles of a triangle are in the ratio 3:4:5, find the smallest angle in degrees and the greatest angle in radians.

19. If $\sin x = \frac{12}{13}$ and x lies in the second quadrant, show that secx + tanx = –5

20. If $\cot\alpha = \frac{1}{2}$, $\sec\beta = \frac{-5}{3}$ where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, find the value of $\tan(\alpha + \beta)$

Prove the following Identities

21. $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta \cos 4\theta$

22. $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$

23. $\dfrac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x \sin x + \cos 6x \cos x} = \tan 2x$

24. $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\dfrac{\theta}{2}$

25. $\tan\alpha . \tan(60° - \alpha) . \tan(60° + \alpha) = \tan 3\alpha$

26. Show that $\cos 20° \cos 40° \cos 80° = \frac{1}{8}$

27. Show that $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$

28. Pr ove that $\dfrac{\cos x}{1 - \sin x} = \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$

29. Draw the graph of cosx in $[0, 2\pi]$

Find the general solution of the following equations (Q.No. 30 to Q. No. 33)

30. $\cos\left(x + \dfrac{\pi}{10}\right) = 0$

31. $\sin 7x = \sin 3x$

32. $\sqrt{3}\cos x - \sin x = 1$

33. $3\tan x + \cot x = 5\operatorname{cosec} x$

34. In any triangle ABC, prove that

$$a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B) = 0$$

35. In any triangle ABC, prove that

$$a = b\cos C + c\cos B$$

36. In any triangle ABC, prove that

$$\frac{a+b}{c} = \frac{\cos\frac{A-B}{2}}{\sin\frac{C}{2}}$$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

37. Prove that

$$\cos A\cos 2A\cos 4A\cos 8A = \frac{\sin 16A}{16\sin A}$$

38. Prove that $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{16}$

39. Find the general solution of

$$\sin 2x + \sin 4x + \sin 6x = 0$$

40. Find the general solution of

$$\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}$$

41. Draw the graph of tanx in $\left(\frac{-3\pi}{2}, \frac{3\pi}{2}\right)$

42. In any triangle ABC, prove that

$$\frac{b^2-c^2}{a^2}\sin 2A + \frac{c^2-a^2}{b^2}\sin 2B + \frac{\left(a^2-b^2\right)}{c^2}\sin 2C = 0$$

ANSWERS

1. $\left(\frac{\pi}{32}\right)^c$

2. $39°22'30''$

3. $\frac{5\pi}{12}$ cm

4. $\sqrt{3}$

5. $\frac{-1}{\sqrt{2}}$

6. $2 - \sqrt{3}$

7. $\frac{-4}{5}$

8. $45°$

9. $2 \sin 8\theta \cos 4\theta$

10. $\sin 6x - \sin 2x$

11. $[-1,1]$

12. $R - \left\{(2n+1)\frac{\pi}{2};\ n \in z\right\}$

13. $\frac{5\pi}{6}, \frac{11\pi}{6}$

14. $(2n+1)\frac{\pi}{2}, n \in z$

15. $-\frac{1}{9}$

16. $\frac{\sqrt{6}}{3}$

17. 70 m

18. $45°, \frac{5\pi}{12}$ radians

20 $\frac{2}{11}$

30. $\left(n\pi + \frac{2\pi}{5}\right), n \in z$

31. $(2n+1)\frac{\pi}{10}, \frac{n\pi}{2}, n \in z$

32. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, n \in z$

33. $2n\pi \pm \frac{\pi}{3}, n \in z$

39. $\frac{n\pi}{4}, n\pi \pm \frac{\pi}{3}, n \in z$

40. $(2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in z$

CHAPTER - 4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY POINTS

- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Principle of Mathematical Induction :

  Let P(n) be any statement involving natural number n such that

  (i) P(1) is true, and

  (ii) If P(k) is true implies that P(k +1) is also true for some natural number k

  then P(n) is true $\forall n \in N$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Using the principle of mathematical induction prove the following for all $n \in N$:

1. 3.6 + 6.9 + 9.12 + ... + 3n (3n + 3) = 3n(n + 1)(n + 2)
2. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)---\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$
3. $n^2 + n$ is an even natural number.
4. $2^{3n} - 1$ is divisible by 7
5. 3^{2n} when divided by 8 leaves the remainder 1.

6. $4^n + 15n - 1$ is divisible by 9

7. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9.

8. $x^{2n-1} - 1$ is divisible by $x - 1$, $x \neq 1$

9. $3^n > n$

10. If x and y are any two distinct integers then $x^n - y^n$ is divisible by $(x - y)$

11. $n < 2^n$

12. $a + (a + d) + (a + 2d) + \text{..........} + [a + (n - 1)d] = \frac{n}{2}\left[2a + (n-1)d\right]$

13. $3x + 6x + 9x + \text{..........}$ to n terms $= \frac{3}{2}n(n+1)x$

14. $11^{n+2} + 12^{2n+1}$ is divisible by 133.

CHAPTER - 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- A number of the form $z = a + ib$, where $a, b \in R$ is called a complex number.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- $a + ib = c + id$ if $a = c$, and $b = d$
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$

  but if $b, d = 0$ and $a > c$ then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- $-z = -a + i(-b)$ is called the Additive Inverse or negative of $z = a + ib$
- $\bar{z} = a - ib$ is called the conjugate of $z = a + ib$

  $z^{-1} = \dfrac{1}{z} = \dfrac{a - ib}{a^2 + b^2} = \dfrac{\bar{z}}{|z|^2}$ is called the multiplicative Inverse of

  $z = a + ib$ $(a \neq 0, b \neq 0)$

- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane
- Polar form of $z = a + ib$ is,

  $z = r(\cos\theta + i\sin\theta)$ where $r = \sqrt{a^2 + b^2} = |z|$ is called the modulus of z,

  θ is called the argument or amplitude of z.
- The value of θ such that, $-\pi < \theta \le \pi$ is called the principle argument of z.
- $|z_1 + z_2| \le |z_1| + |z_2|$
- $|z_1 z_2| = |z_1| . |z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $\left|z^n\right| = |z|^n$, $|z| = |\bar{z}| = |-z| = |-\bar{z}|$, $z\bar{z} = |z|^2$
- $|z_1 - z_2| \le |z_1| + |z_2|$
- $|z_1 - z_2| \ge \left||z_1| - |z_2|\right|$
- For the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in R$, $a \ne 0$,

  if $b^2 - 4ac < 0$ then it will have complex roots given by,

$$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Evaluate, $\sqrt{-16} + 3\sqrt{-25} + \sqrt{-36} - \sqrt{-625}$
2. Evaluate, $i^{29} + \frac{1}{i^{29}}$
3. Find values of x and y if,

   $(3x - 7) + 2iy = -5y + (5 + x)i$

4. Express $\frac{i}{1+i}$ in the form a + ib

5. If $z = \frac{1}{3+4i}$, find the conjugate of z

6. Find the modulus of z = 3 – 2i

7. If z is a purely imaginary number and lies on the positive direction of y-axis then what is the argument of z?

8. Find the multiplicative inverse of 5 + 3i

9. If |z| = 4 and argument of z = $\frac{5\pi}{6}$ then write z in the form x + iy; x, y $\in$R

10. If z = 1 – i, find $\operatorname{Im}\left(\frac{1}{z\,\bar{z}}\right)$

11. Simplify (–i)(3 i) $\left(\frac{-1\ i}{6}\right)^3$

12. Find the solution of the equation $x^2 + 5 = 0$ in complex numbers.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

13. For Complex numbers $z_1 = -1 + i$, $z_2 = 3 - 2i$

show that,

$$\operatorname{Im}(z_1z_2) = \operatorname{Re}(z_1)\operatorname{Im}(z_2) + \operatorname{Im}(z_1)\operatorname{Re}(z_2)$$

14. Convert the complex number $-3\sqrt{2} + 3\sqrt{2}\ i$ in polar form

15. If $x + iy = \sqrt{\frac{1+i}{1-i}}$, prove that $x^2 + y^2 = 1$

16. Find real value of θ such that,

$$\frac{1 + i\cos\theta}{1 - 2i\cos\theta} \text{ is a real number}$$

17. If $\left|\frac{z-5i}{z+5i}\right| = 1$, show that z is a real number.

18. If $(x+iy)^{\frac{1}{3}} = a + ib$, prove that, $\left(\frac{x}{a} + \frac{y}{b}\right) = 4(a^2 - b^2)$

19. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$

20. If $\left(\frac{2+2i}{2-2i}\right)^n = 1$,find the least positive integral value of n.

21. Find the modulus and argument of $z = 2 - 2i$

22. Solve the equation, $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

23. If z_1, z_2 are complex numbers such that, $\left|\frac{z_1 - 3z_2}{3 - z_1\bar{z}_2}\right| = 1$ and $|z_2| \neq 1$ then find $|z_1|$

24. Find the square root of $-3 + 4i$ and verify your answer.

25. If $x = -1 + i$ then find the value of $x^4 + 4x^3 + 4x^2 + 2$

ANSWERS

1. 0	2. 0
3. $x = -1$, $y = 2$	4. $\frac{1}{2} + \frac{1}{2}i$
5. $\bar{z} = \frac{3}{25} + \frac{4i}{25}$	6. $\sqrt{13}$
7. $\frac{\pi}{2}$	8. $\frac{5}{34} - \frac{3i}{34}$
9. $z = -2\sqrt{3} + 2i$	10. 0
11. $\frac{i}{72}$	12. $x = \pm i\sqrt{5}$

14. $z = 6\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

16. $\theta = (2n+1)\frac{\pi}{2}, n \in z$

17. **Hint :** use property $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

19. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

20. n = 4

21. modulus = $2\sqrt{2}$, argument $= \frac{-\pi}{4}$

22. $x = \frac{\sqrt{2} \pm i\sqrt{34}}{2\sqrt{3}}$

23. **Hint :** use $|z|^2 = z.\bar{z}$, $|z_1| = 3$

24. ± (1 + 2i)

25. 6

CHAPTER - 6

LINEAR INEQUALITIES

KEY POINTS

- Two real numbers or two algebraic expressions related by the symbol '$<$', '$>$', '$\leq$' or '$\geq$' form an inequality.
- The inequalities of the form $ax + b > 0$, $ax + b < 0$, $ax + b \geq 0$, $ax + b \leq 0$; $a \neq 0$ are called linear inequalities in one variable x
- The inequalities of the form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$, $ax + by + c \leq 0$, $a \neq 0$, $b \neq 0$ are called linear inequalities in two variables x and y
- Rules for solving inequalities :

  (i) $a \geq b$ then $a \pm k \geq b \pm k$

  where k is any real number.

  (ii) but if $a \geq b$ then ka is not always $\geq$ kb.

  If $k > 0$ (i.e. positive) then $a \geq b \Rightarrow ka \geq kb$

  If $k < 0$ (i.e. negative) then $a \geq b \Rightarrow ka \leq kb$
- **Solution Set :** A solution of an inequality is a number which when substituted for the variable, makes the inequality true. The set of all solutions of an inequality is called the solution set of the inequality.
- The graph of the inequality $ax + by > c$ is one of the half planes and is called the solution region
- When the inequality involves the sign $\leq$ or $\geq$ then the points on the line are included in the solution region but if it has the sign $<$ or $>$ then the points on the line are not included in the solution region and it has to be drawn as a dotted line.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Solve $5x < 24$ when $x \in N$
2. Solve $3x < 11$ when $x \in Z$
3. Solve $3 - 2x < 9$ when $x \in R$
4. Show the graph of the solution of $2x - 3 > x - 5$ on number line.
5. Solve $5x - 8 \geq 8$ graphically
6. Solve $\frac{1}{x-2} \leq 0$
7. Solve $0 < \frac{-x}{3} < 1$

Write the solution in the form of intervals for $x \in R$. for Questions 8 to 10

8. $\frac{2}{x-3} < 0$
9. $-3 \leq -3x + 2 < 4$
10. $3 + 2x > -4 - 3x$
11. Draw the graph of the solution set of $x + y \geq 4$.
12. Draw the graph of the solution set of $x \leq y$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Solve the inequalities for real x

13. $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$
14. $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2$
15. $-5 \leq \frac{2-3x}{4} \leq 9$

16. $|x - 2| \geq 5$

17. $|4 - x| + 1 < 3$

18. $\dfrac{3}{x - 2} < 1$

19. $\dfrac{x}{x - 5} > \dfrac{1}{2}$

20. $\dfrac{x + 3}{x - 2} > 0$

21. $x + 2 \leq 5,\ 3x - 4 > -2 + x$

22. $3x - 7 > 2(x - 6),\ 6 - x > 11 - 2x$

23. The water acidity in a pool is considered normal when the average PH reading of three daily measurements is between 7.2 and 7.8. If the first two PH readings are 7.48 and 7.85, find the range of PH value for the third reading that will result in the acidity level being normal.

24. While drilling a hole in the earth, it was found that the temperature (T °C) at x km below the surface of the earth was given by

    $T = 30 + 25(x - 3)$, when $3 \leq x \leq 15$.

    Between which depths will the temperature be between 200°C and 300°C?

    Solve the following systems of inequalities graphically : (Questions 25, 26)

25. $x + y > 6,\ 2x - y > 0$

26. $3x + 4y \leq 60,\ x + 3y \leq 30,\ x \geq 0,\ y \geq 0$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

Solve the system of inequalities for real x

27. $\dfrac{5x}{4} + \dfrac{3x}{8} > \dfrac{39}{8}$ and

    $\dfrac{2x - 1}{12} - \dfrac{x - 1}{3} < \dfrac{3x + 1}{4}$

Solve the following system of inequalities graphically (Questions 28 to 30)

28. $3x + 2y \leq 24$, $x + 2y \leq 16$, $x + y \leq 10$, $x \geq 0$, $y \geq 0$

29. $2x + y \geq 4$, $x + y \leq 3$, $2x - 3y \leq 6$

30. $x + 2y \leq 2000$, $x + y \leq 1500$, $y \leq 600$, $x \geq 0$, $y \geq 0$

ANSWERS

1. {1,2,3,4}

2. {......, –2, –1, 0,1, 2, 3}

3. $x > -3$

6. $x < 2$

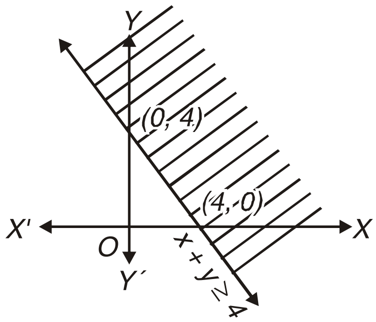
7. $-3 < x < 0$

8. $(-\infty, 3)$

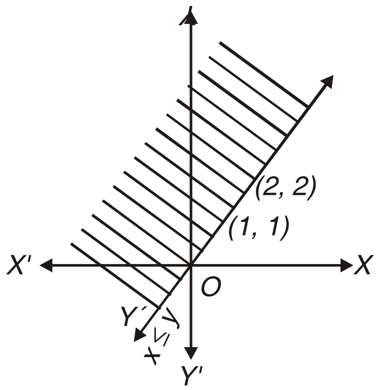
9. $\left(\frac{-2}{3}, \frac{5}{3}\right]$

10. $\left(\frac{-7}{5}, \infty\right)$

11.



12.



13. $\left(-\infty, \frac{63}{10}\right]$

14. $\left(-\infty, \frac{-13}{2}\right)$

15. $\left[\frac{-34}{3}, \frac{22}{3}\right]$

16. $(-\infty, -3] \cup [7, \infty)$

17. $(2, 6)$

18. $(-\infty, 2) \cup (5, \infty)$

19. $(-\infty, -5) \cup (5, \infty)$

20. $(-\infty, -3) \cup (2, \infty)$

21. $(1, 3]$

22. $(5, \infty)$

23. Between 6.27 and 8.07

24. Between 9.8 m and 13.8 m

27. $(3, \infty)$

CHAPTER - 7

PERMUTATIONS AND COMBINATIONS

KEY POINTS

- When a job (task) is performed in different ways then each way is called the permutation.
- **Fundamental Principle of Counting :** If a job can be performed in m different ways and for each such way, second job can be done in n different ways, then the two jobs (in order) can be completed in $m \times n$ ways.
- **Fundamental Principle of Addition :** If there are two events such that they can be performed independently in m and n ways respectively, then either of the two events can be performed in $(m + n)$ ways.
- The number of arrangements (permutations) of n different things taken r at a time is ${}^nP_r$ or $P(n, r)$
- The number of selections (Combinations) of n different things taken r at a time is ${}^nC_r$.
- ${}^nP_r = \dfrac{n!}{(n-r)!}, \quad {}^nC_r = \dfrac{n!}{(n-r)!\,r!}$
- No. of permutations of n things, taken all at a time, of which p are alike of one kind, q are alike of 2^{nd} kind such that $p + q = n$, is $\dfrac{n!}{p!\,q!}$
- $0! = 1$, ${}^nC_o = {}^nC_n = 1$
- ${}^nP_r = r!\ {}^nC_r$

- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_a = {}^nC_b$ if a + b = n or a = b

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Using the digits 1, 2, 3, 4, 5 how many 3 digit numbers (without repeating the digits) can be made?
2. In how many ways 7 pictures can be hanged on 9 pegs?
3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?
4. There are 10 points on a circle. By joining them how many chords can be drawn?
5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?
6. If $\frac{1}{6!} + \frac{1}{8!} = \frac{x}{9!}$ find x
7. If ${}^nP_4 : {}^nP_2 = 12$, find n.
8. How many different words (with or without meaning) can be made using all the vowels at a time?
9. Using 1, 2, 3, 4, 5 how many numbers greater than 10000 can be made? (Repetition not allowed)
10. If ${}^nC_{12} = {}^nC_{13}$ then find the value of ${}^{25}C_n$.
11. In how many ways 4 boys can be choosen from 7 boys to make a committee?
12. How many different words can be formed by using all the letters of word SCHOOL?
13. In how many ways can the letters of the word PENCIL be arranged so that I is always next to L.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

14. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seat?

15. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

16. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

17. In how many ways can one select a cricket team of eleven players from 17 players in which only 6 players can bowl and exactly 5 bowlers are to be included in the team?

18. In how many ways 11 players can be choosen from 16 players so that 2 particular players are always excluded?

19. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

20. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be 182^{nd} word.

21. From a class of 15 students, 10 are to choosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

22. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

23. A polygon has 35 diagnals. Find the number of its sides.

    [**Hint :** Number of diagnals of n sided polygon is given by ${}^nC_2 - n$]

24. How many different products can be obtained by multiplying two or more of the numbers 2, 3, 6, 7, 9?

25. Determine the number of 5 cards combinations out of a pack of 52 cards if atleast 3 out of 5 cards are ace cards?

26. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

LONG ANSWER TYPE QUESTIONS (6 MARKS)

27. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

28. There are 15 points in a plane out of which only 6 are in a straight line, then

    (a) How many different straight lines can be made?

    (b) How many triangles can be made?

29. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that

    (i) No two girls sit together?

    (ii) All the girls never sit together?

30. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

31. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,

    (a) 3 are red and 1 is black.

    (b) All 4 cards are from different suits.

    (c) Atleast 3 are face cards.

    (d) All 4 cards are of the same colour.

32. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

33. How many 5 letter words containing 3 vowels and 2 consonants can be formed using the letters of the word EQUATION so that 3 vowels always occur together?

ANSWERS

1. 60	2. $\frac{9!}{2!}$
3. 100	4. 45

5.	120	6.	513
7.	n = 6	8.	120
9.	120	10.	1
11.	35	12.	360
13.	120	14.	$90 \times {}^{10}P_8$
15.	56	16.	350
17.	2772	18.	364
19.	36	20.	PAANVR
21.	${}^{13}C_{10} + {}^{13}C_8$	22.	5040
23.	10	24.	26
25.	4560	26.	576

27. 420

28. (a) 91 (b) 435

29. (i) $7! \times {}^{8}P_5$ (ii) $12! - 8! \times 5!$

30. 24480

31. ${}^{52}C_4$

(a) ${}^{26}C_1 \times {}^{26}C_3$ (b) $(13)^4$

(c) 9295 (Hint : Face cards : 4J + 4K + 4Q)

(d) $2 \times {}^{26}C_4$

32. 265 (*Hint* : make 3 cases i.e.

(i) All 3 letters are different (ii) 2 are identical 1 different

(iii) All are identical, then form the words.)

33. 1080

CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

- $(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + - - - + {}^nC_n b^n$

  $= \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r, n \in N$

- T_{r+1} = General term

  $= {}^nC_r a^{n-r} b^r \qquad 0 \le r \le n$

- Total number of terms in $(a + b)^n$ is $(n + 1)$

- If n is even, then in the expansion of $(a + b)^n$, middle term is $\left(\frac{n}{2}+1\right)^{th}$ term i.e. $\left(\frac{n+2}{2}\right)^{th}$ term.

- If n is odd, then in the expansion of $(a + b)^n$, middle terms are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$ terms

- In $(a + b)^n$, r^{th} term from the end is same as $(n - r + 2)^{th}$ term from the beginning.

- r^{th} term from the end in $(a + b)^n$

  $= r^{th}$ term from the beginning in $(b + a)^n$

- In $(1 + x)^n$, coefficient of x^r is ${}^nC_r$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Compute $(98)^2$, using binomial theorem.

2. Expand $\left(x - \frac{1}{x}\right)^3$ using binomial theorem.

3. Write number of terms in the expansion of $(1 + 2x + x^2)^{10}$.

4. Write number of terms in $(2a - b)^{15}$

5. Simplify :

$$\frac{{}^nC_r}{{}^nC_{r-1}}$$

6. Write value of

$${}^{2n-1}C_5 + {}^{2n-1}C_6 + {}^{2n}C_7$$

[**Hint :** Use ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$]

7. In the expansion, $(1 + x)^{14}$, write the coefficient of x^{12}

8. Find the sum of the coefficients in $(x + y)^8$

[**Hint :** Put $x = 1$, $y = 1$]

9. If ${}^nC_{n-3} = 120$, find n.

[**Hint :** Express 720 as the product of 3 consecutive positive integers]

10. In $\left(\frac{x}{2} - \frac{2}{x}\right)^8$, write 5^{th} term.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

12. In $\left(3x^2 - \frac{1}{x}\right)^{18}$, which term contains x^{12}?

13. In $\left(2x - \dfrac{1}{x^2}\right)^{15}$, find the term independent of x.

14. Evaluate : $\left(\sqrt{2}+1\right)^5 - \left(\sqrt{2}-1\right)^5$ using binomial theorem.

15. Evaluate $(0.9)^4$ using binomial theorem.

16. Prove that if n is odd, then $a^n + b^n$ is divisible by a + b.

[**Hint :** $a^n = (a + b - b)^n$. Now use binomial theorem]

17. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

18. In $\left(2x - \dfrac{3}{x}\right)^8$, find 7^{th} term from end.

19. In $\left(2x^3 - \dfrac{1}{x^2}\right)^{12}$, find the coefficient of x^{11}.

20. Find the coefficient of x^4 in $(1 - x)^2 (2 + x)^5$ using binomial theorem.

21. Using binomial theorem, show that

$3^{2n+2} - 8n - 9$ is divisible by 8.

[**Hint :** $3^{2n+2} = 9\left(3^2\right)^n = 9(1 + 8)^n$, Now use binomial theorem.]

22. Prove that,

$$\sum_{r=0}^{20} {}^{20}C_{20-r}\,(2-t)^{20-r}\,(t-1)^r = 1$$

23. Find the middle term(s) in $\left(x - \dfrac{1}{x}\right)^8$

24. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that n = 7.

25. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

26. Show that the coefficient of x^5 in the expansion of product $(1 + 2x)^6 (1 - x)^7$ is 171.

27. If the 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n

28. In the expansion of $(1 - x)^{2n-1}$, find the sum of coefficients of x^{r-1} and x^{2n-r}

29. If the coefficients of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$ are equal, then show that ab = 1

ANSWERS

1.	9604	2.	$x^3 - \frac{1}{x^3} - 3x + \frac{3}{x}$
3.	21	4.	16
5.	$\frac{n-r+1}{r}$	6.	${}^{2n+1}C_7$
7.	91	8.	256
9.	n = 10	10.	70
11.	a = 3, b = 2, n = 3	12.	9^{th} term
13.	$-2^{10} \times {}^{15}C_5$	14.	82
15.	0.6561	17.	28
18.	$16128\ x^4$	19.	–101376
20.	10	23.	70
27.	a = 2, x = 1, n = 7	28.	0

CHAPTER - 9

SEQUENCES AND SERIES

KEY POINTS

- A sequence is a function whose domain is the set N of natural numbers.
- A sequence whose range is a subset of R is called a real sequence.
- General A.P. is,

  a, a + d, a + 2d, ...
- $a_n = a + (n - 1)d = n^{th}$ term
- S_n = Sum of first n terms of A.P.

  $= \frac{n}{2}[a + l]$ where l = last term.

  $= \frac{n}{2}[2a + (n - 1)d]$
- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P., ak, bk, ck are also in A.P., $k \neq 0$
- Three numbers in A.P.

  a – d, a, a + d
- Arithmetic mean between a and b is $\frac{a + b}{2}$.
- If $A_1, A_2, A_3, ...A_n$ are inserted between a and b, such that the resulting sequence is A.P. then,

  $$A_n = a + n\left(\frac{b - a}{n + 1}\right)$$

- $S_k - S_{k-1} = a_k$
- $a_m = n$, $a_n = m \Rightarrow a_r = m + n - r$
- $S_m = S_n \Rightarrow S_{m+n} = 0$
- $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$
- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term
- G.P. (Geometrical Progression)

  $a, ar, ar^2, \ldots\ldots\ldots$(General G.P.)

  $a_n = ar^{n-1}$

  $$S_n = \frac{a\left(r^n - 1\right)}{r - 1}, \qquad r \neq 1$$
- Geometric mean between a and b is $\sqrt{ab}$
- Reciprocals of terms in GP always form a G.P.
- If $G_1, G_2, G_3, \ldots\ldots\ldots G_n$ are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $$G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}}, 1 \leq k \leq n$$
- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.
- If each term of a G.P. be raised to some power then the resulting terms are also in G.P.
- Sum of infinite G.P. is possible if $|r| < 1$ and sum is given by $\frac{a}{1-r}$
- $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

- $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If n^{th} term of an A.P. is $6n - 7$ then write its 50^{th} term.
2. If $S_n = 3n^2 + 2n$, then write a_2
3. Which term of the sequence,

   3, 10, 17, ... is 136?
4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.
5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.
6. Which term of the G.P.,

   $2, 1, \frac{1}{2}, \frac{1}{4}$, ... is $\frac{1}{1024}$?
7. If in a G.P., $a_3 + a_5 = 90$ and if $r = 2$ find the first term of the G.P.
8. In G.P. $2, 2\sqrt{2}, 4, \ldots\ldots\ldots, 128\sqrt{2}$, find the 4^{th} term from the end.
9. If the product of 3 consecutive terms of G.P. is 27, find the middle term
10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}, \ldots\ldots$
11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ .... upto infinity.
12. Write the value of $0.\overline{3}$
13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.
14. Write the n^{th} term of the series, $\frac{3}{7.11^2} + \frac{5}{8.12^2} + \frac{7}{9.13^2} + \ldots\ldots$

15. Find a_5 of the series whose n^{th} term is $2^n + 3$.

16. In an infinite G.P., every term is equal to the sum of all terms that follow it. Find r

17. In an A.P.,

    8, 11, 14, ... find $S_n - S_{n-1}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

18. Write the first negative term of the sequence 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, ...

19. Determine the number of terms in A.P. 3, 7, 11, ... 407. Also, find its 11^{th} term from the end.

20. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

21. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

22. Find the sum of the sequence,

$$-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, - - - -, \frac{10}{3}$$

23. If in an A.P. $\frac{a_7}{a_{10}} = \frac{5}{7}$ find $\frac{a_4}{a_7}$

24. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

25. Solve : 1 + 6 + 11 + 16 + ... + x = 148

26. The ratio of the sum of n terms of two A.P.'s is (7n – 1): (3n + 11), find the ratio of their 10^{th} terms.

27. If the I^{st}, 2^{nd} and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

28. If $\frac{b+c-2a}{a}, \frac{c+a-2b}{b}, \frac{a+b-2c}{c}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term]

29. If A = 1 + r^a + r^{2a} + ... up to infinity, then express r in terms of 'a' & 'A'.

30. Insert 5 numbers between 7 and 55 , so that resulting series is A.P.

31. Find the sum of first n terms of the series, 0.7 + 0.77 + 0.777 + ...

32. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

33. Prove that, $0.03\bar{1} = \frac{7}{225}$

[**Hint :** $0.03\bar{1}$ = 0.03 + 0.001 + 0.0001 + ... Now use infinite G.P.]

LONG ANSWER TYPE QUESTIONS (6 MARKS)

34. Prove that the sum of n numbers between a and b such that the resulting series becomes A.P. is $\frac{n(a+b)}{2}$.

35. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of the first square is 15 cm, then find the sum of the areas of all the squares so formed.

36. If a, b, c are in G.P., then prove that

$$\frac{1}{a^2 - b^2} = \frac{1}{b^2 - c^2} - \frac{1}{b^2}$$

[**Hint :** Put b = ar, c = ar^2]

37. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

38. If a is A.M. of b and c and c, G_1, G_2, b are in G.P. then prove that

$$G_1^3 + G_2^3 = 2abc$$

39. Find the sum of the series,

1.3.4 + 5.7.8 + 9.11.12 + ... upto n terms.

40. Evaluate $\sum_{r=1}^{10} (2r-1)^2$

ANSWERS

1. 293
2. 11
3. 20^{th}
4. 0
5. $4n + 5$
6. 12^{th}
7. $\frac{9}{2}$
8. 64
9. 3
10. $20\left(1-\frac{1}{2^{8}}\right)$
11. 5
12. $\frac{1}{3}$
13. $\frac{2}{3}$
14. $\frac{2n+1}{(n+6)(n+10)^{2}}$
15. 35
16. $r = \frac{1}{2}$
17. $3n + 5$
18. $-\frac{1}{4}$
19. 102, 367
20. 33
21. 7999
22. $\frac{63}{2}$
23. $\frac{3}{5}$
24. 11
25. 36
26. 33 : 17
27. $\frac{(b+c-2a)(a+c)}{2(b-a)}$
29. $\left(\frac{A-1}{A}\right)^{1/a}$

30. 15, 23, 31, 39, 47

31. $\frac{7}{81}\left(9n - 1 + 10^{-n}\right)$

32. $\frac{15}{7}\left(2^{n} - 1\right)$

35. 450 cm^2

37. 16, 4

39. $\frac{n(n+1)}{3}\left(48n^{2} - 16n - 14\right)$

40. 1330

CHAPTER - 10

STRAIGHT LINES

- Slope or gradient of a line is defined as $m = \tan\theta$, $(\theta \neq 90°)$, where θ is angle which the line makes with positive direction of x-axis measured in anticlockwise direction, $0 \leq \theta < 180°$
- Slope of x-axis is zero and slope of y-axis is not defined.
- Slope of a line through given points (x_1, y_1) and (x_2, y_2) is given by $\dfrac{y_2 - y_1}{x_2 - x_1}$
- Two lines are parallel to each other if and only if their slopes are equal.
- Two lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$, $1 + m_1 m_2 \neq 0$
- $x = a$ is a line parallel to y-axis at a distance of a units from y-axis. $x = a$ lies on right or left of y-axis according as a is positive or negative.
- $y = b$ is a line parallel to x-axis at a distance of 'b' units from x-axis. $y=b$ lies above or below x-axis, according as b is positive or negative.
- Equation of a line passing through given point (x_1, y_1) and having slope m is given by

  $$y - y_1 = m(x - x_1)$$
- Equation of a line passing through given points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
- Equation of a line having slope m and y-intercept c is given by

  $$y = mx + c$$

- Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

$$\frac{x}{a}+\frac{y}{b}=1$$

- Equation of line in normal form is given by $x \cos\alpha + y \sin\alpha = p$,

  p = Length of perpendicular segment from origin to the line

  α = Angle which the perpendicular segment makes with positive direction of x-axis

- Equation of line in general form is given by $Ax + By + C = 0$, A, B and C are real numbers and at least one of A or B is non zero.

- Distance of a point (x_1, y_1) from line $Ax + By + C = 0$ is given by

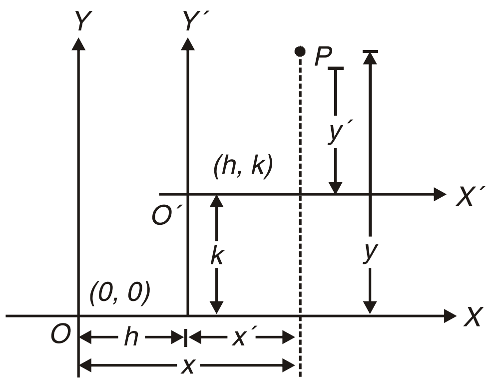
$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

$$d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$$

- Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

  Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane



Let O'X' and O'Y' be drawn parallel to and in same direction as OX and OY respectively. Let coordinates of P referred to new axes O'X' and O'Y' be (x', y') then x = x' + h, y = y' + k

or $\quad x' = x - h,\ y' = y - k$

Thus

(i) The point whose coordinates were (x, y) has now coordinates (x – h, y – k) when origin is shifted to (h, k).

(ii) Coordinates of old origin referred to new axes are (–h, –k).

- Equation of family of lines parallel to Ax + By + C = 0 is given by Ax + By + k = 0, for different real values of k
- Equation of family of lines perpendicular to Ax + By + C = 0 is given by Bx – Ay + k = 0, for different real values of k.
- Equation of family of lines through the intersection of lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is given by $(A_1x + B_1y + C_1) + k\,(A_2x + B_2y + C_2) = 0$, for different real values of k.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.
2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?
3. The mid point of the segment joining (a, b) and (–3, 4b) is (2, 3a + 4). Find a and b.
4. Coordinates of centroid of ΔABC are (1, –1). Vertices of ΔABC are A(–5, 3), B(p, –1) and C(6, q). Find p and q.
5. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1) ?
6. What are the possible slopes of a line which makes equal angle with both axes?
7. Determine x so that slope of line through points (2, 7) and (x, 5) is 2.
8. Show that the points (a, 0), (0, b) and (3a – 2b) are collinear.

9. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).

10. Find k so that the line $2x + ky - 9 = 0$ may be perpendicular to $2x + 3y - 1 = 0$

11. Find the acute angle between lines $x + y = 0$ and $y = 0$

12. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.

13. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

14. On shifting the origin to (p, q), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

15. If the image of the point (3, 8) in the line $px + 3y - 7 = 0$ is the point (–1, –4), then find the value of p.

16. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines $x + 2y = 5$ and $x - 3y + 5 = 0$

17. The line $2x - 3y = 4$ is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

18. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

19. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

20. Find the equation of a line with slope –1 and whose perpendicular distance from the origin is equal to 5.

21. If a vertex of a square is at (1, –1) and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

22. Find the coordinates of the orthocentre of a triangle whose vertices are (–1, 3) (2, –1) and (0, 0). [Orthocentre is the point of concurrency of three altitudes].

23. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

24. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

25. Find points on the line $x + y + 3 = 0$ that are at a distance of $\sqrt{5}$ units from the line $x + 2y + 2 = 0$

26. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point(–5, 0) and is at a perpendicular distance of 3 units from origin.

27. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1,1). Find the equation of other three sides.

28. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

29. Find the equations of the straight lines which cut off intercepts on x–axis twice that on y–axis and are at a unit distance from origin.

30. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

ANSWERS

1. (1, 2)	2. $k = 3$
3. $a = 7, b = 10$	4. $p = 2, q = -5$
5. 3 : 2 (internally)	6. ± 1
7. 1	9. $x + y = 7$

10. $\frac{-4}{3}$

11. $\frac{\pi}{4}$

12. $\frac{2\pi}{3}$

13. (–3, –1)

14. p = –3, q = –3

15. 1

16. $\frac{10}{\sqrt{26}}$

17. (1, –5)

18. c = –4, (2,0), (4, 4)

19. 49 square units

20. $x + y + 5\sqrt{2} = 0$, $x + y - 5\sqrt{2} = 0$

21. 4 square units

22. (–4, –3)

23. $x + 2y = 1$

24. $x + y - 5 = 0$

25. (1, –4), (–9 , 6)

26. $3x - 4y + 15 = 0$

27. $4x + 7y - 11 = 0$, $7x - 4y + 25 = 0$

    $7x - 4y - 3 = 0$

28. $x - 2y + 3 = 0$, $2x + y - 14 = 0$,

    $x - 2y + 13 = 0$, $2x + y - 4 = 0$

    $3x - y - 1 = 0$, $x + 3y - 17 = 0$

29. $x + 2y + \sqrt{5} = 0$, $x + 2y - \sqrt{5} = 0$

30. $x = y$

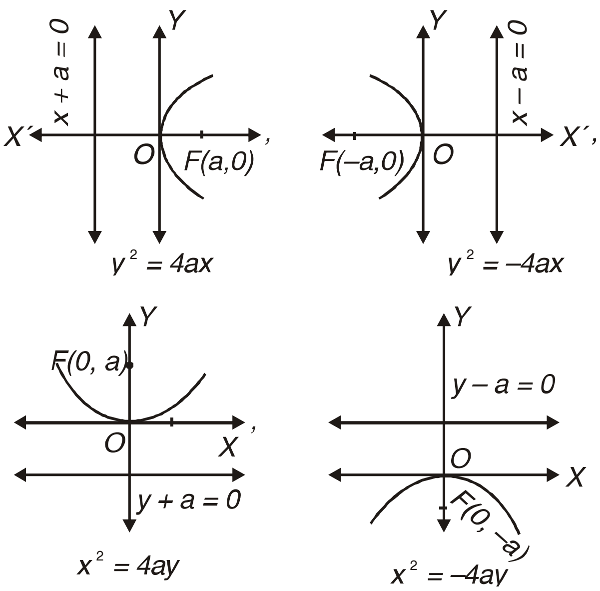
CHAPTER - 11

CONIC SECTIONS

KEY POINTS

- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions
- **Circle :** It is the set of all points in a plane that are equidistant from a fixed point in that plane
- Equation of circle : $(x - h)^2 + (y - k)^2 = r^2$

  Centre (h, k), radius = r
- **Parabola :** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line.

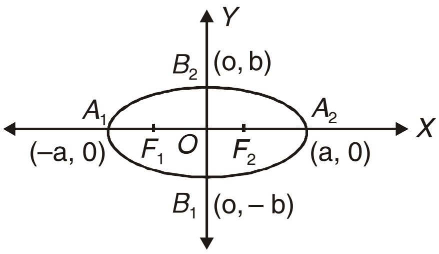


Main facts about the Parabola

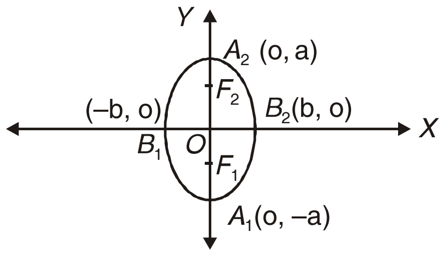
Equation	$y^2 = 4ax$ (a > 0) Right hand	$y^2 = -4ax$ a > 0 Left hand	$x^2 = 4ay$ a > 0 Upwards	$x^2 = -4ay$ a > 0 Downwards
Axis	y = 0	y = 0	x = 0	x = 0
Directrix	x + a = 0	x − a = 0	y + a = 0	y − a = 0
Focus	(a, 0)	(–a, 0)	(0, a)	(0, –a)
Length of latus–rectum	4a	4a	4a	4a
Equation of latus–rectum	x − a = 0	x + a = 0	y − a = 0	y + a = 0

- **Latus Rectum :** A chord through focus perpendicular to axis of parabola is called its latus rectum.

- **Ellipse :** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

a > b > 0, a> b > 0

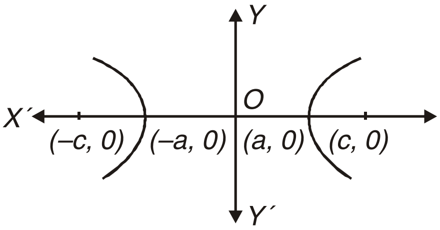
$$c = \sqrt{a^2 - b^2}$$

Main facts about the ellipse

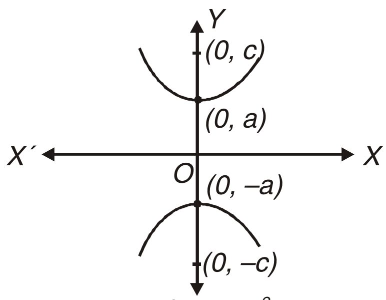
Equation	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a > 0, b > 0	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ a > 0, b > 0
Centre	(0,0)	(0,0)
Major axis lies along	x–axis	y–axis
Length of major axis	2a	2a
Length of minor axis	2b	2b

Foci	(–c, 0), (c, 0)	(0, –c),(0, c)
Vertices	(–a, 0), (a, 0)	(0, –a), (0, a)
Eccentricity e	$\frac{c}{a}$	$\frac{c}{a}$
Length of latus–rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

- **Latus rectum :** Chord through foci perpendicular to major axis called latus rectum.
- **Hyperbola :** It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$c = \sqrt{a^2 + b^2}$$

Main facts about the Hyperbola

Equation	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $a > 0, b > 0$	$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ $a > 0, b > 0$
Centre	(0,0)	(0,0)
Transverse axis lies along	x–axis	y–axis
Length of transverse axis	2a	2a
Length of conjugate axis	2b	2b
Foci	(–c, 0), (c, 0)	(0, –c),(0, c)
Vertices	(–a, 0), (a, 0)	(0, –a), (0, a)
Eecentricity e	$\frac{c}{a}$	$\frac{c}{a}$
Length of latus–rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

- **Latus Rectum :** Chord through foci perpendicular to transverse axis is called latus rectum.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the centre and radius of the circle

$$3x^2 + 3y^2 + 6x - 4y - 1 = 0$$

2. Does $2x^2 + 2y^2 + 3y + 10 = 0$ represent the equation of a circle? Justify.
3. Find equation of circle whose end points of one of its diameter are (–2, 3) and (0, –1).
4. Find the value(s) of p so that the equation $x^2 + y^2 - 2px + 4y - 12 = 0$ may represent a circle of radius 5 units.
5. If parabola $y^2 = px$ passes through point (2, –3), find the length of latus rectum.
6. Find the coordinates of focus, and length of latus rectum of parabola $3y^2 = 8x$.
7. Find the eccentricity of the ellipse

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

8. One end of diameter of a circle $x^2 + y^2 - 6x + 5y - 7 = 0$ is (7, –8). Find the coordinates of other end.
9. Find the equation of the ellipse coordinates of whose foci are (±2, 0) and length of latus rectum is $\frac{10}{3}$.
10. Find the equation of ellipse with eccentricity $\frac{3}{4}$, centre at origin, foci on y-axis and passing through point (6, 4).
11. Find the equation of hyperbola with centre at origin, transverse axis along x-axis, eccentricity $\sqrt{5}$ and sum of lengths of whose axes is 18.

12. Two diameters of a circle are along the lines $x - y - 9 = 0$ and $x - 2y - 7 = 0$ and area of circle is 154 square units, find its equation.

13. Find equation(s) of circle passing through points (1,1), (2,2) and whose radius is 1 unit.

14. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

15. Find the equation of a circle whose centre is at (4, –2) and $3x - 4y + 5 = 0$ is tangent to circle.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

16. Show that the four points (7,5), (6, –2) (–1,–1) and (0,6) are concyclic. [Concylic points : Four or more points which lie on a circle].

ANSWERS

1. $\left(-1, \frac{2}{3}\right)$, $\frac{4}{3}$

2. No

3. $x^2 + y^2 + 2x - 2y - 3 = 0$ or $(x + 1)^2 + (y - 1)^2 = 5$

4. –3, +3

5. $\frac{9}{2}$

6. $\left(\frac{2}{3}, 0\right)$, $\frac{8}{3}$

7. $\frac{4}{5}$

8. (–1, 3)

9. $\frac{x^2}{9} + \frac{y^2}{5} = 1$

10. $16x^2 + 7y^2 = 688$

11. $4x^2 - y^2 = 36$

12. $x^2 + y^2 - 22x - 4y + 76 = 0$

    [**Hint :** Point of intersection of two diameters is the centre]

13. $x^2 + y^2 - 2x - 4y + 4 = 0$, $x^2 + y^2 - 4x - 2y + 4 = 0$

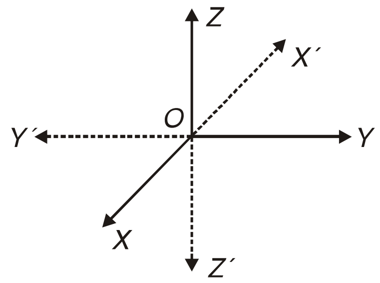
14. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

15. $x^2 + y^2 - 8x + 4y - 5 = 0$

CHAPTER - 12

INTRODUCTION TO THREE DIMENSIONAL COORDINATE GEOMETRY

- Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.



Coordinate axes :	XOX', YOY', ZOZ'
Coordinate planes :	XOY, YOZ, ZOX or
	XY, YX, ZX planes
Octants :	OXYZ, OX'YZ, OXY'Z, OXYZ'
	OX' Y'Z, OXY'Z', OX'YZ', OX'Y'Z'

- Coordinates of a point P are the perpendicular distances of P from three coordinate planes YZ, ZX and XY respectively.

- The distance between the point $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points in space and let R be a point on line segment PQ such that it divides PQ in the ratio $m_1 : m_2$

(i) internally, then the coordinates of R are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

(ii) externally, then coordinates of R are

$$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

- Coordinates of centroid of a triangle whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) are

$$\left(\frac{x_1 + y_1 + z_1}{3}, \frac{x_2 + y_2 + z_2}{3}, \frac{x_3 + y_3 + z_3}{3}\right)$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find image of (–2, 3, 5) in YZ plane.
2. Name the octant in which (–5, 4, –3) lies.
3. Find the distance of the point P(4, –3, 5) from XY plane.
4. Find the distance of point P(3, –2,1) from z–axis.
5. Write coordinates of foot of perpendicular from (3, 7, 9) on x axis.
6. Find the distance between points (2, 3, 4) and (–1, 3, –2).

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

7. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.
8. Find the point on y–axis which is equidistant from the point (3, 1, 2) and (5, 5, 2).
9. Find the coordinates of a point equidistant from four points (0,0,0), (2,0,0), (0,3,0) and (0,0,8), if it exists.
10. The centroid of $\triangle$ABC is at (1,1,1). If coordinates of A and B are (3,–5,7) and (–1, 7, –6) respectively, find coordinates of point C.

11. If the extremities (end points) of a diagonal of a square are (1,–2,3) and (2,–3,5) then find the length of the side of square.

12. Determine the point in XY plane which is equidistant from the points A (1, –1, 0) B(2, 1, 2) and C(3, 2, –1).

13. If the points A(1, 0, –6), B(–5, 9, 6) and C(–3, p, q) are collinear, find the value of p and q.

14. Show that the points A(3,3,3), B(0,6,3), C(1,7,7) and D(4,4,7) are the vertices of a square.

15. The coordinates of mid point of sides of Δ ABC are (–2, 3, 5), (4, –1, 7) and (6, 5, 3). Find the coordinates of vertices of ΔABC.

16. Find the coordinates of the point P which is five-sixth of the way from A(2, 3, –4) to B(8, –3, 2).

ANSWERS

1.	(2,3,5)	2.	OX' YZ'
3.	5 units	4.	$\sqrt{13}$ units
5.	(3,0,0)	6.	$\sqrt{45}$ units
8.	(0, 5, 0)	9.	$\left(1, \frac{3}{2}, 4\right)$
10.	(1,1,2)	11.	$\sqrt{3}$ units
12.	$\left(\frac{3}{2}, 1, 0\right)$	13.	p = 6, q = 2
15.	$\left[\begin{array}{l}(0, 9, 1), \\ (-4, -3, 9), \\ (12, 1, 5)\end{array}\right.$	16.	(7,–2,1)

CHAPTER - 13

LIMITS AND DERIVATIVES

KEY POINTS

- $\lim_{x \to c} f(x) = l$ if and only if

  $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

- $\lim_{x \to c} \alpha = \alpha$, where α is a fixed real number.
- $\lim_{x \to c} x^n = c^n$, for all $n \in N$
- $\lim_{x \to c} f(x) = f(c)$, where f(x) is a real polynomial in x.

Algebra of limits

Let f, g be two functions such that $\lim_{x \to c} f(x) = l$ and $\lim_{x \to c} g(x) = m$, then

- $\lim_{x \to c} [\alpha\ f(x)] = \alpha \lim_{x \to c} f(x)$

  $= \alpha\ l$ for all $\alpha \in R$

- $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = l \pm m$
- $\lim_{x \to c} [f(x).g(x)] = \lim_{x \to c} f(x). \lim_{x \to c} g(x) = lm$
- $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \dfrac{l}{m}$, $m \neq 0$ $g(x) \neq 0$

- $\lim_{x \to c} \frac{1}{f(x)} = \frac{1}{\lim_{x \to c} f(x)} = \frac{1}{l}$ provided $l \neq 0$ $f(x) \neq 0$

- $\lim_{x \to c} [(f(x)]^n = \left[\left(\lim_{x \to c} f(x)\right)\right]^n = l^n$, for all $n \in N$

Some important theorems on limits

- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(-x)$

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$ where x is measured in radians.

- $\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right) = 1$

- $\lim_{x \to 0} \frac{\tan x}{x} = 1 \left[\text{Note that } \lim_{x \to 0} \frac{\cos x}{x} \neq 1\right]$

- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

- $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

- $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$

- $\lim_{x \to 0} (1 + x)^{1/x} = e$

Derivative of a function at any point

- A function f is said to have a derivative at any point x if it is defined in some neighbourhood of the point x and $\lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$ exists.

  The value of this limit is called the derivative of f at any point x and is denoted by f'(x) i.e.

  $$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Algebra of derivatives :

- $\dfrac{d}{dx}\left(cf(x)\right) = c.\dfrac{d}{dx}\left(f(x)\right)$ where c is a constant
- $\dfrac{d}{dx}\left(f(x) \pm g(x)\right) = \dfrac{d}{dx}\left(f(x)\right) \pm \dfrac{d}{dx}\left(g(x)\right)$
- $\dfrac{d}{dx}\left(f(x).g(x)\right) = f(x).\dfrac{d}{dx}\left(g(x)\right) + g(x)\dfrac{d}{dx}\left(f(x)\right)$
- $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{g(x)\dfrac{d}{dx}\left(f(x)\right) - f(x).\dfrac{d}{dx}\left(g(x)\right)}{\left(g(x)\right)^2}$
- If y = f(x) is a given curve then slope of the tangent to the curve at the point (h, k) is given by $\left.\dfrac{dy}{dx}\right]_{(h,k)}$ and is denoted by 'm'.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Evaluate the following Limits :

1. $\lim\limits_{x \to 3} \dfrac{\sqrt{2x + 3}}{x + 3}$

2. $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x}$

3. $\lim_{x \to 0} \frac{\tan^2 3x}{x^2}$

4. $\lim_{x \to 2} \left(x^2 - 5x + 1\right)$

Differentiate the following functions with respect to x :

5. $\frac{x}{2} + \frac{2}{x}$

6. $x^2 \tan x$

7. $\frac{x}{\sin x}$

8. $\log_x x$

9. 2^x

10. If $f(x) = x^2 - 5x + 7$, find $f'(3)$

11. If $y = \sin x + \tan x$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{3}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

12. If $f(x) = \begin{cases} 5x - 4, & 0 < x \le 1, \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$ show that $\lim_{x \to 1} f(x)$ exists.

13. If $f(x) = \begin{cases} \frac{x - |x|}{x}, & x \ne 0, \\ 2, & x = 0 \end{cases}$ show that $\lim_{x \to 0} f(x)$ does not exist.

14. Let f(x) be a function defined by

$$f(x) = \begin{cases} 4x - 5, & \text{If } x \le 2, \\ x - \lambda, & \text{If } x > 2, \end{cases}$$ Find λ, if $\lim_{x \to 2} f(x)$ exists

Evaluate the following Limits :

15. $\lim\limits_{x \to 3} \dfrac{x^2 - 4x + 3}{x^2 - 2x - 3}$

16. $\lim\limits_{x \to 0} \dfrac{\sqrt{2 + x} - \sqrt{2}}{x}$

17. $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{1 + x} - \sqrt{1 - x}}$

18. $\lim\limits_{x \to a} \dfrac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}}$

19. $\lim\limits_{x \to a} \dfrac{(x + 2)^{\frac{5}{2}} - (a + 2)^{\frac{5}{2}}}{x - a}$

20. $\lim\limits_{x \to 0} \dfrac{1 - \cos 2mx}{1 - \cos 2nx}$

21. $\lim\limits_{x \to 0} \dfrac{\tan x - \sin x}{x^3}$

22. $\lim\limits_{x \to 0} \dfrac{x \tan x}{1 - \cos x}$

23. $\lim\limits_{x \to a} \dfrac{\sin x - \sin a}{x - a}$

24. $\lim\limits_{x \to a} \dfrac{\cos x - \cos a}{\cot x - \cot a}$

25. $\lim\limits_{x \to \pi} \dfrac{1 + \sec^3 x}{\tan^2 x}$

26. $\lim\limits_{x \to 0} \dfrac{e^x - e^{-x}}{x}$

27. $\lim_{x \to 1} \frac{x - 1}{\log_e x}$

28. $\lim_{x \to e} \frac{\log x - 1}{x - e}$

29. $\lim_{x \to 4} \frac{3 - \sqrt{5 + x}}{1 - \sqrt{5 - x}}$

30. $\lim_{x \to a} \frac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}}$

31. $\lim_{x \to 0} \frac{\sin(2 + x) - \sin(2 - x)}{x}$

Differentiate the following functions with respect to x from first principle:

32. $\sqrt{2x + 3}$

33. $\frac{x^2 + 1}{x}$

34. e^x

35. $\log x$

36. cosec x

37. cot x

38. a^x

Differentiate the following functions with respect to x :

39. $\frac{(3x + 1)\,(2\sqrt{x} - 1)}{\sqrt{x}}$

40. $\left(x - \frac{1}{\sqrt{x}}\right)^3$

41. $\left(x - \frac{1}{x}\right)\left(x^2 - \frac{1}{x^2}\right)$

42. $\frac{\sin x - x \cos x}{x \sin x + \cos x}$

43. $x^3 e^x \sin x$

44. $x^n \log_a x\ e^x$

45. $\dfrac{e^x + \log x}{\sin x}$

46. $\dfrac{1 + \log x}{1 - \log x}$

47. $e^x \sin x + x^n \cos x$

48. If $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$, prove that $2x\dfrac{dy}{dx} + y = 2\sqrt{x}$

49. If $y = \sqrt{\dfrac{1 - \cos 2x}{1 + \cos 2x}}$ find $\dfrac{dy}{dx}$

50. If $y = \sqrt{\dfrac{x}{a}} + \sqrt{\dfrac{a}{x}}$, prove that

$(2xy)\dfrac{dy}{dx} = \dfrac{x}{a} - \dfrac{a}{x}$

51. For the curve $f(x)=(x^2 + 6x-5)(1-x)$, find the slope of the tangent at $x = 3$.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

Differentiate the following functions with respect to x from first principle:

52. $\dfrac{\cos x}{x}$

53. $x^2 \sin x$

Evaluate the following limits :

54. $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

55. $\lim\limits_{x \to 0} \dfrac{\cos 2x - \cos 3x}{\cos 4x - 1}$

ANSWERS

1.	$\frac{1}{2}$	2.	3
3.	9	4.	–5
5.	$\frac{1}{2}-\frac{2}{x^2}$	6.	$2x\tan x + x^2 \sec^2 x$
7.	$\operatorname{cosec}x - x\cot x \operatorname{cosec}x$	8.	0
9.	$2^x \log_e 2$	10.	1
11.	$\frac{9}{2}$	14.	$\lambda = -1$
15.	$\frac{1}{2}$	16.	$\frac{1}{2\sqrt{2}}$
17.	1	18.	$\frac{5}{2}a^{\frac{3}{7}}$
19.	$\frac{5}{2}(a+2)^{\frac{3}{2}}$	20.	$\frac{m^2}{n^2}$
21.	$\frac{1}{2}$	22.	2
23.	$\cos a$	24.	$\sin^3 a$
25.	$-\frac{3}{2}$	26.	2
27.	1	28.	$\frac{1}{e}$
29.	$-\frac{1}{3}$	30.	$\frac{2}{3\sqrt{3}}$
31.	$2\cos 2$	32.	$\frac{1}{\sqrt{2x+3}}$

33. $\dfrac{x^2 - 1}{x^2}$

34. e^x

35. $\dfrac{1}{x}$

36. $-$ cosecx. cotx

37. $-\operatorname{cosec}^2 x$

38. $a^x \log_e a$

39. $6 - \dfrac{3}{2} x^{-\frac{1}{2}} + \dfrac{1}{2} x^{-\frac{3}{2}}$

40. $3x^2 + \dfrac{3}{2x^{5/2}} - \dfrac{9}{2}\sqrt{x}$

41. $3x^2 + \dfrac{1}{x^2} - 1 - \dfrac{3}{x^4}$

42. $\dfrac{x^2}{(x \sin x + \cos x)^2}$

43. $x^2 e^x$ (3 sinx + x sinx + x cosx)

44. $e^x x^{n-1}\{n \log_a x + \log a + x \log_a x\}$

45. $\dfrac{\left(e^x + \frac{1}{x}\right)\sin x - \left(e^x + \log x\right)\cos x}{\sin^2 x}$

46. $\dfrac{2}{x(1 - \log x)^2}$

47. $e^x \left(1 + \dfrac{1}{x} + x + \log x\right)$

49. $\sec^2 x$

51. -46

52. $\dfrac{-(x \sin x + \cos x)}{x^2}$

53. $2x \sin x + x^2 \cos x$

54. -3

55. $-\dfrac{5}{16}$

CHAPTER – 14

MATHEMATICAL REASONING

KEY POINTS

- A sentence is called a statement if it is either true or false but not both.
- The denial of a statement p is called its negative and is written as ~p and read as not p.
- Compound statement is made up of two or more simple statements. These simple statements are called component statements.
- 'And', 'or', 'If–then', 'only if' 'If and only if' etc. are connecting words, which are used to form a compound statement.
- Compound statement with **'And'** is
  - (a) true if all its component statements are true
  - (b) false if any of its component statement is false
- Compound statement with **'Or'** is
  - (a) true when at least one component statement is true
  - (b) false when any of its component statement is false.
- A statement with **"If p then q"** can be rewritten as
  - (a) p implies q
  - (b) p is sufficient condition for q
  - (c) q is necessary condition for p
  - (d) p only if q
  - (e) (~q) implies (~p)

- Contrapositive of the statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- Converse of the statement $p \Rightarrow q$ is the statement $q \Rightarrow p$
- "For all", "For every" are called universal quantifiers
- A statement is called valid or invalid according as it is true or false.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Identify which of the following are statements (Q. No 1 to 7)

1. Prime factors of 6 are 2 and 3.
2. $x^2 + 6x + 3 = 0$
3. The earth is a planet.
4. There is no rain without clouds.
5. All complex numbers are real numbers.
6. Tomorrow is a holiday.
7. Answer this question.

Write negation of the following statements (Q. No 8 to 12)

8. All men are mortal.
9. π is not a rational number.
10. Every one in Spain speaks Spanish.
11. Zero is a positive number.

Write the component statements of the following compound statements

12. 7 is both odd and prime number.
13. All integers are positive or negative.
14. 36 is a multiple of 4, 6 and 12.
15. Jack and Jill went up the hill.

Identify the type 'Or' (Inclusive or Exclusive) used in the following statements (Q. No. 16 to 19)

16. Students can take French or Spanish as their third language.

17. To enter in a country you need a visa or citizenship card.

18. $\sqrt{2}$ is a rational number or an irrational number.

19. 125 is a multiple of 5 or 8.

Which of the following statements are true or false. Give Reason. (Question No. 20 to 23)

20. 48 is a multiple of 6, 7 and 8

21. $\pi > 2$ and $\pi < 3$.

22. Earth is flat or it revolves around the moon.

23. $\sqrt{2}$ is a rational number or an irrational number.

Identify the quantifiers in the following statements (Q. No. 24 to 26)

24. For every integer p, $\sqrt{p}$ is a real number.

25. There exists a capital for every country in the world.

26. There exists a number which is equal to its square.

Write the converse of the following statements (Q. No. 27 to 30)

27. If a number x is even then x^2 is also even.

28. If $3 \times 7 = 21$ then $3 + 7 = 10$

29. If n is a prime number then n is odd.

30. Some thing is cold implies that it has low temperature.

Write contrapositive of the following statements (Q. No. 31 and 32)

31. If $5 > 7$ then $6 > 7$.

32. x is even number implies that x^2 is divisible by 4.

33. Check the validity of the statement 'An integer x is even if and only if x^2 is even.

ANSWERS

1. Statement
2. Not a statement
3. Statement
4. Statement
5. Statement
6. Not a Statement
7. Not a statement
8. All men are not mortal
9. π is a rational number.
10. Everyone in Spain doesn't speak Spanish.
11. Zero is not a positive number.
12. 7 is an odd number. 7 is a prime number.
13. All integer are positive. All integers are negative.
14. 36 is a multiple of 4.

    36 is a multiple of 6.

    36 is a multiple of 12.
15. Jack went up the hill.

    Jill went up the hill.
16. Exclusive
17. Inclusive
18. Exclusive
19. Exclusive
20. False, 48 is not a multiple of 7
21. False, π lies between 3 and 4
22. False
23. True
24. For every
25. For every, there exists
26. There exists
27. If x^2 is even then x is even

28. If $3 + 7 = 10$ then $3 \times 7 = 21$

29. If n is odd then n a prime number.

30. If some thing has low temperature then it is cold.

31. If $6 \leq 7$ then $5 \leq 7$

32. If x^2 is not divisible by 4 then x is not even.

33. Valid

CHAPTER - 15

STATISTICS

- Range = Largest observation – smallest observation.
- Mean deviation for ungrouped data or raw data

$$M.D.(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}$$

$$M.D.(M) = \frac{\sum |x_i - M|}{n}, \qquad M = \text{Median}$$

- Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution).

$$M.D.(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$M.D.(M) = \frac{\sum f_i |x_i - M|}{N}$$

where $N = \Sigma f_i$

- Standard deviation 'σ' is positive square root of variance.

$$\sigma = \sqrt{\text{Variance}}$$

- Variance σ^2 and standard deviation (SD) σ for ungrouped data

$$\sigma^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$$

$$SD = \sigma = \sqrt{\frac{1}{n}\sum (x_i - \bar{x})^2}$$

- Standard deviation of a discrete frequency distribution

$$\sigma = \sqrt{\frac{1}{N}\sum f_i \left(x_i - \bar{x}\right)^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^{\,2} - \left(\sum f_i x_i\right)^2}$$

- Standard deviation of a continuous frequency distribution

$$\sigma = \sqrt{\frac{1}{N}\sum f_i \left(x_i - \bar{x}\right)^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^{\,2} - \left(\sum f_i x_i\right)^2}$$

  where x_i are the midpoints of the classes.

- Short cut method to find variance and standard deviation

$$\sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^{\,2} - \left(\sum f_i y_i\right)^2\right]$$

$$\sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^{\,2} - \left(\sum f_i y_i\right)^2}$$

  where $y_i = \dfrac{x_i - A}{h}$

- Coefficient of variation $(\text{C.V}) = \frac{\sigma}{\bar{x}} \times 100,\ \bar{x} \neq 0$

- If each observation is multiplied by a positive constant k then variance of the resulting observations becomes k^2 times of the original value and standard deviation becomes k times of the original value.

- If each observation is increased by k, where k is positive or negative, the variance and standard deviation remains same.

- Standard deviation is independent of choice of origin but depends on the scale of measurement.

- The series having higher coefficient of variation is called more variable than the other. While the series having lesser coefficient of variation is called more consistent or more stable. For series with equal means the series with lesser standard deviation is more stable.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Define dispersion.

2. What is the range of the data

   7, 12, 18, 22, 11, 6, 26?

3. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what are the new mean and the new variance?

4. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what are the new mean and the new standard deviation?

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Calculate the mean deviation about mean for the following data

5. 7, 6, 10, 12, 13, 4, 8, 20

6. 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Calculate the mean deviation about median for the following data

7. 40, 42, 44, 46, 48

8. 22, 24, 30, 27, 29, 35, 25, 28, 41, 42

Calculate the mean, variance and standard deviation of the following data

9. 6, 7, 10, 12, 13, 4, 8 12

10. 15, 22, 27, 11, 9, 21, 14, 9

11. Coefficients of variation of two distribution are 60 and 80 and their standard deviations are 21 and 36. What are their means?

12. On study of the weights of boys and girls in an institution following data are obtained.

	Boys	*Girls*
Number	100	50
Mean	60 kgs.	45 kgs.
Variance	9	4

Whose weight is more variable?

13. Mean of 5 observations is 6 and their standard deviation is 2. If the three observations are 5, 7 and 9 then find the other two observations.

14. Calculate the possible values of x if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.

15. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

Calculate the mean deviation about mean for the following data.

16.

Size	2	4	6	8	10	12	14	16
Frequency	2	2	4	5	3	2	1	1

17.

Marks	10	30	50	70	90
Frequency	4	24	28	16	8

Calculate the mean deviation about median for the following data

18.

Marks	10	11	12	13	14
Frequency	3	12	18	12	5

19.

x	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	4

20. Calculate the mean and standard deviation for the following data

Wages in Rs/hour	45	50	55	60	65	70	75	80
Number of Workers	3	5	8	7	9	7	4	7

21. Calculate the standard deviation for the following data

Weight	18	19	20	21	22	23	24	25	26	27
Number of students	3	7	11	14	18	17	13	8	5	4

Calculate the mean deviation about mean for the following data

22.

Classes	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	2	3	8	14	8	3	2

23.

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	5	8	15	16	6

24. Find the mean deviation about the median

Weight (in kg.)	30-40	40-50	50-60	60-70	70-80	80-90
Number of Persons	8	10	10	16	4	2

25. Calculate the mean deviation about median for the following distribution

Classes	0-10	10-20	20-30	30-40	40-50
Frequency	5	10	20	5	10

26. Find the mean and standard deviation for the following

C.I.	25-35	35-45	45-55	55-65	65-75	75-85	85-95
Frequency	21	12	30	45	50	37	5

27. Find the mean and standard deviation of the following data

Ages under (in years)	10	20	30	40	50	60	70	80
Number of members	15	30	53	75	100	110	115	125

28. Find the coefficient of variation of the following data

Classes	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	5	12	15	20	18	10	6	4

29. Which group of students is more stable- Group A or Group B?

Classes	5-15	15-25	25-35	35-45	45-55	55-65	65-75
Number in Group A	4	12	22	30	23	5	4
Number in Group B	5	15	20	33	15	10	2

30. For a group of 200 candidates, the mean and standard deviation of scores were found to be 40 and 15 respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53 respectively. Find the correct mean and correct standard deviation.

ANSWERS

1. Dispersion is scattering of the observations around the central value of the observations.

2. 20
3. 48, 256
4. 35, 4
5. 3.75
6. 2.33
7. 2.4
8. 4.7
9. 9, 9.25, 3.04
10. 16. 38.68. 6.22
11. 35, 45
12. Boys weight
13. 3 and 6
14. 3, 7/3
15. 6.5, 2.5
16. 2.8
17. 16
18. 0.8
19. 10.1

20. 63.6, 10.35

21. 2.1807

22. 10

23. 9.44

24. 11.44

25. 9

26. 61.1, 15.93

27. 35.16, 19.76

28. 31.24

29. Group A

30. 39.955, 14.9.

CHAPTER - 16

PROBABILITY

- **Random Experiment :** If an experiment has more than one possible out come and it is not possible to predict the outcome in advance then experiment is called random experiment.

- **Sample Space :** The collection of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space(set) is called a sample point.

- **Some examples of random experiments and their sample spaces**

  (i) A coin is tossed

  S = {H, T}, n(S) = 2

  Where n(S) is the number of elements in the sample space S.

  (ii) A die is thrown

  S = { 1, 2, 3, 4, 5, 6], n(S) = 6

  (iii) A card is drawn from a pack of 52 cards

  n (S) = 52.

  (iv) Two coins are tossed

  S = {HH, HT, TH, TT}, n(S) = 4.

  (v) Two dice are thrown

$$S = \begin{Bmatrix} 11, 12, 13, 14, 15, 16, \\ 21, 22, -----, 26, \\ \vdots \\ 61, 62, -----, 66 \end{Bmatrix}$$

$$n(S) = 36$$

(vi) Two cards are drawn from a well shuffled pack of 52 cards

(a) with replacement $n(S) = 52 \times 52$

(b) without replacement $n(S) = {}^{52}C_2$

- **Event :** A subset of the sample space associated with a random experiment is called an event.
- **Simple Event :** Simple event is a single possible outcome of an experiment.
- **Compound Event :** Compound event is the joint occurrence of two or more simple events.
- **Sure Event :** If event is same as the sample space of the experiment, then event is called sure event.
- **Impossible Event :** Let S be the sample space of the experiment, $\phi \subset S$, ϕ is an event called impossible event.
- **Exhaustive and Mutually Exclusive Events :** Events E_1, E_2, E_3 ------ E_n are mutually exclusive and exhaustive if

  $E_1 \cup E_2 \cup E_3 \cup$ ------- $\cup E_n = S$ and $E_i \cap E_j = \phi$ for all $i \neq j$
- **Probability of an Event :** For a finite sample space S with equally likely outcomes, probability of an event A is $P(A) = \frac{n(A)}{n(S)}$, where n(A) is number of elements in A and n(S) is number of elements in set S and $0 \leq P(A) \leq 1$.
- (a) If A and B are any two events then

  $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  $= P(A) + P(B) - P(A \text{ and } B)$

  (b) If A and B are mutually exclusive events then

  $P(A \cup B) = P(A) + P(B)$

  (c) $P(A) + P(\overline{A}) = 1$

  or $P(A) + P(\text{not } A) = 1$

(d) P (Sure event) = 1

(e) P (impossible event) = 0

- $P(A - B) = P(A) - P(A \cap B) = P(A \cap \overline{B})$
- If $S = \{w_1, w_2, \ldots\ldots\ldots, w_n\}$ then
  (i) $0 \leq P(w_i) \leq 1$ for each $w_i \in S$
  (ii) $P(w_1) + P(w_2) + \ldots\ldots\ldots + P(w_n) = 1$
  (iii) $P(A) = \Sigma P(w_i)$ for any event A containing elementary events w_i.
- $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B)$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Describe the Sample Space for the following experiments (Q. No. 1 to 4)

1. A coin is tossed twice and number of heads is recorded.
2. A card is drawn from a deck of playing cards and its colour is noted.
3. A coin is tossed repeatedly until a tail comes up for the first time.
4. A coin is tossed. If it shows head we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.
5. Write an example of an impossible event.
6. Write an example of a sure event.
7. Three coins are tossed. Write three events which are mutually exclusive and exhaustive.
8. A coin is tossed n times. What is the number of elements in its sample space?

   If E, F and G are the subsets representing the events of a sample space S. What are the sets representing the following events? (Q No 9 to 12).

9. Out of three events atleast two events occur.
10. Out of three events only one occurs.

11. Out of three events only E occurs.

12. Out of three events exactly two events occur.

13. If probability of event A is 1 then what is the type of event 'not A'?

14. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?

15. What is the probability that a given two digit number is divisible by 15?

16. If $P(A \cup B) = P(A) + P(B)$, then what can be said about the events A and B?

17. If A and B are mutually exclusive events then what is the probability of $A \cap B$?

18. If A and B are mutually exclusive and exhaustive events then what is the probability of $A \cup B$?

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

19. The letters of the word EQUATION are arranged in a row. Find the probability that

    (i) all vowels are together

    (ii) the arrangement starts with a vowel and ends with a consonant.

20. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.

21. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

22. Find the probability of almost two tails or atleast two heads in a toss of three coins.

23. A, B and C are events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$ $P(A \cap C) = 0.28$ and $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$ then prove that $P(B \cap C)$ lies in the interval [0.23, 0.48]

[***Hint :*** $P(A \cup B \ A \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$].

24. For a post three persons A, B and C appear in the interview. The probability of A being selected is twice that of B and the probability of B being selected is twice that of C. The post is filled. What are the probabilities of A, B and C being selected?

25. A and B are two candidates seeking admission in college. The probability that A is selected is 0.5 and the probability that both A and B are selected is utmost 0.3. Show that the probability of B being selected is utmost 0.8.

26. S = {1, 2, 3, -----, 30}, A = {x : x is multiple of 7} B = { x : x is multiple of 5}, C = {x : x is a multiple of 3}. If x is a member of S chosen at random find the probability that

    (i) $x \in A \cup B$

    (ii) $x \in B \cap C$

    (iii) $x \in A \cap C'$

27. A number of 4 different digits is formed by using 1, 2, 3, 4, 5, 6, 7. Find the probability that it is divisible by 5.

28. A bag contains 5 red, 4 blue and an unknown number of m green balls. Two balls are drawn. If probability of both being green is $\frac{1}{7}$ find m.

29. A ball is drawn from a bag containing 20 balls numbered 1 to 20. Find the probability that the ball bears a number divisible by 5 or 7?

30. What is the probability that a leap year selected at random will contain 53 Tuesdays?

ANSWERS

1. {0, 1, 2}
2. {Red, Black}
3. {T, HT, HHT, HHHT.........}
4. {HR_1, HR_2, HB_1, HB_2, HB_3, TH, TT}
5. Getting a number 8 when a die is rolled

6. Getting a number less then 7 when a die is rolled

7. A = {HHH, HHT, HTH, THH}

   B = {HTT, THT, HTT}

   C = {TTT}

8. 2^n

9. $(E \cap F \cap G) \cup (E' \cap F \cap G) \cup (E \cap F' \cap G) \cap (E \cap F \cap G')$

10. $(E \cap F' \cap G) \cup (E' \cap F \cap G') \cup (E' \cap F' \cap G)$

11. $(E \cap F' \cap G')$

12. $(E \cap F \cap G') \cup (E \cap F' \cap G) \cup (E' \cap F \cap G)$

13. Impossible event

14. $\frac{8}{21}$

15. $\frac{1}{15}$

16. Mutually exclusive events.

17. 0

18. 1

19. (i) $\frac{1}{14}$ (ii) $\frac{15}{56}$

20. 3

21. $\frac{23}{28}$

22. $\frac{7}{8}$

23. $0.23 \leq P(B) \leq 0.48$

24. $\frac{4}{7}, \frac{2}{7}, \frac{1}{7}$

26. (i) $\frac{1}{3}$, (ii) $\frac{1}{15}$, (iii) $\frac{1}{10}$

27. $\frac{1}{7}$

28. 6

29. $\frac{3}{10}$.

30. $\frac{2}{7}$.

MODEL TEST PAPER – I

Time : 3 hours **Maximum Marks : 100**

General Instructions :

(i) All questions are compulsory.

(ii) Q. 1 to Q. 10 of Section A are of 1 mark each.

(iii) Q. 11 to Q. 22 of Section B are of 4 marks each.

(iv) Q. 23 to Q. 29 of Section C are of 6 marks each.

(v) There is no overall choice. However an internal choice has been provided in some questions.

SECTION A

1. A = {1, 2, 3, 4, 5, 6}, B = {2, 3, 5, 7, 9}

   U = {1, 2, 3, 4, ...10}, Write $(A - B)'$

2. Express $(1 - 2i)^{-2}$ in the standard form a + ib.

3. Find 20th term from end of the A.P. 3, 7, 11, ... 407.

4. Evaluate $5^2 + 6^2 + 7^2 + ... + 20^2$

5. Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$

6. Evaluate $\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x}$

7. A bag contains 9 red, 7 white and 4 black balls. If two balls are drawn at random, find the probability that both balls are red.

8. What is the probability that an ordinary year has 53 Sundays?

9. Write the contrapositive of the following statement :

   "it two lines are parallel, then they do not intersect in the same plane."

10. Check the validity of the compound statement "80 is a multiple of 5 and 4."

SECTION B

11. Find the derivative of $\frac{\sin x}{x}$ with respect to x from first principle.

OR

Find the derivative of $\frac{\sin x - x\cos x}{x\sin x + \cos x}$ with respect to x.

12. Two students Ajay and Aman appeared in an interview. The probability that Ajay will qualify the interview is 0.16 and that Aman will quality the interview is 0.12. The probability that both will qualify is 0.04. Find the probability that—

    (a) Both Ajay and Aman will not qualify.

    (b) Only Aman qualifies.

13. Find domain and range of the real function $f(x) = \frac{3}{1 - x^2}$

14. Let R be a relation in set A = {1, 2, 3, 4, 5, 6, 7} defined as R = {(a, b): a divides b, a ≠ b}. Write R in Roster form and hence write its domain and range.

OR

Draw graph of f(x) = 2 + |x − 1|.

15. Solve : $\sin^2 x - \cos x = \frac{1}{4}$.

16. Prove that $\cos 2\theta . \cos\frac{\theta}{2} - \cos 3\theta \cos\frac{9\theta}{2} = \sin 5\theta \sin\frac{5\theta}{2}$.

17. If x and y are any two distinct integers, then prove by mathematical induction that $x^n - y^n$ is divisible by $(x - y)$ $\forall n \in N$.

18. If $x + iy = (a + ib)^{1/3}$, then show that $\frac{a}{x} + \frac{b}{y} = 4\left(x^2 - y^2\right)$

OR

Find the square roots of the complex number 7 – 24i

19. Find the equation of the circle passing through points (1, –2) and (4, –3) and has its centre on the line $3x + 4y = 7$.

OR

The foci of a hyperbola coincide with of the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. Find the equation of the hyperbola, if its eccentricity is 2.

20. Find the coordinates of the point, at which yz plane divides the line segment joining points (4, 8, 10) and (6, 10, –8).

21. How many words can be made from the letters of the word 'Mathematics', in which all vowels are never together.

22. From a class of 20 students, 8 are to be chosen for an excusion party. There are two students who decide that either both of them will join or none of the two will join. In how many ways can they be choosen?

SECTION C

23. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had taken

    (i) atleast one of the three subjects,

    (ii) only one of the three subjects.

24. Prove that $\cos^3 A + \cos^3\left(\frac{2\pi}{3} + A\right) + \cos^3\left(\frac{4\pi}{3} + A\right) = \frac{3}{4}\cos 3A.$

25. Solve the following system of inequations graphically

$$x + 2y \le 40,\ 3x + y \ge 30,\ 4x + 3y \ge 60,\ x \ge 0,\ y \ge 0$$

OR

A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

26. Find n, it the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left[\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right]^n$ is $\sqrt{6} : 1$.

27. The sum of two numbers is 6 times their geometric mean. Show that the numbers are in the ratio $\left(3 + 2\sqrt{2}\right) : \left(3 - 2\sqrt{2}\right)$.

28. Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

29. Calculate mean and standard deviation for the following data

Age	***Number of persons***
20 – 30	3
30 – 40	51
40 – 50	122
50 – 60	141
60 – 70	130
70 – 80	51
80 – 90	2

OR

The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking it was found that an observation 12 was misread as 8. Calculate correct mean and correct standard deviation.

MODEL TEST PAPER – I

Time : 3 hours **Maximum Marks : 100**

SOLUTIONS AND MARKING SCHEME

SECTION A

Note : For 1 mark questions in Section A, full marks are given if answer is correct (i.e. the last step of the solution). Here, solution is given for your help.

Marks

1. A – B = {1, 4, 6}

   $(A - B)^c = \{2, 3, 5, 7, 8, 9, 10\}$ ...(1)

2. $(1-2i)^{-2} = \dfrac{1}{(1-2i)^2}$

   $= \dfrac{1}{1+4i^2-4i} = \dfrac{1}{-3-4i} \times \dfrac{-3+4i}{-3+4i}$

   $= \dfrac{-3+4i}{9-16i^2}$

   $= \dfrac{-3}{25} + \dfrac{4}{25}i$ ...(1)

3. The given A.P. can be written in reverse order as 407, 403, 399, ...

   Now 20th term = a + 19d

   = 407 + 19 × (–4)

   = 407 – 76

   = 331 ...(1)

4. $5^2 + 6^2 + 7^2 + ... + 20^2$

   $= \sum_{r=1}^{20} r^2 - \sum_{k=1}^{4} k^2$ $\qquad \because \Sigma n^2 = \dfrac{n(n+1)(2n+1)}{6}$

Marks

$$= \frac{20 \times 21 \times 41}{6} - \frac{4 \times 5 \times 9}{6}$$

$= 2870 - 30 = 2840$ ...(1)

5. $\lim_{x \to 0} \left(\frac{e^x - e^{-x}}{x} \right)$

$$= \lim_{x \to 0} \left(\frac{e^{2x} - 1}{e^x x} \right) \times \frac{2}{2}$$

$= 2$ $\because \lim_{x \to 0} \frac{e^x - 1}{x} = 1$ ...(1)

6. $\lim_{x \to 0} \frac{\sqrt{1 + x + x^2} - 1}{x}$

$$= \lim_{x \to 0} \frac{x + x + x^2 - \cancel{1}}{x\left(\sqrt{1 + x + x^2} + 1\right)}$$

$$= \lim_{x \to 0} \frac{x + 1}{\sqrt{1 + x + x^2} + 1} = \frac{1}{2}$$ ...(1)

7. Required Probability $= \frac{{}^9C_2}{{}^{20}C_2} = \frac{36}{190} = \frac{18}{95}$ ...(1)

8. 365 days = (7 × 52 + 1) days

   After 52 weeks 1 day can be Sunday or Monday or .... Saturday. i.e., (7 cases)

   P (53 Sundays) $= \frac{1}{7}$. ...(1)

9. If two lines intersect in same plane then they are not parallel. ...(1)

10. 5 and 4 both divide 80.

    So, given statement is true. ...(1)

SECTION B

11. By definition,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ ...(1)

Marks

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{\sin(x+h)}{x+h}-\frac{\sin x}{x}\right)$$

$$= \lim_{h\to 0}\frac{x\sin(x+h)-(x+h)\sin x}{hx(x+h)}$$

$$= \lim_{h\to 0}\frac{x[\sin(x+h)-\sin x]-h\sin x}{hx(x+h)} \qquad ...(1)$$

$$= \lim_{h\to 0}\left[\frac{\cancel{x}\,\cancel{2}\cos\left(x+\frac{h}{2}\right).\sin\left(\frac{h}{2}\right)}{\cancel{x}(x+h)\frac{h}{2}\times\cancel{2}}-\frac{\sin x}{x(x+h)}\right] \qquad ...(1)$$

$$= \frac{\cos x}{x}-\frac{\sin x}{x^2}=\frac{x\cos x-\sin x}{x^2} \qquad ...(1)$$

OR

$$\frac{d}{dx}\left(\frac{\sin x - x\cos x}{x\sin x+\cos x}\right)$$

$$= \frac{(x\sin x+\cos x)(\cancel{\cos x}+x\sin x-\cancel{\cos x}) - (\sin x - x\cos x)(x\cos x+\cancel{\sin x}-\cancel{\sin x})}{(x\sin x+\cos x)^2} \qquad ...(2)$$

$$= \frac{x^2\sin^2 x + x\,\cancel{\sin x}\cos x - x\,\cancel{\sin x}\cos x + x^2\cos^2 x}{(x\sin x+\cos x)^2} \qquad ...(1)$$

$$= \frac{x^2}{(x\sin x+\cos x)^2} \qquad ...(1)$$

12. Let A = Event that Ajay will qualify.

B = Event that Aman will qualify.

Then P(A) = 0.16, P(B) = 0.12, P $(A \cap B)$ = 0.04 ...(1)

Now

(a) $P\left(\overline{A}\cap\overline{B}\right)=P\left(\overline{A\cup B}\right)=1-P(A\cup B)$

Marks

$= 1 - (P(A) + P(B) - P(A \cap B))$

$= 1 - (0.16 + 0.12 - 0.04)$

$= 1 - 0.24 = 0.76$ ...(1½)

(b) $P(B \cap A^c) = P(B) - P(A \cap B)$

$= 0.12 - 0.04$

$= 0.08$ ...(1½)

13. $f(x) = \dfrac{3}{1-x^2}$

Clearly, f(x) is not defined for $x^2 = 1$ i.e., $x = \pm 1$

So, $D_f = R - \{-1, 1\}$ 2

For Range, Let $y = \dfrac{3}{1-x^2}$ then $y \neq 0$

$\Rightarrow \quad 1 - x^2 = \dfrac{3}{y}$

$\Rightarrow \quad x^2 = 1 - \dfrac{3}{y} = \dfrac{y-3}{y}$

$x = \pm\sqrt{\dfrac{y-3}{y}}$ ...(1)

for $x \in D_f$, $\dfrac{y-3}{y} \geq 0$

$y - 3 \geq 0,\ y > 0$ or $y - 3 \leq 0,\ y < 0$

$y \geq 3,\ y > 0$ $\Rightarrow y < 0$

$\Rightarrow y \geq 3.$

$\therefore R_f = (-\infty, 0) \cup [3, \infty)$ ...(1)

14. R = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (1, 7), (2, 4) (2, 6), (3, 6)}...(2)

Domain = {1, 2, 3} ...(1)

Marks

Range = {2, 3, 4, 5, 6, 7} ...(1)

OR

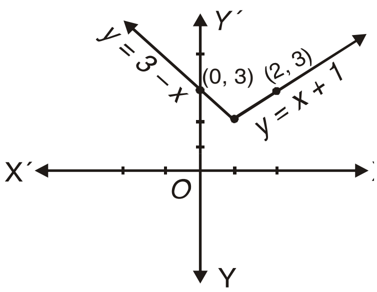
f(x) = 2 + |x − 1|

when $x \geq 1$, f(x) = 2 + x − 1 = x + 1

when x < 1, f(x) = 2 + 1 − x = 3 − x ...(2)

x	1	2	0	–1	–2
y	2	3	3	4	5

...(1)



...(1)

15. $\sin^2 x - \cos x = \dfrac{1}{4}$

$\Rightarrow$ $1 - \cos^2 x - \cos x = \dfrac{1}{4}$

$\Rightarrow$ $4 - 4\cos^2 x - 4\cos x = 1$ ...(1)

$\Rightarrow$ $4\cos^2 x + 4\cos x - 3 = 0$

$\Rightarrow$ $(2\cos x + 3)(2\cos x - 1) = 0$ ...(1)

$\Rightarrow$ $\cos x = -3/2$, $\cos x = 1/2 = \cos(\pi/3)$

Impossible $x = 2n\pi \pm \pi/3,\ n \in Z$ ...(2)

16. L.H.S. = $\cos 2\theta \cos\dfrac{\theta}{2} - \cos 3\theta \cos\dfrac{9\theta}{2}$

$= \dfrac{1}{2}\left[2\cos 2\theta \cos\dfrac{\theta}{2} - 2\cos 3\theta \cos\dfrac{9\theta}{2}\right]$ ...(1)

Marks

$$= \frac{1}{2}\left[\cos\left(2\theta + \frac{\theta}{2}\right) + \cos\left(2\theta - \frac{\theta}{2}\right) - \cos\left(3\theta + \frac{9\theta}{2}\right) - \cos\left(3\theta - \frac{9\theta}{2}\right)\right]$$ ...(1)

$$= \frac{1}{2}\left[\cos\frac{5\theta}{2} + \cancel{\cos\frac{3\theta}{2}} - \cos\frac{15\theta}{2} - \cancel{\cos\left(\frac{3\theta}{2}\right)}\right] \quad \because \cos(-\theta) = \cos\theta$$

$$= \frac{1}{2}\left[-2\sin\left(\frac{\frac{5\theta}{2} + \frac{15\theta}{2}}{2}\right)\sin\left(\frac{\frac{5\theta}{2} - \frac{15\theta}{2}}{2}\right)\right]$$ ...(1)

$$= -\sin 5\theta . \sin\left(-\frac{5\theta}{2}\right)$$

$$= \sin(5\theta)\sin\left(\frac{5\theta}{2}\right) = \text{R.H.S.}$$ ...(1)

17. P(n) : $x^n - y^n$ is divisible by $(x - y)$

P(1) : $x - y$ is divisible by $(x - y)$.

This is true.

Hence P(1) is true. ...(1)

Let us assume that P(k) be true for some natural number k.

i.e., $x^k - y^k$ is divisible by $x - y$.

So, $x^k - y^k = t(x - y)$ where *t* is an integer. ...(1)

Now we want to prove that P(k + 1) is also true.

i.e., $x^{k+1} - y^{k+1}$ is divisible by $x - y$.

Now $x^{k+1} - y^{k+1}$

$= x . x^k - y . y^k$

$= x [t(x - y) + y^k] - y . y^k$ using (i).

$= tx (x - y) + (x - y) y^k$.

Marks

$= (x - y)\ (tx + y^k)$

$= (x - y)\ .\ m$ where $m = tx + y^k$ is an integer.

So, $x^{k+1} - y^{k+1}$ is divisible by $(x - y)$

i.e., P(k + 1) is true whenever P(k) is true.

Hence by P.M.I., P(n) is true $\forall\ n \in N$. ...(2)

18. $x + iy = (a + ib)^{1/3}$

$\Rightarrow \quad (x + iy)^3 = a + ib$

$\Rightarrow \quad x^3 + i^3 y^3 + 3xyi\ (x + iy) = a + ib$ ...(1)

$\Rightarrow \quad x^3 - iy^3 + 3x^2yi - 3xy^2 = a + ib$

$\Rightarrow \quad (x^3 - 3xy^2) + i\ (3x^2y - y^3) = a + ib$ ...(1)

Comparing real and imaginary parts,

$x\ (x^2 - 3y^2) = a$ and $y\ (3x^2 - y^2) = b$

$x^2 - 3y^2 = \frac{a}{x}$ (i) $\qquad 3x^2 - y^2 = \frac{b}{y}$ (ii) ...(1)

Adding (i) and (ii) we get.

$4\left(x^2 - y^2\right) = \frac{a}{x} + \frac{b}{y}$ ...(1)

OR

Let the square root of $7 - 24i$ be $x + iy$

Then $\sqrt{7 - 24i} = x + iy$

$\Rightarrow \quad 7 - 24i = x^2 - y^2 + 2xyi$ ...(1)

Comparing real and imaginary parts.

$x^2 - y^2 = 7$ (i), $\qquad xy = -12$ (ii) ...(1)

Marks

We know that

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$$

$\Rightarrow \quad (x^2 + y^2)^2 = 49 + 4\ (144)$

$$x^2 + y^2 = 25 \qquad \text{(iii)}$$

Solving (i), (ii) we get $x = \pm 4$, $y = \pm 3$ ...(1)

From equation (ii) we conclude that $x = 4$, $y = -3$ and $x = -4$, $y = 3$.

Required square roots are,

$$4 - 3i \quad \text{and} \quad -4 + 3i$$ ...(1)

19. Let the equation of circle be,

$$(x - h)^2 + (y - k)^2 = r^2 \qquad \text{(i)}$$

$\because \quad$ (1, –2) and (4, –3) lie on (i).

So, $\quad (1 - h)^2 + (-2 - k)^2 = r^2$

and $\quad (4 - h)^2 + (-3 - k)^2 = r^2$ ...(1)

So, equating value of r^2, we get.

$$1 + h^2 - 2h + 4 + k^2 + 4k = 16 + h^2 - 8h + 9 + k^2 + 6k$$

$\Rightarrow \quad 6h - 2k = 20$

$$3h - k = 10 \qquad \text{(ii)}$$

As centre lies on $3x + 4y = 7$

So, $\quad 3h + 4k = 7 \qquad$ (iii) ...(1)

Solving (ii) and (iii) we get

$$k = \frac{-3}{5}, h = \frac{47}{15}$$ ...(1)

So, $\quad r = \dfrac{\sqrt{1465}}{15} \qquad$ Put in (i)

Marks

Hence required equation is

$$15x^2 + 15y^2 - 94x + 18y + 55 = 0 \qquad ...(1)$$

OR

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$a = 5, b = 3 \quad \Rightarrow \quad \sqrt{a^2 - b^2} = c = 4$$

$\Rightarrow$ foci of ellipse is ($\pm$4, 0) ...(1)

So, foci of required hyperbola are ($\pm$4, 0)

Distance between foci = 2ae = 8

$\because$ e = 2, a = 2 ...(1)

Using $b^2 = a^2 (e^2 - 1)$

$\Rightarrow$ $b^2 = 4 (4 - 1) = 12$ ...(1)

Hence equation of hyperbola is,

$$\frac{x^2}{4} - \frac{y^2}{12} = 1 \qquad ...(1)$$

20. Let yz plane divides the line joining A(4, 8, 10) and B(6, 10, –8) in the ratio λ : 1. So by section formula, the point of intersection is

$$R\left(\frac{6\lambda + 4}{\lambda + 1}, \frac{10\lambda + 8}{\lambda + 1}, \frac{-8\lambda + 10}{\lambda + 1}\right) \qquad ...(1)$$

Because this point lies on yz plane i.e., x = 0

So, $\frac{6\lambda + 4}{\lambda + 1} = 0$

$\Rightarrow$ $\lambda = -2/3$. ...(1)

$\therefore$ Ratio = 2 : 3 externally.

$\therefore$ R(0, 4, 46) ...(2)

Marks

21. 'MATHEMATICS'

Vowels in above word = A, A, E, I

Consonants in above word = M, M, T, T, C, S, H

Total arrangements of letters of above word

$$= \frac{11!}{2!\ 2!\ 2!} = \frac{10 \times 11 \times 9 \times \cancel{8} \times 7 \times 720}{\cancel{8}}$$

= 990 × 5040

= 4989600 ...(2)

Consider all the vowels as one letter. Now we have 8 letters, which can be arranged in $\frac{8!}{2!\ 2!}$ ways. Vowels can be arranged among themselves in $\frac{4!}{2!}$ ways. Total arrangements when all vowels are always together

$$= \frac{8!}{2!\ 2!} \times \frac{4!}{2!}$$

$$= \frac{\cancel{8} \times 7 \times 6 \times 120 \times 24}{\cancel{8}} = 1{,}20{,}960$$ ...(1)

The number of arrangements when all the vowels never come together

= 4989600 − 120960

= 4868640. ...(1)

22. **Case I :** If 2 particular students always join party then remaining 6 out of 18 can be choosen in ${}^{18}C_6$ ways. ...(1½)

Case II : If 2 particular students always do not join the excursion party then selection of 8 students out of 18 can be done in ${}^{18}C_8$ ways.

So, Required number of ways ...(1½)

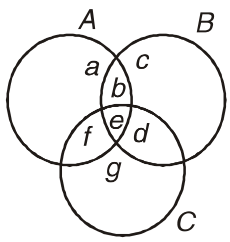
$$= {}^{18}C_6 + {}^{18}C_8$$

= 62322 ...(1)

Marks

SECTION C

23. Let A, B, C denote the sets of those students who take Maths, Physics, Chemistry respectively. ...(1)



...(1)

By given condition,

$a + b + e + f = 15$

$b + c + e + d = 12,\ f + e + d + g = 11$

$e + f = 5,\ b + e = 9,\ e + d = 4,\ e = 3$ ...(1)

Solving above equations, we obtain.

$e = 3,\ d = 1,\ b = 6,\ f = 2,\ g = 5,\ c = 2,\ a = 4$ ...(1)

(i) No. of students who had taken atleast one of the three subjects $= n\,(A \cup B \cup C)$

$= a + b + c + d + e + f + g$

$= 23.$ ...(1)

(ii) No. of Students who had taken only one of the three subjects

$= a + c + g$

$= 4 + 2 + 5 = 11$ ...(1)

24. $\cos 3x = 4\cos^3 x - 3\cos x$

$\Rightarrow \quad 4\cos^3 x = \cos 3x + 3\cos x$ (i) ...(1)

Using (i)

$$\cos^3 A = \frac{1}{4}\cos 3A + \frac{3}{4}\cos A$$

Marks

$$\cos^3\left(\frac{2\pi}{3}+A\right)=\frac{1}{4}\cos\left(3\left(\frac{2\pi}{3}+A\right)\right)+\frac{3}{4}\cos\left(\frac{2\pi}{3}+A\right)$$

$$\cos^3\left(\frac{4\pi}{3}+A\right)=\frac{1}{4}\cos(4\pi+3A)+\frac{3}{4}\cos\left(\frac{4\pi}{3}+A\right) \qquad ...(1)$$

Now L.H.S. of given result becomes

$$=\frac{1}{4}[\cos 3A+\cos(2\pi+3A)+\cos(4\pi+3A)]$$
$$+\frac{3}{4}\left[\cos A+\cos\left(\frac{2\pi}{3}+A\right)+\cos\left(\frac{4\pi}{3}+A\right)\right] \qquad ...(1)$$

$$=\frac{3}{4}\cos 3A+\frac{3}{4}\left[\cos A+2\cos(\pi+A)\cos\left(-\frac{\pi}{3}\right)\right] \qquad ...(1)$$

$$=\frac{3}{4}\cos 3A+\frac{3}{4}\left[\cos A-\cancel{2}\times\frac{1}{\cancel{2}}\cos A\right] \qquad ...(1)$$

$$=\frac{3}{4}\cos 3A=\text{R.H.S.} \qquad ...(1)$$

25. $x + 2y \le 40$ ...(i), $3x + y \ge 30$ ...(ii), $4x + 3y \ge 60$ ...(iii), $x, y \ge 0$. ...(1)

The corresponding equations are

$x + 2y = 40$

x	0	40
y	20	0

$3x + y = 30$

x	10	0
y	0	30

$4x + 3y = 60$

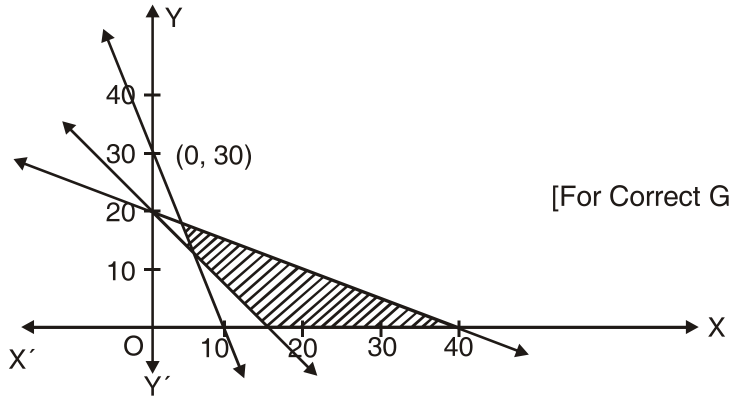
x	0	15
y	20	0

...(2)

Putting x = 0 = y in (i), (ii), (iii) we get result True, false, false respectively. So, the shades will be made accordingly.

$x, y \ge 0$ shows I quadrant.

Marks



...(3)
[For Correct Graph]

Common shaded portion is required solution set.

OR

Quantity of 12% acid solution = 600 litres.

Quantity of acid $= 600 \times \frac{12}{100} = 72$ litres. ...(1)

Let x litres of 30% acid solution be mixed. Then according to given question. ...(1)

$$15\% \text{ of } (600 + x) < 72 + \frac{30}{100} \times x < 18\% \text{ of } (600 + x) \qquad ...(1)$$

$\Rightarrow$ 15 (600 + x) < 7200 + 30x < 18 (600 + x)

9000 + 15x < 7200 + 30x, 7200 + 30x < 10800 + 18x

15x > 1800 , 12x < 3600

x > 120 , x < 300 ...(2)

So, 120 < x < 300

So, 30% acid solution must be between 120 litres and 300 litres. ...(1)

26. 5th term from beginning in $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$

$$= {}^nC_4 2^{\frac{n-4}{4}} . \left(\frac{1}{3}\right) \qquad ...(1½)$$

Marks

5th term from the end in $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{2}}\right)^n$ is

$$= {}^nC_4\,(2)\,.\left(\frac{1}{3}\right)^{\frac{n-4}{4}} \qquad ...(1½)$$

According to given question,

$$\frac{{}^nC_4\,2^{\frac{n-4}{4}}.\left(\frac{1}{3}\right)}{{}^nC_4\,(2)\,.\left(\frac{1}{3}\right)^{\frac{n-4}{4}}}=\frac{\sqrt{6}}{1} \qquad ...(1½)$$

$$2^{\frac{n-4}{4}-1}\times 3^{\frac{n-4}{4}-1}=\sqrt{6}$$

$$6^{\frac{n-8}{4}}=6^{1/2}$$

$\Rightarrow$ $\quad \frac{n-8}{4}=\frac{1}{2} \quad \Rightarrow \quad n=10$ ...(1½)

27. Let the numbers be a and b.

So, $\quad a+b=6\sqrt{ab}$ ...(1)

$$a^2 + b^2 + 2ab = 36\ ab$$

$$\left(\frac{a}{b}\right)^2-34\left(\frac{a}{b}\right)+1=0 \qquad ...(1½)$$

$$\Rightarrow \quad \frac{a}{b}=\frac{34\pm\sqrt{1156-4\times1\times1}}{2}$$

$$=\frac{34\pm24\sqrt{2}}{2}=17\pm12\sqrt{2} \qquad ...(1½)$$

So, $\quad \frac{a}{b}=\frac{17+12\sqrt{2}}{1}$ taking +ve sign

$$=\frac{\left(3+2\sqrt{2}\right)^2}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$$

Marks

$$= \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

So, $\quad a : b = (3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$ ...(2)

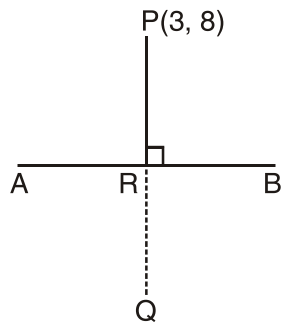
28. Slope of given line = –1/3

Slope of line PQ = 3

Equation of line PQ is

$$y - 8 = 3(x - 3)$$

$\Rightarrow \quad y = 3x - 1$ ...(1)



...(1)

Solving equations of AB and PQ we get coordinates of R (foot of perpendicular)

So, $\quad R(1, 2)$ ...(2)

Let Q (x', y') be image of P.

then as R is mid point of PQ. We have,

$$\frac{x'+3}{2} = 1 \qquad \text{and} \qquad \frac{y'+8}{2} = 2$$

$\Rightarrow \quad x' = -1 \qquad\qquad y' = -4$

$\therefore \quad Q(-1, -4)$ ...(2)

Marks

29.

C.I.	x (mid values)	f	$u = \frac{x - A}{i}$	fu	fu^2
20-30	25	3	–3	–9	27
30-40	35	51	–2	–102	204
40-50	45	122	–1	–122	122
50-60	55 A	141	0	0	0
60-70	65	130	1	130	130
70-80	75	51	2	102	204
80-90	85	2	3	6	18
		$\Sigma f = 500$		**$\Sigma fu = 5$**	**$\Sigma fu^2 = 705$**

...(2)

$$\bar{x} = A + \frac{\Sigma fu}{\Sigma f} \times i$$

$$= 55 + \frac{5}{500} \times 10 = 55.1 \quad \text{...(1)}$$

$$\text{S.D.} = \sigma = i \times \sqrt{\frac{1}{N}\Sigma fu^2 - \left(\frac{1}{N}\Sigma fu\right)^2} \quad \text{...(1)}$$

$$= 10\sqrt{\frac{1}{500}(705) - \left(\frac{5}{500}\right)^2}$$

$$= 10\sqrt{1.41 - 0.0001} = \sqrt{1.4099} \times 10$$

$$= 11.874 \quad \text{...(2)}$$

OR

$N = 20, \quad \bar{x} = 10, \quad \sigma = 2$

Using $\bar{x} = \frac{\Sigma x}{N}$ ...(1)

$\Rightarrow$ Incorrect $\Sigma x = 10 \times 20 = 200$

Correct $\Sigma x = 200 + 12 - 8 = 204$

Marks

Correct Mean $= \frac{204}{20} = 10.2$ ...(1½)

Using $\sigma^2 = \frac{1}{N}\Sigma x^2 - (\bar{x})^2$ ...(1)

$$4 = \frac{1}{20}\Sigma x^2 - (10)^2$$

$\Rightarrow$ Incorrect $\Sigma x^2 = 2080$

Correct $\Sigma x^2 = 2080 + (12)^2 - (8)^2$

$= 2160$ ...(1)

Correct S.D.

$$= \sqrt{\frac{1}{N}\Sigma x^2 - (\bar{x})^2}$$

$$= \sqrt{\frac{1}{20}(2160) - (10.2)^2}$$

$= \sqrt{108 - 104.04} = \sqrt{3.96} = 1.99.$ ...(1½)

MODEL TEST PAPER – II

Time : 3 hours **Maximum Marks : 100**

General Instructions :

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three Sections A, B and C.

(iii) Section A comprises of 10 questions of one mark each. Section B comprises of 12 questions of four marks each and Section C comprises of 7 questions of six marks each.

(iv) There is no overall choice. However, an internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

SECTION A

1. Let A = {1, 2} and B = {3, 4}. Find the number of relations from A to B.

2. Find the value of sin 1845°.

3. Write the negation of the following statement : 'Sum of 2 and 3 is 6'.

4. Write the converse of the statement : 'If the sum of digits of a number is divisible by 9 then the number is divisible by 9'.

5. Write the solution of $3x^2 - 4x + \frac{20}{3} = 0$.

6. Find the sum of the series

   $(1^2 + 1) + (2^2 + 2) + (3^2 + 3) + ...$ to n terms.

7. A die is thrown. Find the probability of getting a number less than or equal to 6.

8. Five marbles are drawn from a bag which contains 7 blue marbles and 4 black marbles. What is the probability that all will be blue?

9. Find the general solution of $\cos 3\theta = -\frac{1}{2}$.

10. What is y-intercept of the line passing through the point (2, 2) and perpendicular to the line $3x + y = 3$?

SECTION B

11. Evaluate : $\lim\limits_{x \to 0} \dfrac{\tan x - \sin x}{x^3}$

OR

$\lim\limits_{x \to 0} \dfrac{\cos ax - \cos bx}{x^2}$

12. Differentiate $\cot x$ with respect to x by the first principle.

13. Find the square root of $-5 + 12\,i$

14. How many diagonals are there in a polygon with n sides?

15. Prove the following by the principle of mathematical induction

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1},\ n \in N$$

OR

Using principle of mathematical induction prove that

$4^n + 15n - 1$ is divisible by 9 for all $n \in N$.

16. Find the domain and range of $f(x) = \dfrac{1}{\sqrt{x - 5}}$

17. Find the value of n so that $\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b.

OR

Find the sum of the following series upto n terms :

$.6 + .66 + .666 + \ldots\ldots$

18. If p is the length of perpendicular from the origin to the line whose intercepts on the axes are a and b, then show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

19. Find the length of the axes, eccentricity and length of the latus-rectum of the hyperbola $25x^2 - 36y^2 = 225$.

OR

Find the equation of the circle passing through the point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$.

20. Using section formula, prove that the three points (–4, 6, 10), (2, 4, 6) and (14, 0, –2) are collinear.

21. On her vacations Veena visits four cities (A, B, C, D) in a random order. What is the probability that she visits.

    (i) A before B? (ii) A before B and B before C?

22. Prove that

$\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1.$

SECTION C

23. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find :

    (i) How many read none of the three magazines?

    (ii) How many read magazine C only?

24. The 3rd, 4th and 5th terms in the expansion of $(x + a)^n$ are respectively 84, 280 and 560, find the values of x, a and n.

OR

The coefficients of $(r - 1)^{th}$, r^{th} and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio 1 : 3 : 5. Find n and r.

25. Find the sum of the following series upto n terms :

$$\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + ...$$

26. Prove that

$$\cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$$

27. Solve the following system of inequalities graphically :

$$x + 2y \leq 10,\ x + y \geq 1,\ x - y \leq 0,\ x \geq 0,\ y \geq 0.$$

28. Find the general solution of

$$\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$$

OR

If $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$, find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$

29. Find the mean deviation about the median for the following data :

Marks	0–10	10–20	20–30	30–40	40–50	50–60
Number of girls	8	10	10	16	4	2

ANSWERS

1. 16
2. $\frac{1}{\sqrt{2}}$
3. It is false that sum of 2 and 3 is 6.
4. If a number is divisible by 9 then the sum of the digits of the number is divisible by 9.
5. $\frac{2 \pm 4i}{3}$
6. $\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$
7. 1
8. $\frac{1}{22}$

9. $\frac{2n\pi}{3} \pm \frac{2\pi}{9}, n \in z.$

10. $\frac{4}{3}$

11. $\frac{1}{2}$ or $\frac{b^2 - a^2}{2}$

12. $-\text{cosec}^2 x$

13. $\pm (2 + 3i)$

14. $\frac{n(n-3)}{2}$

16. $(5, \infty); (0, \infty)$

17. $n = -\frac{1}{2}$ or $\frac{2n}{3} - \frac{2}{27}\left(1 - 10^{-n}\right)$

19. Length of transverse axis = 6, lengths of conjugate axis = 5, $e = \frac{\sqrt{61}}{6}$,

Length of latus rectum $= \frac{25}{6}$

OR $x^2 + y^2 + 4x - 2y = 0.$

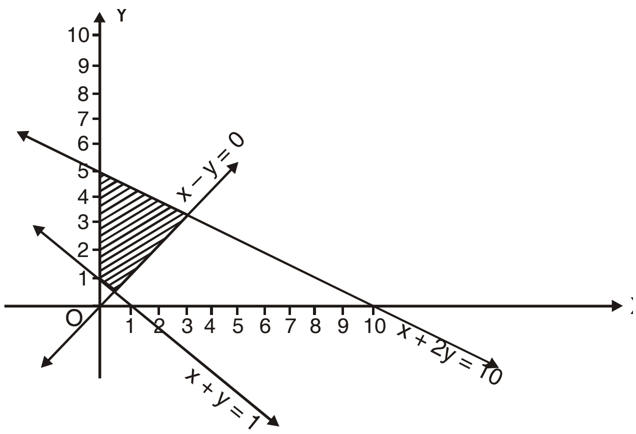
21. (i) $\frac{1}{2}$ (ii) $\frac{1}{6}$

23. (i) 20 (ii) 30

24. $n = 7$, $a = 2$, $x = 1$ **OR** $n = 7$ and $r = 3$.

25. $\frac{n}{24}\left(2n^2 + 9n + 13\right)$

27.



28. $\theta = (2n+1)\frac{\pi}{8}, n\pi \pm \frac{\pi}{3}, n \in z$ or $\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}, 3.$

29. 11.44