17

Similarity of Triangle

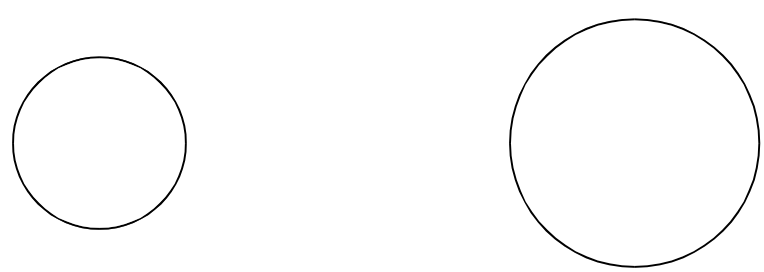
17.1 INTRODUCTION

Looking around you will see many objects which are of the same shape but of same or different sizes. For examples, leaves of a tree have almost the same shape but same or different sizes. Similarly, photographs of different sizes developed from the same negative are of same shape but different sizes, the miniature model of a building and the building itself are of same shape but different sizes. **All those objects which have the same shape but different sizes are called similar objects.**

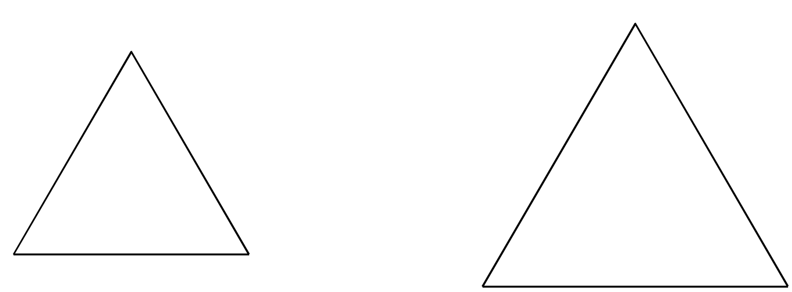
Let us examine the similarity of plane figures :

(i) Two line-segments of the same length are congruent but of different lengths are similar.

(ii) Two circles of the same radius are congruent but circles of different radii are similar.



(iii) Two equilateral triangles of different sides are similar.



(iv) Two squares of different sides are similar.

In this lesson, we shall study about the concept of similarity, especially similarity of triangles and the conditions thereof. We shall also study about various results related to them.

17.2 OBJECTIVES

After studying this lesson, the learner will be able to :

- identify similar figures
- distinguish between congruent and similar plane figures
- cite the criteria for similarity of triangles viz. AAA, SSS and SAS.
- verify and use unstarred results given in the curriculum based on similarity experimentally
- prove the Baudhayan/Pythagoras Theorem
- apply these results in verifying experimentally (or proving logically) problems based on similar triangles.

17.3 EXPECTED BACKGROUND KNOWLEDGE

Knowledge of

- plane figures like triangles, quadrilaterals, circles, rectangles, squares, etc.
- criteria of congruency of triangles
- finding squares and square-roots of numbers
- ratio and proportion
- internal and external bisectors of angles of a triangle.

17.4 SIMILAR PLANE FIGURES

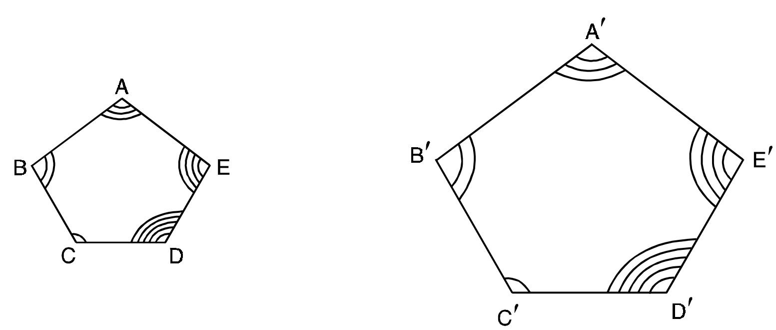


Fig. 17.2

In Fig. 17.2, the two pentagon seem to be of the same shape.

We can see that $\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$, $\angle D = \angle D'$ and $\angle E = \angle E'$ and $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DE}{D'E'} = \frac{EA}{E'A'}$. We say that the two pentagons are similar. Thus we say that

Any two polygons, with corresponding angles equal and corresponding sides proportional, are similar

Thus, two polygons are similar, if they satisfy the following two conditions :

(i) Corresponding angles are equal

(ii) The corresponding sides are proportional.

Even if one of the conditions does not hold, the polygons are not similar as in the case of a rectangle and square given in Fig. 17.3. Here all the corresponding angles are equal but the corresponding sides are not proportional.

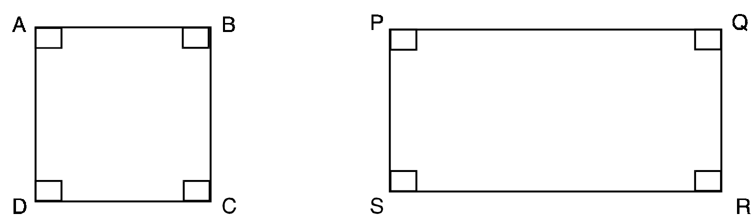


Fig. 17.3

17.5 SIMILARITY OF TRIANGLES

Triangles are special type of polygons and therefore the conditions of similarity of polygons also hold for triangles. Thus,

Two triangles are similar if

(i) their corresponding angles are equal, and

(ii) their corresponding sides are proportional

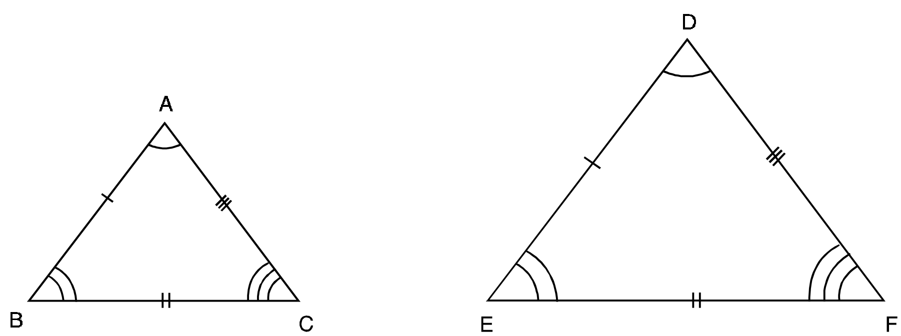


Fig. 17.4

We say that ΔABC is similar to ΔDEF and denote it by writing

$\Delta ABC \sim \Delta DEF$

The symbol '~' stands for the phrase " is similar to"

If ΔABC ~ ΔDEF, then by definition

$\angle A = \angle D$, $\angle B = \angle E$, $\angle D = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

17.5.1 AAA criterion for similarity

We shall show that if either of the above two conditions is satisfied then the other automatically holds in the case of triangles.

Let us perform the following experiment.

Construct two Δ's ABC and PQR in which $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$ as shown in Fig. 17.5.

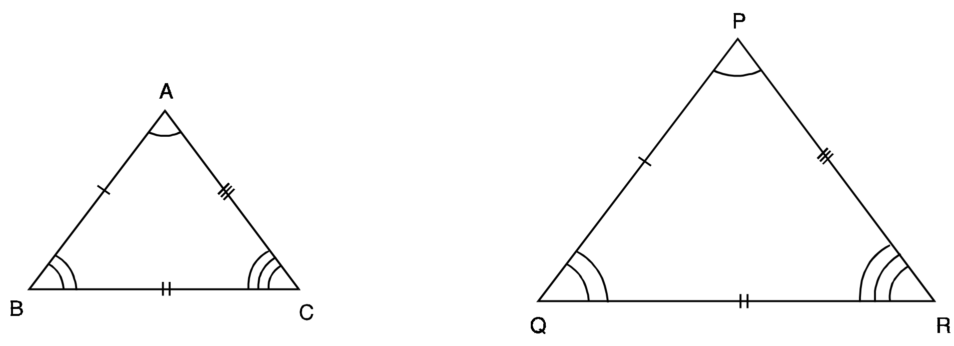


Fig. 17.5

Measure the sides AB, BC and CA of ΔABC and also measure the sides PQ, QR and RP of ΔPQR.

Now find the ratio $\frac{AB}{PQ}$, $\frac{BC}{QR}$ and $\frac{CA}{RP}$.

What do you find ? You will find that all the three ratios are equal and therefore the triangles are similar.

Try this with different triangles with equal corresponding angles. You will find the same result.

Thus, we can say that

If in two triangles, the corresponding angles are equal the triangles are similar.

This is called AAA similarity criterion.

17.5.2 SSS criterion for similarity.

Let us now perform the following experiment :

Draw a triangle ABC with AB = 3 cm, BC = 4.5 cm and CA = 3.5 cm

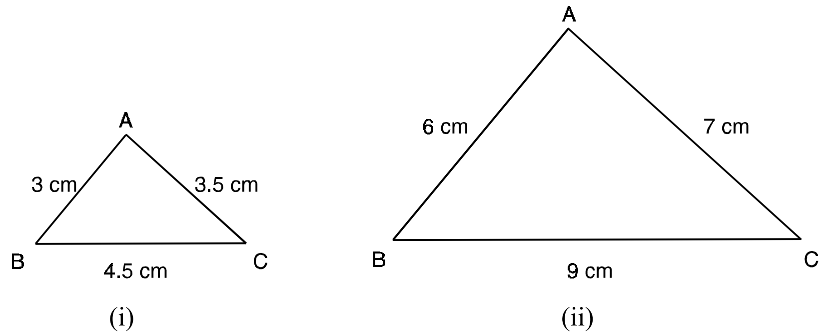


Fig. 17.6

Draw another ΔPQR as shown in Fig. 17.6 (ii)

We can see that $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

i.e., the sides of the two triangles are proportional

Now measure ∠A, ∠B and ∠C of ΔABC and ∠P, ∠Q and ∠R of ΔPQR.

You will find that ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R.

Repeat the experiment with another two triangles having corresponding sides proportional, you will find that the corresponding angles are equal and so the triangle are similar.

Thus, we can say that

> **If the corresponding sides of two triangles are proportional the triangles are similar.**

17.5.3 SAS Criterian for Similarity

Let us conduct the following experiment.

Take a line AB = 3 cm and at A construct an angle of 60°. Cut off AC = 4.5 cm. Join BC

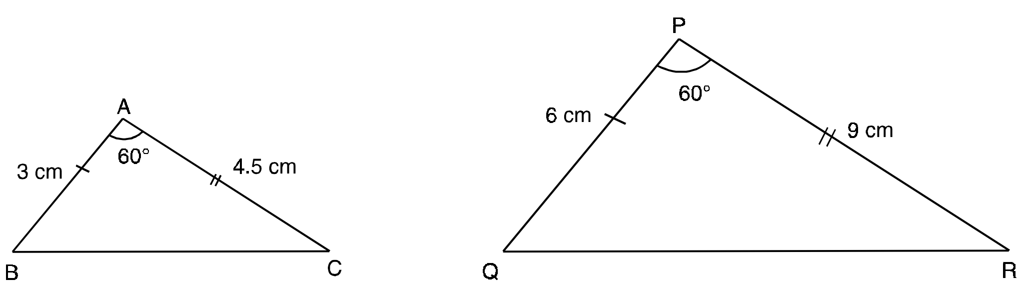


Fig. 17.7

Now take PQ = 6 cm. At P, draw an angle of 60° and cut off PR = 9 cm.

Measure ∠B, ∠C, ∠Q and ∠R. We shall find that ∠B = ∠Q and ∠C = ∠R

Thus, ΔABC ~ ΔPQR

Thus, we conclude that

If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangles are similar.

Thus, we have three important criteria for the similarity of triangles. They are given below:

(i) If in two triangles, the corresponding angles are equal, the triangles are similar.

(ii) If the corresponding sides of two triangles are proportional, the triangles are similar.

(iii) If one angle of a triangle is equal to one angle of the other triangle and the sides containing these angles are proportional, the triangle are similar.

Example 17.1 : In Fig. 17.8 are given two triangles ABC and PQR

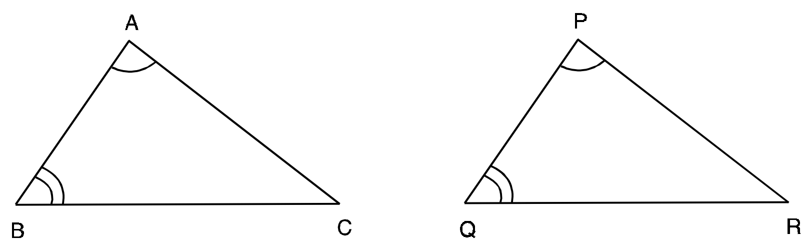


Fig. 17.8

Is $\Delta ABC \sim \Delta PQR$?

Solution : We are given that

$$\angle A = \angle P \text{ and } \angle B = \angle Q$$

We also know that

$$\angle A + \angle B + \angle C = \angle P + \angle Q + \angle R = 180°$$

Thus, according to first criterion of similarity

$$\Delta ABC \sim \Delta PQR.$$

Example 17.2 :

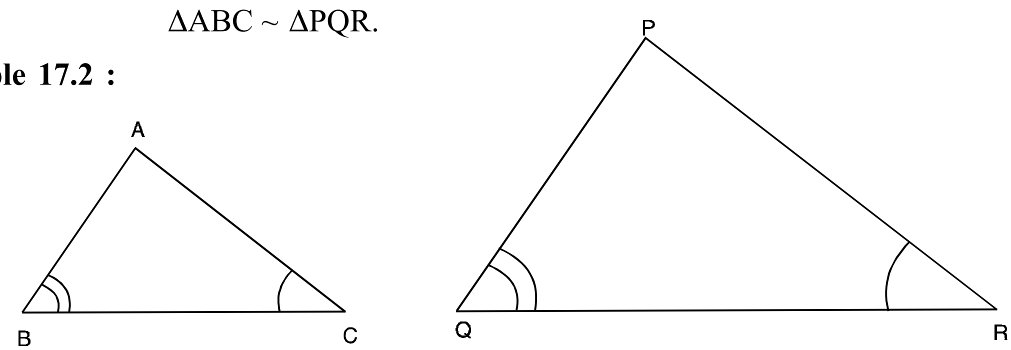


Fig. 17.9

In Fig. 17.9, $\Delta ABC \sim \Delta PQR$. If AC = 4.8 cm, AB = 4 cm and PQ = 9 cm, find PR.

Solution : It is given that $\Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{AB}{PQ} = \frac{AC}{PR}$$

Let $\quad PR = x \text{ cm}$

$$\therefore \quad \frac{4}{9} = \frac{4.8}{x}$$

$\Rightarrow \quad 4x = 9 \times 4.8$

$\Rightarrow \quad x = 10.8$

i.e., $\quad PR = 10.8$ cm.

CHECK YOUR PROGRESS 17.1

Find the values of x and y if $\Delta ABC \sim \Delta PQR$

(i)

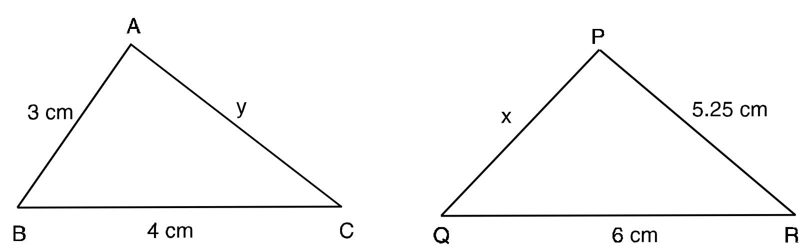


Fig. 17.10

(ii)

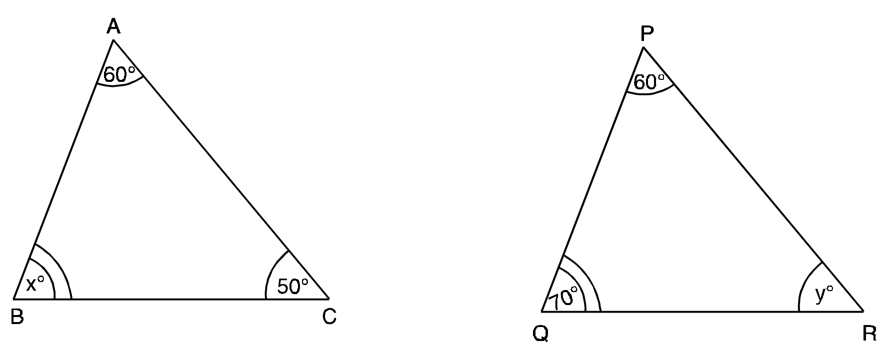


Fig. 17.11

(iii)

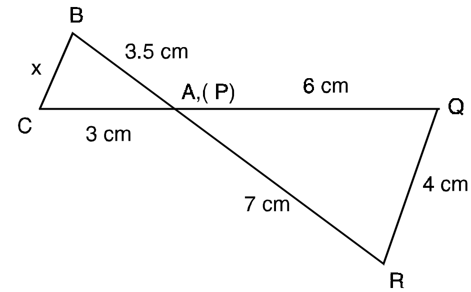


Fig. 17.12

17.6 BASIC PROPORTIONALITY THEOREM

We state below the Basic Proportionality Theorem :

> **If a line is drawn parallel to one side of a triangle, the other two sides of the triangle are divided proportionally.**

Thus, in Fig. 17.13, DE || BC, According to the above result

$\frac{AD}{DB} = \frac{AE}{EC}$

We can easily verify this by measuring AD, DB, AE and EC. You will find that

$$\frac{AD}{DB} = \frac{AE}{EC}$$

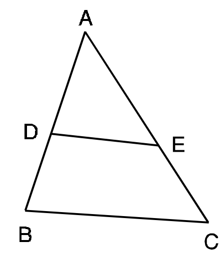


Fig. 17.13

We state the converse of the above result as follows :

> **If a line divides any two sides of a triangle in the same ratio, the line is parallel to third side of the triangle.**

Thus, in Fig. 17.13, if DE divides sides AB and AC of ΔABC such that $\frac{AD}{DB} = \frac{AE}{EC}$, then DE || BC.

We can verify this by measuring $\angle ADE$ and $\angle ABC$ and finding that

$$\angle ADE = \angle ABC$$

These being alternate angles, the lines DE and BC are parallel.

We can verify the above two results by taking different triangles.

Let us solve some examples based on these.

Example 17.3 : In Fig. 17.14, DE || BC. If AD = 3 cm, DB = 5 cm and AE = 6 cm, find AC.

Solution : DE || BC (Given). Let EC = x

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC}$$

$$\therefore \quad \frac{3}{5} = \frac{6}{x}$$

$$\Rightarrow \quad 3x = 30$$

$$\Rightarrow \quad x = 10$$

$$\therefore \quad EC = 10 \text{ cm}$$

$$\therefore \quad AC = AE + EC = 16 \text{ cm.}$$

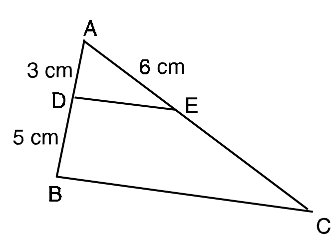


Fig. 17.14

Example 17.4 : In Fig. 17.15, AD = 4 cm, DB = 5 cm, AE = 4.5 cm and EC = $5\frac{5}{8}$ cm. Is DE || BC ? Given reasons for your answer.

Solution : We are given that AD = 4 cm and DB = 5 cm.

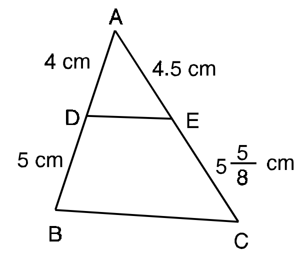


Fig. 17.15

$$\therefore \qquad \frac{AD}{DB} = \frac{4}{5}$$

$$\text{Similarly,} \qquad \frac{AE}{EC} = \frac{4.5}{\frac{45}{8}} = \frac{9}{2} \times \frac{8}{45} = \frac{4}{5}$$

∴ According to converse of Basic Proportionality Theorem

DE || BC.

CHECK YOUR PROGRESS 17.2

1. In Fig. 17.16 (i), (ii) and (iii), PQ || BC. Find the value of x in each case.

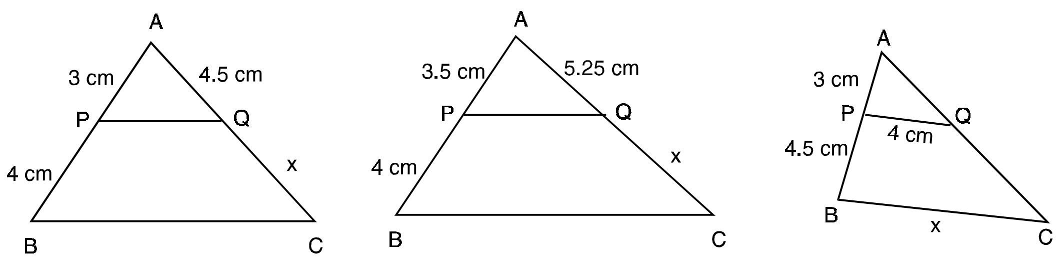


Fig. 17.16

2. In Fig. 17.17 [(i), (ii) and (iii)], find whether DE is parallel to BC or not ? Give reasons for your answer.

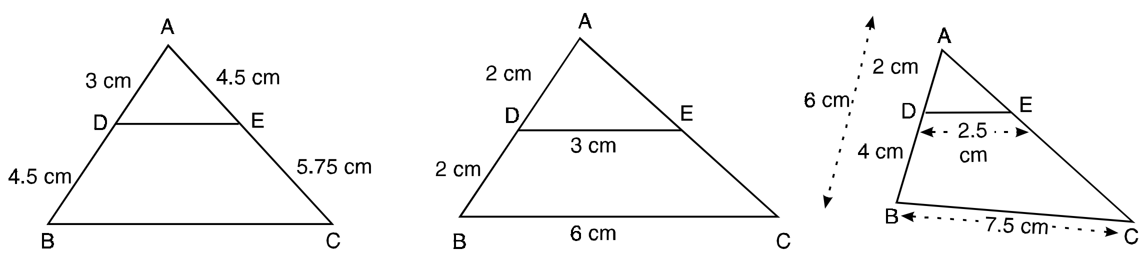


Fig. 17.17

17.7 BISECTOR OF AN ANGLE OF A TRIANGLE

We now state an important result as given below :

The internal bisector of an angle of a triangle divides the opposite side in the ratio of sides containing the angle

Thus, according to the above result, if AD is the internal bisector of $\angle A$ of ΔABC, then

$$\frac{BD}{DC} = \frac{AB}{AC}$$

We can easily verify this by measuring BD, DC, AB and AC and finding the ratios. We will find that

$$\frac{BD}{DC} = \frac{AB}{AC}$$

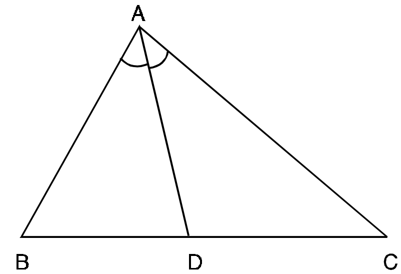


Fig. 17.18

Repeating the same activity with other triangles, we may verify the result.

Let us solve some examples to illustrate this.

Example 17.5 : The sides AB and AC of a triangle are 6 cm and 8 cm. The bisector AD of $\angle A$ intersects the opposite side BC in D such that BD = 4.5 cm. Find the length of segment CD.

Solution : According to the above result, we have

$$\frac{BD}{DC} = \frac{AB}{AC}$$

($\because$ AD is the internal bisector of $\angle A$ of ΔABC)

or $$\frac{4.5}{x} = \frac{6}{8}$$

$$\Rightarrow \quad 6x = 4.5 \times 8$$

$$x = 6$$

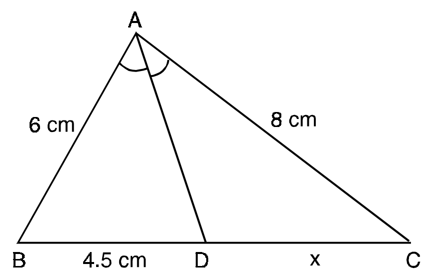


Fig. 17.19

i.e., the length of line-segment CD = 6 cm.

Example 17.6 : The sides of a triangle are 28 cm, 36 cm and 48 cm. Find the lengths of the line-segments into which the smallest side is divided by the bisector of the angle opposite to it.

Solution : The smallest side is of length 28 cm and the sides forming the angle. A opposite to it are 36 cm and 48 cm. Let the angle bisector AD meet BC in D.

$$\therefore \quad \frac{BD}{DC} = \frac{36}{48} = \frac{3}{4}$$

$$\Rightarrow \quad 4BD = 3DC \text{ or } BD = \frac{36}{48}DC = \frac{3}{4}DC$$

$$BC = BD + DC = 28 \text{ cm}$$

$$\therefore \quad DC + \frac{3}{4}DC = 28$$

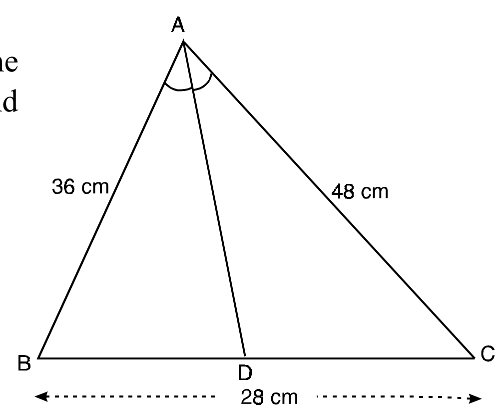


Fig. 17.20

$\therefore \qquad DC = \left(28 \times \frac{4}{7}\right) \text{ cm} = 16$

$\therefore \qquad BD = 12 \text{ cm and } DC = 16 \text{ cm}$

CHECK YOUR PROGRESS 17.3

1. In Fig. 17.21, AD is the bisector of $\angle A$, meeting BC in D. If AB = 4.5 cm, BD = 3 cm, DC = 5 cm, find x.

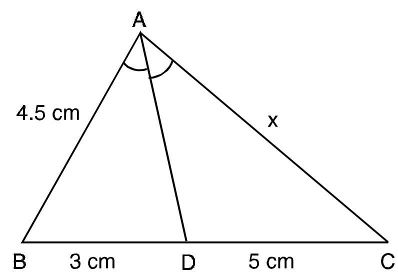


Fig. 17.21

2. In Fig. 17.22, PS is the internal bisector of $\angle P$ of ΔPQR. The dimensions of some of the sides are given in Fig. 17.22. Find x.

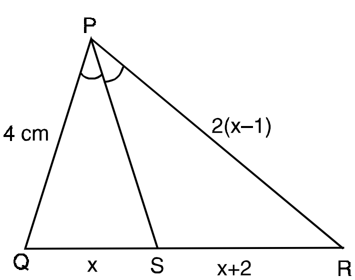


Fig. 17.22

3. In Fig. 17.23, RS is the internal bisector of $\angle R$ of ΔPQR. For the given dimensions, express p, the length of QS in terms of x, y and z.

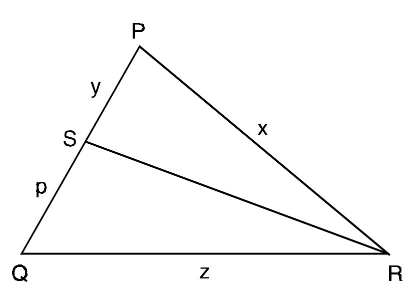


Fig. 17.23

17.8 SOME MORE IMPORTANT RESULTS

Let us study another important result on similarity in connection with a right triangle and the perpendicular from the vertex of the right angle to the opposite side. We state the result below and try to verify the same.

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to each other and to the triangle.

Let us try of verify this by an activity.

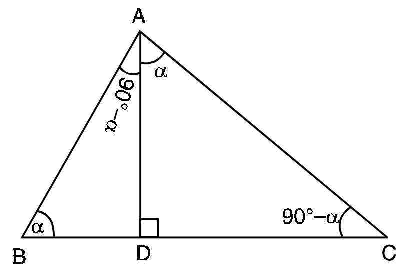


Fig. 17.24

Draw a ΔABC, right angled at A. Draw $AD \perp$ to the hypotenuse BC, meeting it in D.

Let $\angle DBA = \alpha$, As $\angle ADB = 90°$,

$\therefore$ $\angle BAD = 90° - \alpha$

As $\angle BAC = 90°$ and $\angle BAD = 90° - \alpha$,

Therefore $\angle DAC = \alpha$

Similarly, $\angle DCA = 90° - \alpha$

$\therefore$ ΔADB and ΔCDA are similar, as it has all the corresponding angles equal.

Also, the angles of ΔBAC are α, $90°$ and $90° - \alpha$

$$\therefore \quad \Delta ADB \sim \Delta CDA \sim \Delta CAB$$

Another important result is about relation between sides and areas of similar triangles.

It states that

The ratio of the areas of similar triangles is equal to the ratio of the squares on their corresponding sides

Let us verify this result by the following activity. Draw two triangles ABC and PQR which are similar i.e., their sides are proportional.

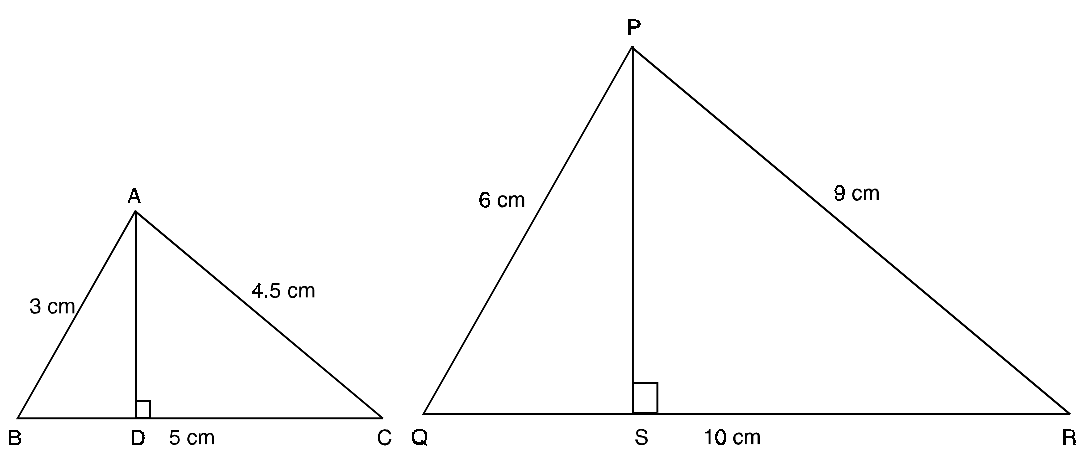


Fig. 17.25

Draw $AD \perp BC$ and $PS \perp QR$

Measure the lengths of AD and PS.

Find the product AD × BC and PS × QR

You will find that AD × BC = BC^2 and PS × QR = QR^2

Now AD × BC = 2. Area of ΔABC

PS × QR = 2. Area of ΔPQR

$$\therefore \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{AD \times BC}{PS \times QR} = \frac{BC^2}{QR^2} \qquad \text{...(i)}$$

$$\text{As} \quad \frac{BC}{QR} = \frac{AB}{PQ} = \frac{AC}{PR}$$

$$\therefore \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{BC^2}{QR^2} = \frac{AB^2}{PQ^2} = \frac{AC^2}{PR^2}$$

The activity may be repeated by taking different pairs of similar triangles.

Let us illustrate these results with the help of examples.

Example 17.7 : Find the ratio of the area of two similar triangles if one pair of their corresponding sides are 2.5 cm and 5.0 cm.

Solution : Let the two triangles be ABC and PQR

Let BC = 2.5 cm and QR = 5.0

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{BC^2}{QR^2} = \frac{(2.5)^2}{(5.0)^2} = \frac{1}{4}$$

Example 17.8 : In a ΔABC, PQ || BC and intersects AB and AC at P and Q respectively. If $\frac{AP}{BP} = \frac{2}{3}$, find the ratio of areas of ΔAPQ and ΔABC.

Solution : In Fig. 17.26,

$$PQ \parallel BC$$

$$\therefore \quad \frac{AP}{BP} = \frac{AQ}{QC} = \frac{2}{3}$$

$$\therefore \quad \frac{BP}{AP} = \frac{3}{2}$$

$$\therefore \quad 1 + \frac{BP}{AP} = 1 + \frac{3}{2} = \frac{5}{2}$$

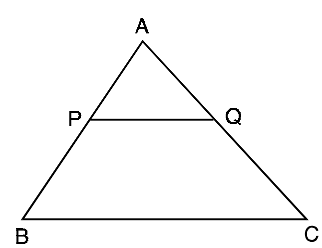


Fig. 17.26

$$\Rightarrow \quad \frac{AB}{AP} = \frac{5}{2} \quad \Rightarrow \quad \frac{AP}{AB} = \frac{2}{5}$$

$$\therefore \quad \frac{\text{Area}(\Delta APQ)}{\text{Area}(\Delta ABC)} = \frac{AP^2}{AB^2} = \left(\frac{AP}{AB}\right)^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}.$$

CHECK YOUR PROGRESS 17.4

1. In Fig. 17.27, ABC is a right triangle with $\angle A = 90°$ and $\angle C = 30°$. Show that $\Delta DAB \sim \Delta DCA \sim \Delta ACB$.

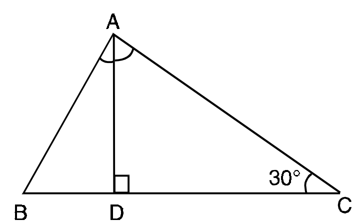


Fig. 17.27

2. Find the ratio of the areas of two similar triangles if the corresponding sides are of lengths 3 cm and 5 cm.

3. In Fig. 17.28, ABC is a triangle in which DE || BC. If AB = 6 cm and AD = 2 cm, find the ratio of the area of ΔADE and trapezium DBCE.

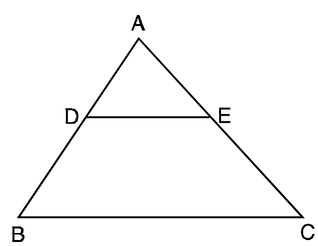


Fig. 17.28

4. P, Q and R are the mid-points of the sides AB, BC and CA of the ΔABC respectively. Show that the area of ΔPQR is one-fourth the area of ΔABC.

5. In two similar triangles ABC and PQR, if the corresponding altitudes AD and PS are in the ratio of 4 : 9, find the ratio of the areas of ΔABC and ΔPQR.

$$\left[\text{Hint : Use } \frac{AB}{PQ} = \frac{AD}{PS} = \frac{BC}{QR} = \frac{CA}{PR}\right]$$

6. If the ratio of the areas of two similar triangles is 16 : 25, find the ratio of their corresponding sides.

17.9 BAUDHAYAN/PYTHAGORAS THEOREM

We know prove an important theorem, called Baudhayan/Phythagorus Theorem using the concept of similarity.

Theorem : In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Given. A right triangle ABC, in which $\angle B = 90°$.

To Prove : $AC^2 = AB^2 + BC^2$

Construction. From B, draw $BD \perp AC$ (See Fig. 17.29)

Proof : $BD \perp AC$

$\therefore \quad \Delta ADB \sim \Delta ABC \quad ...(i)$

and $\quad \Delta BDC \sim \Delta ABC \quad ...(ii)$

From (i), we get $\quad \dfrac{AB}{AC} = \dfrac{AD}{AB}$

$\Rightarrow \quad AB^2 = AC.AD \quad ...(A)$

From (ii), we get $\quad \dfrac{BC}{AC} = \dfrac{DC}{BC}$

$\Rightarrow \quad BC^2 = AC.DC \quad ...(B)$

Adding (A) and (B), we get

$$AB^2 + BC^2 = AC\ (AD + DC)$$
$$= AC.\ AC = AC^2$$

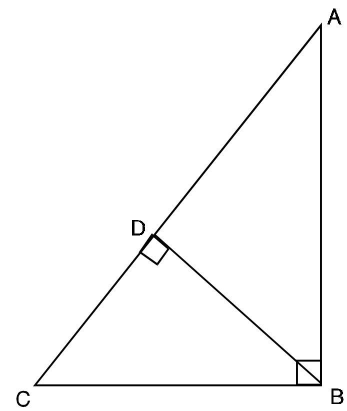


Fig. 17.29

The theorem is known after the name of famous Greek Mathematician Pythagoras. This was originally stated by the Indian Mathematician. Baudhayan about 200 years before Pythagoras.

17.9.1 Converse of Pythagoras Theorem

The conserve of the above theorem states :

In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angle opposite to first side is a right angle.

This result can be verified by the following activity.

Draw a triangle ABC with side 3 cm, 4 cm and 5 cm.

i.e., $\quad AB = 3$ cm, $BC = 4$ cm

and $AC = 5$ cm.

You can see that $AB^2 + BC^2 = (3)^2 + (4)^2$

$= 9 + 16 = 25$

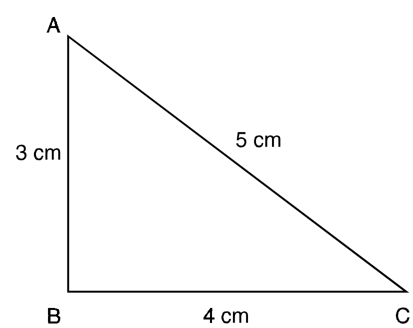


Fig. 17.30

$$AC^2 = (5)^2 = 25$$

$$\therefore \quad AB^2 + BC^2 = AC^2$$

The triangle in Fig. 17.30 satisfies the condition of the above result.

Measure $\angle ABC$, you will find that $\angle ABC = 90°$. Construct triangles of sides 5 cm, 12 cm and 13 cm, and of sides 7 cm, 24 cm, 25 cm. You will again find that the angles opposite to side of length 13 cm and 25 cm are 90° each.

Let us solve some examples using above results.

Example 17.9 : In a right triangle, the sides containing the right angle are of length 5 cm and 12 cm. Find the length of the hypotenuse.

Solution : Let ABC be the right triangle, right angled at B

$$\therefore \quad AB = 5 \text{ cm}, BC = 12 \text{ cm}$$

$$\text{Also,} \quad AC^2 = BC^2 + AB^2$$

$$= (12)^2 + (5)^2$$

$$= 144 + 125$$

$$= 169$$

$$\therefore \quad AC = 13$$

i.e., the length of the hypotenuse is 13 cm.

Example 17.10 : Find the length of diagonal of a rectangle the lengths of whose sides are 3 cm and 4 cm.

Solution : In Fig. 17.31, is a rectangle ABCD. Join the diagonal BD. Now DCB is a right triangle.

$$\therefore \quad BD^2 = BC^2 + CD^2$$

$$= 4^2 + 3^2$$

$$= 16 + 9 = 25$$

$$BD = 5$$

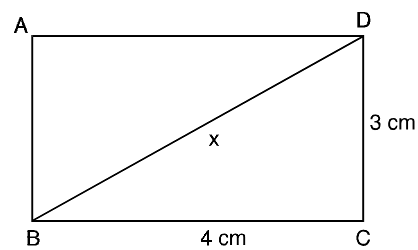


Fig. 17.31

i.e., the length of diagonal of rectangle ABCD is 5 cm.

Example 17.11 : In an equilateral triangle, verify that three times the square on one side is equal to four times the square on its altitude.

Solution : The altitude $AD \perp BC$

and $\quad BD = CD$

Let $\quad AB = BC = CA = 2a$

and $\quad BD = CD = a$

Let $\quad AD = x$

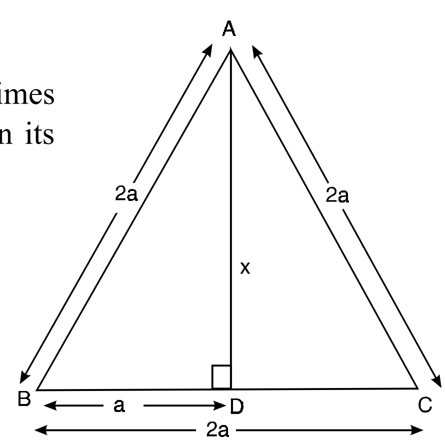


Fig. 17.32

$\therefore \quad x^2 = (2a)^2 - (a)^2 = 3a^2$

$3.(\text{Side})^2 = 3.\ (2a)^2 = 12\ a^2$

$4.(\text{Altitude}) = 4.\ 3a^2 = 12a^2$

Hence the result.

Example 17.12 : ABC is a right triangle, right angled at C. If CD, the length of perpendicular from C on AB is p, BC = a, AC = b and AB = c, show that

(i) $pc = ab$

(ii) $\dfrac{1}{p^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$

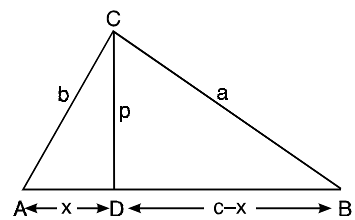


Fig. 17.33

Solution : (i) $CD \perp AB$

$\therefore \quad \Delta ABC \sim \Delta ACD$

$\therefore \quad \dfrac{c}{b} = \dfrac{a}{p}$

$\Rightarrow \quad pc = ab.$

(ii) $\quad AB^2 = AC^2 + BC^2$

or $\quad c^2 = b^2 + a^2$

$$\left(\frac{ab}{p}\right)^2 = b^2 + a^2$$

or $\quad \dfrac{1}{p^2} = \dfrac{a^2 + b^2}{a^2 b^2} = \dfrac{1}{a^2} + \dfrac{1}{b^2}$

CHECK YOUR PROGRESS 17.5

1. The sides of certain triangles are given below. Determine which of them are right triangles : [AB = c, BC = a, CA = b]

(i) a = 4 cm, b = 5 cm, c = 3 cm

(ii) a = 1.6 cm, b = 3.8 cm, c = 4 cm

(iii) a = 9 cm, b = 16 cm, c = 18 cm

(iv) a = 7 cm, b = 24 cm, c = 25 cm

2. Two poles of height 6 m and 11 m, stand on a plane ground. If the distance between their feet is 12 m, find the distance between their tops.

3. Find the length of the diagonal of a square of side 10 cm.

4. In Fig. 17.34, $\angle C$ is acute and $AD \perp BC$. Show that $AB^2 = AC^2 + BC^2 - 2BC.\ DC$

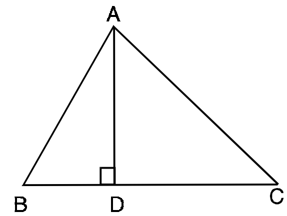


Fig. 17.34

5. L and M are the mid-points of the sides AB and AC of ΔABC, right angled at B. Show that

$4LC^2 = AB^2 + 4BC^2$

6. P and Q are points on the sides CA and CB respectively of ΔABC, right angled at C. Prove that

$AQ^2 + BP^2 = AB^2 + PQ^2$

7. PQR is an isosceles right triangle with $\angle Q = 90°$. Prove that $PR^2 = 2PQ^2$.

8. A ladder is placed against a wall such that its top reaches upto a height of 4 m of the wall. If the foot of the ladder is 3 m away from the wall, find the length of the ladder.

LET US SUM UP

- Objects which have the same shape but different sizes are called similar objects.
- Any two polygons, with corresponding angles equal and corresponding sides proportional, are similar.
- Two triangles are said to be similar, if

  (a) their corresponding angles are equal and

  (b) their corresponding sides are proportional
- Criteria of similarity
  - AAA criterion
  - SSS criterion
  - SAS criterion
- If a line is drawn parallel to one-side of a triangle, it divides the other two sides in the same ratio and its converse.

- The internal bisector of an angle of a triangle divides the opposite side in the ratio of sides containing the angle.
- If a perpendicular is drawn from the vertex of the right angle of a right angled triangle to the hypotenuse, the triangles so formed are similar to each other and to the given triangle.
- The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides.
- In a right triangle, the square on the hypotenuse is equal to sum of the squares on the remaining two sides – (Baudhyan) Pythagoras Theorem
- In a triangle, if the square on one side is equal to the sum of the squares on the remaining two sides, then the angle opposite to the first side is a right angle – converse of (Baudhayan) Pythagoras Theorem.

TERMINAL EXERCISE

1. Write the criteria for the similarity of two polygons.

2. Enumerate different criteria for the similarity of the two triangles.

3. In which of the following cases, Δ's ABC and PQR are similar

(i) $\angle A = 40°$, $\angle B = 60°$, $\angle C = 80°$, $\angle P = 40°$, $\angle Q = 60°$ and $\angle R = 80°$

(ii) $\angle A = 50°$, $\angle B = 70°$, $\angle C = 60°$, $\angle P = 50°$, $\angle Q = 60°$ and $\angle R = 70°$

(iii) AB = 2.5 cm, BC = 4.5 cm, CA = 3.5 cm

PQ = 5.0 cm, QR = 9.0 cm, RP = 7.0 cm

(iv) AB = 3 cm, BC = 4 cm, CA = 5.0 cm

PQ = 4.5 cm, QR = 7.5 cm, RP = 6.0 cm.

4. In Fig. 17.35, AD = 3 cm, AE = 4.5 cm, DB = 4.0 cm, find CE, given that DE || BC.

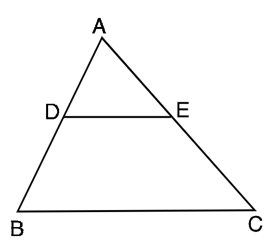


Fig. 17.35

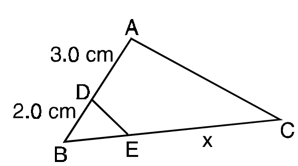


Fig. 17.36

5. In Fig. 15.36, DE || AC. From the dimension given in the figure, find the value of x.

6. In Fig. 17.37 is shown a ΔABC in which AD = 5 cm, DB = 3 cm, AE = 2.50 cm and EC = 1.5 cm. Is DE || BC ? Give reasons for your answer.

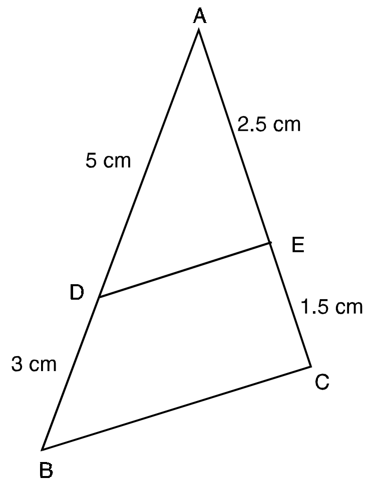


Fig. 17.37

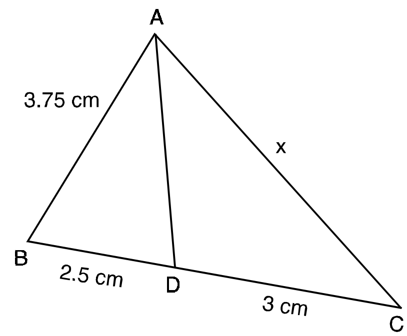


Fig. 17.38

7. In Fig. 17.38, AD is the internal bisector of $\angle$A of ΔABC. From the given dimension, find x.

8. The perimeter of two similar Δ's ABC and DEF are 12 cm and 18 cm. Find the ratio of the area of ΔABC to that of ΔDEF.

9. The altitudes AD and PS of two similar Δ's ABC and PQR are of length 2.5 cm and 3.5 cm. Find the ratio of area of ΔABC to that of ΔPQR.

10. Which of the following are right triangles ?

(i) AB = 5 cm, BC = 12 cm, CA = 13 cm

(ii) AB = 8 cm, BC = 6 cm, CA = 10 cm

(iii) AB = 10 cm, BC = 5 cm, CA = 6 cm

(iv) AB = 25 cm, BC = 24 cm, CA = 7 cm

(v) AB = $a^2 + b^2$, BC = 2ab, CA = $a^2 - b^2$

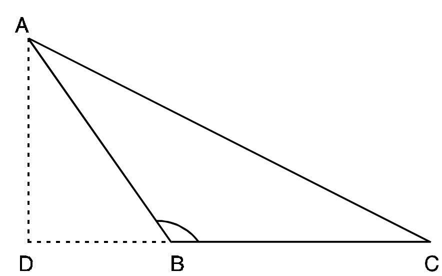


Fig. 17.39

11. Find the area of an equilateral triangle of side 2a.

12. Two poles of height 12 m and 17 m, stand on a plane ground and the distance between their feet is 12 m. Find the distance between their tops.

13. In Fig. 17.39, show that

$AB^2 = AC^2 + BC^2 + 2BC.CD$

14. A ladder is placed against a wall and its top reaches a point at a height of 8 m from the ground. If the distance between the wall and foot of the ladder is 6 m, find the length of the ladder.

15. In an equilateral triangle, show that three times the square of a side equals four times the square on medians.

ANSWERS

Check Your Progress 17.1

1. (i) $x = 4.5, y = 3.5$ (ii) $x = 70, y = 50$ (iii) $x = 2$ cm, $y = 7$ cm

Check Your Progress 17.2

1. (i) 6 (ii) 6 (iii) 10 cm

2. (i) No (ii) Yes (iii) Yes

Check Your Progress 17.3

1. 7.5 cm **2.** 4 cm

3. $\frac{yz}{x}$ ($x = -1$ is not possible)

Check Your Progress 17.4

2. 9 : 25 **3.** 1 : 8 **5.** 16 : 81

6. 4 : 5

Check Your Progress 17.5

1. (i) Yes (ii) No (iii) No

(iv) Yes

2. 13 m **3.** $10\sqrt{2}$ cm **8.** 5 m

Terminal Exercise

3. (i) and (iii) **4.** 6 cm **5.** 4.5 cm

6. Yes : $\frac{AD}{DB} = \frac{AE}{EC}$ **7.** 4.5 cm

8. 4 : 9 **9.** 25 : 49

10. (i), (ii), (iv) and (v)

11. $\sqrt{3}a^2$ **12.** 13 m **14.** 10 m