Class 10 Quadratic Equation solved Test Paper

1. Solve the following quadratic equation for x:

$4\sqrt{3}x^2 + 5x + 2\sqrt{3} = 0$

Solution:

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$
$$\Rightarrow 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$\Rightarrow 4x \ \sqrt{3}x + 2 \ - \sqrt{3} \ \sqrt{3}x + 2 \ = 0$$
$$\Rightarrow \ 4x - \sqrt{3} \ \ \sqrt{3}x + 2 \ = 0$$
$$\therefore x = \frac{\sqrt{3}}{4} \text{ or } x = -\frac{2}{\sqrt{3}}$$

2. For what values of k, the roots of the quadratic equation $(k + 4)\ x^2 + (k+ 1)x + 1 = 0$ are equal?

$k + 4)x^2 + (k + 1)x + 1 = 0$

$a = k + 4,\ b = k + 1,\ c = 1$

For equal roots, dicriminant, $D = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (k + 1)^2 - 4(k + 4) \times 1 = 0$

$\Rightarrow k^2 + 2k + 1 - 4k - 16 = 0$

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow k(k - 5) + 3(k - 5) = 0$

$\Rightarrow (k - 5)\ (k + 3) = 0$

$\Rightarrow k = 5 \text{ or } k = -3$

Thus, for $k = 5$ or $k = -3$,

the given quadratic equation has equal roots.

3. Solve that following for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

Solution:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

$$\Rightarrow \frac{2x - 2a - b - 2x}{2x \;\; 2a+b+2x} = \frac{b+2a}{2ab}$$

$$\Rightarrow \frac{-\;2a+b}{2x \;\; 2a+b+2x} = \frac{b+2a}{2ab}$$

$$\Rightarrow \frac{-1}{x \;\; 2a+b+2x} = \frac{1}{ab}$$

$$\Rightarrow 2x^2 + 2ax + bx + ab = 0$$

$$\Rightarrow 2x \;\; x+a \; + b \;\; x+a \; = 0$$

$$\Rightarrow \; x+a \;\; 2x+b \; = 0$$

$$\Rightarrow x+a = 0 \text{ or } 2x+b = 0$$

$$\Rightarrow x = -a, \text{ or } x = \frac{-b}{2}$$

4. Sum of the areas of two squares is 400 cm2. If the difference of their perimeters is 16 cm, find the sides of the two squares.

Solution:

Let the sides of the two squares be x cm and y cm where x > y.

Then, their areas are x^2 and y^2 and their perimeters are 4x and 4y.

By the given condition:

$x^2 + y^2 = 400$ ... (1)

and 4x - 4y = 16

$\Rightarrow 4(x - y) = 16 \Rightarrow x - y = 4$

$\Rightarrow x = y + 4$ ... (2)

Substituting the value of x from (2) in (1), we get:

$(y + 4)^2 + y^2 = 400$

$\Rightarrow y^2 + 16 + 8y + y^2 = 400$

$\Rightarrow 2y^2 + 16 + 8y = 400$

$\Rightarrow y^2 + 4y - 192 = 0$

$\Rightarrow y^2 + 16y - 12y - 192 = 0$

$\Rightarrow y(y + 16) - 12(y + 16) = 0$

$\Rightarrow (y + 16)(y - 12) = 0$

$\Rightarrow y = -16$ or $y = 12$

Since, y cannot be negative, y = 12.

So, x = y + 4 = 12 + 4 = 16

Thus, the sides of the two squares are 16 cm and 12 cm.

5. Find the values of k for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots.

Solution:

Given: Quadratic equation $9x^2 - 3kx + k = 0$ has equal roots

Let β be the equal roots of the equation

Thus $2\beta = \frac{3k}{9} = \frac{k}{3}$ (Sum of the roots is equal to –b/a)

We get $\beta = \frac{k}{6}$ (i)

Or $\beta^2 = \frac{k^2}{36}$

Also, that $\beta^2 = \frac{k}{9}$ (Product of the roots is equal to c/a) (ii)

$\frac{k^2}{36} = \frac{k}{9}$

For $k \neq 0$, $\frac{k}{36} = \frac{1}{9}$

Thus k = 4

6. Solve for x: $\frac{16}{x} - 1 = \frac{15}{x-1}$

Solution:

Given: $\frac{16}{x} - 1 = \frac{15}{x+1};\ x \# 0, -1$

$\therefore \frac{16-x}{x} = \frac{15}{x+1}$

$\frac{16-x}{x} = \frac{15}{x+1}$

$\frac{(16-x)(x+1)}{1} = \frac{15x}{1}$

$16x + 16 - x^2 - x = 15x$

$16 = x^2$

$x = 4$

7. A motorboat whose speed in still water is 18 km/h, takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Solution:

Let speed of stream = x km/h

Speed f boat in steel water = 18 km/h

Speed f boat in upstream = (18 –x)km/h

Speed f boat in downstream = (18 + x) km/h

Distance = 24 km

As per question, it takes 1 hour more to go upstream 24 km, than downstream

$$\frac{24}{18-x} = \frac{24}{18+x} + 1$$

$x^2 + 48x - 324 = 0$

$x = 6 \text{ or } x = -54$

But, as speed can not be negative

Hence, the speed of stream = 6 km/h

8. Solve for x: $\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3}$

Solution:

$$\frac{x-3}{x-4}+\frac{x-5}{x-6}=\frac{10}{3}$$

$$\frac{[(x-3)(x-6)+(x-4)(x-5)]}{(x-4)(x-6)}=\frac{10}{3}$$

$$\frac{x^2-9x+18+x^2-9x+20}{(x-4)(x-6)}=\frac{10}{3}$$

$$\frac{2x^2-18x+38}{(x-4)(x-6)}=\frac{10}{3}$$

$$\frac{2(x^2-9x+19)}{1}=\frac{10}{3}(x^2-10x+24)$$

$$\frac{x-9x+19}{1}=\frac{5}{3}[x^2-10x+24]$$

$$3x^2-27x+57=5x^2-50x+120$$

$$2x^2-23x+63=0$$

$$\therefore x=7 \text{ or } x=\frac{9}{2}$$

9. Solve for x : $x^2-(\sqrt{3}+1)x+3=0$

$x^2-\sqrt{3}x-x+\sqrt{3}=0$

$x(x-\sqrt{3})-1(x-\sqrt{3})=0$

$(x-\sqrt{3})(x-1)=0$

$x=\sqrt{3},1$

10. Solve for x : $x^2+5x-(a^2+a-6)=0$

Solution: $x=\frac{-5\pm\sqrt{25+4(a^2+a-6)}}{2}=\frac{-5\pm(2a+1)}{2}=\frac{2a-4}{2},\frac{-2a-6}{2}$

$x=a-2,-a-3$

11. If x = – 2 is a root of the equation $3x^2 + 7x + p = 0$, find the values of k so that the roots of the equation $x^2 + k (4x + k – 1) + p = 0$ are equal.

Solution: x = – 2 is root of the equation $3x^2 + 7x + p = 0$

$\Rightarrow$ $3 (– 2)^2 + 7 (– 2) + p = 0$

$\Rightarrow$ p = 2

Roots of the equation $x^2 + 4k x + k^2 – k + 2 = 0$ are equal

$16k^2 – 4 (k^2 – k + 2) = 0$

$3k^2 + k – 2 = 0$

$(3k – 2) (k + 1) = 0$

k = 2/3 , – 1

12. The total cost of a certain length of a piece of cloth is Rs. 200. If the piece was 5 m longer and each meter of cloth costs Rs. 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per meter?

Solution:

Let length of cloth be x m.

Cost per meter = Rs$\frac{200}{x}$

New lenght of cloth = (x + 5) m.

New cost per meter = Rs. $\frac{200}{x} - 2$

$(x + 5)\left(\frac{200}{x} - 2\right) = 200$

$x^2 + 5x – 500 = 0$

$(x + 25)(x - 20) = 0$

$x = 20, x = -25$

13. Solve the following for x : $4x^2 - 4a^2x + (a^4 - b^4) = 0$

Solution:

The given quadratic equation can be written as

$$(4x^2 - 4a^2x + a^2) - b^4 = 0$$

$$\text{or } (2x - a^2)^2 - (b^2)^2 = 0$$

$$\therefore \ (2x - a^2 + b^2)(2x - a^2 - b^2)^2 = 0$$

$$\Rightarrow \ x = \frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$$

14. Find that non-zero value of k, for which the quadratic equation $kx^2 + 1 - 2(k-1)x + x^2 = 0$ has equal roots. Hence find the roots of the equation.

Solution:

The given quadratic eqn. can be written as

$$(k + 1)x^2 - 2(k - 1)x + 1 = 0$$

For qual roots $4(k-1)^2 - 4(k+1) = 0$ or $k^2 - 3k = 0$

$$\Rightarrow \ k = 0, 3$$

$$\therefore \ \text{Non-zero value of } k = 3 : \text{Roots are } \frac{1}{2}, \frac{1}{2}$$

15. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

Solution:

Let the fraction be $\frac{x-3}{x}$

By the given condition, new fraction $\frac{x-3+2}{x+2} = \frac{x-1}{x+2}$

$$\therefore \quad \frac{x-3}{x} + \frac{x-1}{x+2} = \frac{29}{20}$$

$$\Rightarrow \quad 20\left[(x-3)(x+2) + x(x-1)\right] = 29\left(x^2+2x\right)$$

$$= \quad 20\left(x^2 - x - 6 + x^2 - x\right) = 29x^2 + 58x$$

or $11x^2 - 98x - 120 = 0$

or $11x^2 - 110x - 12x - 120 = 0$

$(11x + 12)(x - 10) = 0 \quad \Rightarrow \quad x = 10$

16.Solve for x: $\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}$

$$5x\left[4(x-2) + 3x + 3\right] = 46(x+1)(x-2)$$

$$5x(7x-5) = 46\left(x^2 - x - 2\right) \Rightarrow 11x^2 - 21x - 92 = 0$$

$$\Rightarrow \quad x = \frac{21 \pm \sqrt{441+4048}}{22} = \frac{21 \pm 67}{22} = 4, \frac{-23}{11}$$

17. Solve the following quadratic equation for x :

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

Solution:

$$4x^2 + 4bx + b^2 - a^2 = 0 \quad \Rightarrow \quad (2x+b)^2 - (a)^2 = 0$$

$$\Rightarrow \quad (2x + b + a)\ (2x + b - a) = 0$$

$$\Rightarrow \quad x = -\frac{a+b}{2},\ x = \frac{a-b}{2}$$

18. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field.

Soltion:

Let the length of shorter side be x m.

$\therefore$ length of diagonal = (x + 16) m

and, lenght of longer side = (x + 14) m

$$\therefore \; x^2 + (x+14)^2 = (x+16)^2$$

$$\Rightarrow \; x^2 - 4x - 6 = 0 \quad \Rightarrow \; x = 10 \text{ m.}$$

$\therefore$ length of sides are 10m and 24m.

19. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 $\frac{\text{km}}{\text{h}}$ more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed ?

Solution: Let the original speed of first train = x km/hr $So, \; \frac{54}{x} + \frac{63}{x+6} = 3$

$\Rightarrow 54x + 324 + 63x = 3x(x+6) \Rightarrow x^2 - 33x - 108 = 0$

On solving, we get, x = 36 . Hence,the original speed of first train = 36km/hr

20. A takes 6 days less than B to finish a piece of work. If both A and B together can finish the work in 4 days, find the time taken by B to finish the work.

Solution: Let B alone takes x days to finish the work. Then, A alone can finish it in (x - 6) days.

Now, (A's one day's work) + (B's one day's work) = $\frac{1}{x} + \frac{1}{x+6}$ But, (A + B)'s one day's work = $\frac{1}{4}$

Therefore, $\frac{1}{x} + \frac{1}{x+6} = \frac{1}{4} \Rightarrow 8x - 24 = x^2 - 6x \Rightarrow x^2 - 14x + 24 = 0 \Rightarrow x^2 - 12x - 2x + 24 = 0$

$\Rightarrow (x - 12)(x - 2) = 0 \Rightarrow x = 12$ or $x = 2$

But, x cannot be less than 6. So, x = 12.