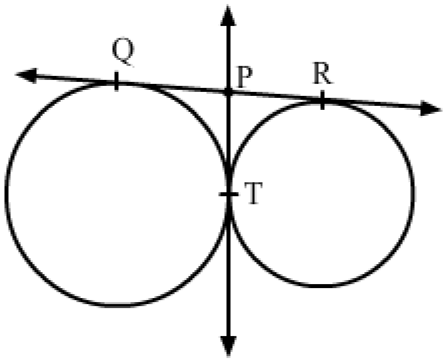
Summative Assessment –II Subject Mathematics class 10 CBSE Board 2014[Delhi] Set 1:

Q1 The first three terms of an AP respectively are $3y - 1$, $3y + 5$ and $5y + 1$. Then y equals:

(A) –3 (B) 4 (C) 5 (D) 2

Solution: (C) 5

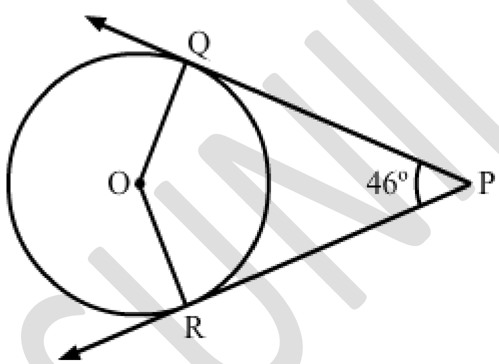
Q2 In Fig. 1, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, then the length of QR (in cm) is :



(A) 3.8 (B) 7.6 (C) 5.7 (D) 1.9

Solution: (B) 7.6

Q3 In Fig. 2, PQ and PR are two tangents to a circle with centre O. If ∠QPR = 46°, then ∠QOR equals:



(A) 67° (B) 134° (C) 44° (D) 46°

Solution: (B) 134°

Q4. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall, then the length of the ladder (in metres) is:

(A) 43 (B) 43 (C) 22 (D) 4

Solution: (D) 4

Q5 If two different dice are rolled together, the probability of getting an even number on both dice, is:

(A) 136 (B) 12 (C) 16 (D) 14

Solution: (D) 14

Q6 A number is selected at random from the numbers 1 to 30. The probability that it is a prime number is:

(A) 23 (B) 16 (C) 13 (D) 1130

Solution: (C) 13

Q7 If the points A(x, 2), B(−3, −4) and C(7, − 5) are collinear, then the value of x is:

(A) −63 (B) 63 (C) 60 (D) −60

Solution: (A) −63

Q8 The number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm, is:

(A) 3 (B) 5 (C) 4 (D) 6

Solution: (B) 5

SECTION- B

Q9 Solve the quadratic equation $2x^2 + ax - a^2 = 0$ for x.

Solution: Comparing the given equation with the standard quadratic equation ($ax^2 + bx + c = 0$),

We get:

a = 2, b = a and c =$-a^2$

Using the quadratic formula,

= - b ± $\sqrt{b2-4ac}$/2a,

we get: x = - a ± $\sqrt{a2-\ 4\times 2\times -a2}$/ 2×2 =- a±$\sqrt{9a2}$ /4 = (-a ± 3a)/4⇒ x= (- a+3a)/4 ⇒

⇒ x = a/2 or x= - a -3a / 4 = - a

So, the solutions of the given quadratic equation are x=a/2 or x = -a.

Q10. The first and the last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find its common difference.

Let a be the first term and d be the common difference.

Given: $a = 5$ $T_n = 45$ $S_n = 400$

We know:

$T_n = a + (n-1)d$ ⇒ $45 = 5 + (n-1)d$ ⇒ $40 = (n-1)d$ ... (i) and S_n = n/2[a + Tn]

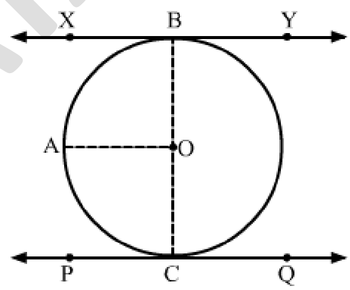
⇒400=n/2[5+45]⇒n/2=400/50 ⇒ n = 16

On substituting n = 16 in (i), we get:

40 = (16 − 1)d ⇒ 40 = (15)d ⇒ d = 40/15=8/3 Thus, the common difference is 8/3.

Q11 Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

Solution:



Now, XB∥AO

⇒ ∠XBO + ∠AOB = 180° sum of adjacent interior angles is 180°)

Now, ∠XBO = 90° (A tangent to a circle is perpendicular to the radius through the point of contact)

⇒ 90° + ∠AOB = 180°

⇒ ∠AOB = 180° − 90° = 90°

Similarly, ∠AOC = 90°

∴ ∠AOB + ∠AOC = 90° + 90° = 180°

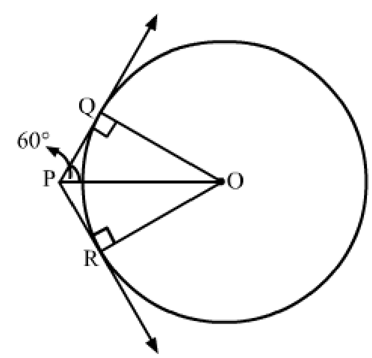
Hence, BOC is a straight line passing through O.

Thus, the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

Q12 If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that ∠QPR = 120°, prove that 2PQ = PO.

Solution: We know that the radius is perpendicular to the tangent at the point of contact.

∴∠OQP=90∘



We also know that the tangents drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point.

∴∠QPO=60∘

Now, in ΔQPO:

cos60∘ =PQ/ PO⇒1/2=PQ/PO⇒2PQ=PO

Q13 Rahim tosses two different coins simultaneously. Find the probability of getting at least one tail.

Solution: Rahim tosses two coins simultaneously. The sample space of the experiment is {HH, HT, TH, TT}.

Total number of outcomes = 4

Outcomes in favour of getting at least one tail on tossing the two coins = {HT, TH, TT}

Number of outcomes in favour of getting at least one tail = 3

∴ Probability of getting at least one tail on tossing the two coins

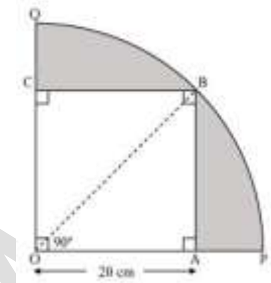
= Number of favourable outcomesTotal number of outcomes = 3/4

Q14 In fig. 3, a square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 20 cm, find the area of the shaded region. (Use π = 3.14)

Solution: In ΔOAB:

$OB^2 = OA^2 + AB^2 = (20)^2 + (20)^2 = 2 \times (20)^2$

$\Rightarrow OB = 20\sqrt{2}$



Radius of the circle, $r = 20\sqrt{2}$ cm

Area of quadrant OPBQ = θ360°×πr2

=90°360° × 3.14 × 2022 cm2

= 14 × 3.14 ×800 cm2

= 628 cm^2

Area of square OABC = $(Side)^2 = (20)^2\ cm^2 = 400\ cm^2$

∴ Area of the shaded region = Area of quadrant OPBQ − Area of square OABC

$= (628 - 400)\ cm^2 = 228\ cm^2$

SECTION- C

Q15 Solve the equation 4x-3=52x+3; for *x*.

The solutions of the given equation is − 2 or 1.

Q16 If the seventh term of an AP is 19 and its ninth term is 17, find its 63rd term.

Solution: Let *a* be the first term and *d* be the common difference of the given A.P.

Given:

a7 = 19 ⇒ a + 6d = 19 ... (1)

a9 = 17 a + 8d = 17 ...(2)

Subtracting equation (1) from (2), we get:

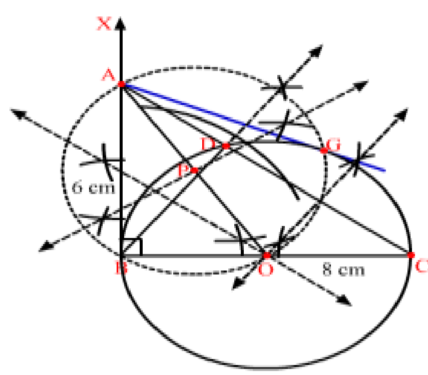
2d = - 2 ⇒ d = -1

Putting d = -1 in equation (1), we get:

a + 6 ×-1 = 19⇒ a = 19 + 6= 25

∴ a_{63} = a + 62 d = 25 + 62 x -1 = 25 – 62 = - 37

Thus, the 63rd term of the given A.P. is 1.



Q17 Draw a right triangle ABC in which AB = 6 cm, BC = 8 cm and ∠B = 90°. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

Q18 If the point A(0, 2) is equidistant from the points B(3, *p*) and C(*p*, 5), find *p*. Also find the length of AB.

Solution: The given points are A(0, 2), B(3, *p*) and C(*p*, 5).

It is given that A is equidistant from B and C.

∴ AB = AC

⇒ $AB^2 = AC^2$

⇒ $(3 - 0)^2 + (p - 2)^2 = (p - 0)^2 + (5 - 2)^2$

⇒ $9 + p^2 + 4 - 4p = p^2 + 9$

⇒ $4 - 4p = 0$

⇒ $4p = 4$

⇒ $p = 1$

Thus, the value of p is 1.

Length of AB = 3-02+1-22=32+-12=9+1=10 units

Q19 Two ships are there in the sea on either side of a light house in such a way that the ships and the light house are in the same straight line. The angles of depression of two ships as observed from the top of the light house are 60° and 45°. If the height of the light house is 200 m, find the distance between the two ships. [Use π =1.73]

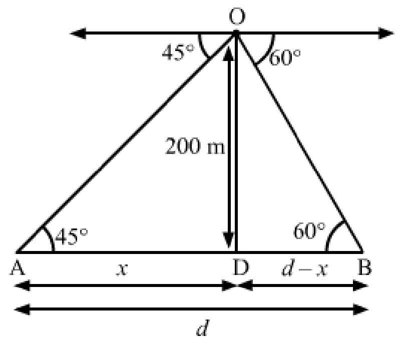
Solution: Let d be the distance between the two ships. Suppose the distance of one of the ships from the tower is x metres, then the distance of the other ship from the tower is d-x metres.

In right-angled Δ ADO, we have:

tan45°=OD/AD⇒1=200/x⇒x=200 ...(1)

In right-angled Δ BDO, we have:

tan60°=OD/BD=200/(d-x)



⇒√3 =200/(d-x) ⇒d-x=200/√3 ⇒d =200/√3 + x ⇒d =200/√3 + 200 =316

Thus, the distance between two ships is 316 m.

Q20 If the points A(−2, 1), B(a, b) and C(4, −1) are collinear and $a - b = 1$, find the values of a and b.

Solution: The given points are A (−2, 1), B (a, b) and C (4, −1).

Since the given points are collinear, the area of the triangle ABC is 0.

⇒12x1y2-y3+x2y3-y1+x3y1-y2=0

Here, x1= -2, y1 =1, x2 =a, y2 =b and x3 = 4, y3 = -1

∴ ½[-2(b+1)+a(-1-1)+4(1-b)=0

⇒a+3b=1 --------(i)

Given: a-b=1 ⇒ a -1 = b

putting this value in (i) ⇒ a + 3(a -1) = 1 ⇒ a = 1

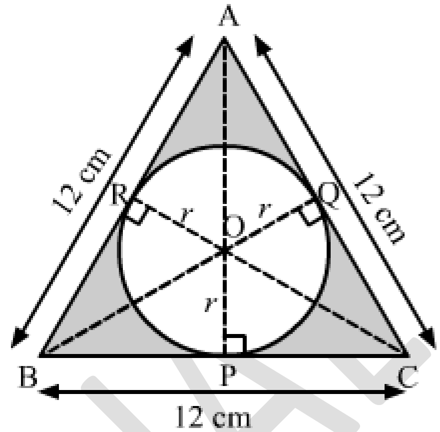
Thus, the values of a and b are 1 and 0, respectively.

Q21 In Fig 4, a circle is inscribed in an equilateral triangle ABC of side 12 cm. Find the radius of inscribed circle and the area of the shaded region. [Use π = 3.14 and 3=1.73]

Construction:

Join OA, OB and OC.

Draw: OP⊥BC ; OQ⊥AC ; OR⊥AB



Let the radius of the circle be *r* cm.

Area of ΔAOB + Area of ΔBOC + Area of ΔAOC = Area of ΔABC

⇒ 1/2×r[AB + BC + AC]= √3/4× $(\text{Side})^2$

⇒ 1/2× 12 [12 + 12 + 12] = √34× $(12)^2$

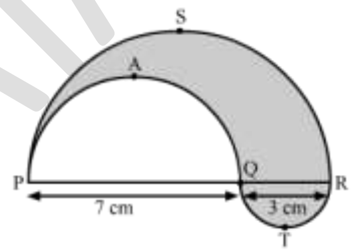
⇒ $r = 2\sqrt{3}$ cm.

Now, area of the shaded region = Area of ΔABC − Area of the inscribed circle

= $[\sqrt{3}/4\times (12)^2 - \pi(2\sqrt{3})^2]\ cm^2$

= 24.6 cm^2

Q22 In Fig.5, PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of the shaded region. [Use π = 3.14]



Solution: Radius of semicircle PSR = 12×10 cm= 5 cm

Radius of semicircle RTQ = 12×3 cm = 1.5 cm

Radius of semicircle PAQ = 12×7 cm = 3.5 cm

Perimeter of the shaded region = Circumference of semicircle PSR + Circumference of semicircle RTQ + Circumference of semicircle PAQ = [π(5) + π(1.5) + π(3.5)] cm = 3.14 × 10 cm = 31.4 cm

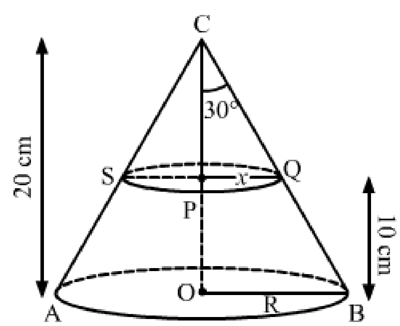
Q23 A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled completely?

Solution: Rate of flow of water = *v* = 4 km/h=4000 m/h

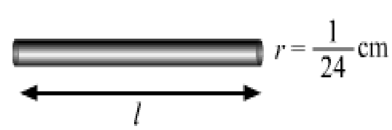
Let *t* be the time taken to fill the tank.

So, the water flows through the pipe in *t* hours will be equal to the volume of the tank.

∴$\pi r^2 \times v \times t = \pi R^2 H$ ⇒ 0.1 x 0.1 ×4000 × t = 5 x 5 ×2 t = 50/40 = 1hr 15min



Q24 A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter 1/12 cm, find the length of the wire.



Solution:

Let ACB be the cone whose vertical angle ∠ACB = 60°. Let *R* and *x* be the radii of the lower and upper end of the frustum. Here, height of the cone, OC = 20 cm

Let us consider P as the mid-point of OC. After cutting the cone into two parts through P,

OP = 20/2=10 cm Also, ∠ACO and ∠OCB = $1/2\times 60^o = 30^o$

After cutting cone CQS from cone CBA, the remaining solid obtained is a frustum.

Now, in triangle CPQ: $\tan 30^\circ = x/10 \Rightarrow 1/\sqrt{3} = x/10 \Rightarrow x = 10/\sqrt{3}$ cm

In triangle COB: $\tan 30^\circ = R/CO \Rightarrow 1/\sqrt{3} = R/20 \Rightarrow R = 20/\sqrt{3}$ cm

Volume of the frustum, $V = 1/3\ \pi\ h\ [R^2 + x^2 - rx] = 1/3\pi \times 10\ [(20/\sqrt{3})^2 + (10/\sqrt{3})^2 + 20/\sqrt{3} \times 10/\sqrt{3}\ ] = 7000\pi/9$

The volumes of the frustum and the wire formed are equal.

$\pi \times (1/24)^2 \times l = 7000\ \pi/9 \Rightarrow l = (7000 \times 24 \times 24)/9 \Rightarrow l = 448000$ cm $= 4480$ m

Q25 The difference of two natural numbers is 5 and the difference of their reciprocals is 110. Find the numbers.

Solution: Let one natural number be *x.*

Given: Difference between the natural numbers = 5

∴ Other natural number = $x + 5$

Difference of their reciprocals = 110 (given)

∴ $(1/x) - 1/(x+5) = 110 \Rightarrow x^2 + 5x - 50 = 0 \Rightarrow (x+10)(x-5) = 0 \Rightarrow x + 10 = 0$ or $x - 5 = 0 \Rightarrow x = -10$ or $x = 5$

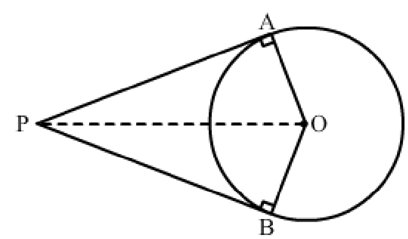
$\Rightarrow x = 5$ ∵ x is a natural number

∴ One natural number = 5   Other natural number = $x + 5 = 5 + 5 = 10$

Thus, the two natural numbers are 5 and 10.

Q26 Prove that the length of the tangents drawn from an external point to a circle are equal.

Let AP and BP be the two tangents to the circle with centre O.



To prove: AP = BP

Proof:

In ΔAOP and ΔBOP:

OA = OB   (radii of the same circle)

$\angle OAP = \angle OBP = 90^\circ$ (since tangent at any point of a circle is perpendicular to the radius through the point of contact)

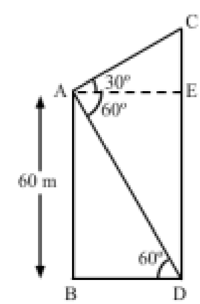
OP = OP   (common)

∴ $\Delta AOP \cong \Delta BOP$ (by R.H.S. congruence criterion)

∴ AP = BP   (corresponding parts of congruent triangles)

Hence, the length of the tangents drawn from an external point to a circle are equal.

Q27 The angles of elevation and depression of the top and the bottom of a tower from the top of a building, 60 m high, are 30° and 60° respectively. Find the difference between the heights of the building and the tower and the distance between them.



Solution:

Let AB be the building and CD be the tower.

In right ΔABD: $AB/BD = \tan 60^\circ \Rightarrow 60/BD = \sqrt{3} \Rightarrow BD = 60/\sqrt{3} \Rightarrow BD = 20\sqrt{3}$

In right ΔACE: $CE/AE = \tan 30^\circ \Rightarrow CE/BD = 1/\sqrt{3}$ ∵ $AE = BD \Rightarrow CE = 20\sqrt{3}/\sqrt{3} = 20$

Height of the tower = CE + ED = CE + AB = 20 m + 60 m = 80 m

Difference between the heights of the tower and the building = 80 m − 60 m = 20 m

Distance between the tower and the building = BD = $20\sqrt{3}$ m

Q28 A bag contains cards numbered from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is:

(i) an odd number (ii) a multiple of 5 (iii) a perfect square (iv) an even prime number

Solution: Total number of cards = 49

(i) The odd numbers form 1 to 49 are 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47 and 49.

Total number of favourable outcomes = 25

∴ Required probability = Total number of favourable outcomesTotal number of outcomes= 25/49

(ii) The numbers 5, 10, 15, 20, 25, 30, 35, 40 and 45 are multiples of 5.

Total number of favourable outcomes = 9

∴ Required probability =Total number of favourable outcomesTotal number of outcomes =9/49

(iii) The numbers 1, 4, 9, 16, 25, 36 and 49 are perfect squares.

Total number of favourable outcomes = 7

∴ Required probability = Total number of favourable outcomesTotal number of outcomes= 7/49=1/7

(iv) We know that there is only one even prime number, which is 2.

Total number of favourable outcomes = 1

∴ Required probability = Total number of favourable outcomesTotal number of outcomes= 1/49

Q29 Find the ratio in which the point P(x, 2) divides the line segment joining the points A(12, 5) and B(4, – 3). Also find the value of x.

Solution: Let the point P (x, 2) divide the line segment joining the points A (12, 5) and B (4, −3) in the ratio k:1.

Then, the coordinates of P are (4k+12)/(k+1),(-3k+5)/(k+1)

Now, the coordinates of P are (x, 2).

(4k+12)/(k+1) =x and (-3k+5)/(k+1) = 2 ⇒-3k+5=2k+2 ⇒5k=3 ⇒k=3/5

Substituting k = 3/5 in (4k+12)/(k+1) =x, we get: ⇒ x = 9

Thus, the value of x is 9.

Also, the point P divides the line segment joining the points A(12, 5) and (4, −3) in the ratio 3/5:1, i.e. 3:5.

Q30 Find the values of k for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots. Also find these roots.

Solution: $(k + 4)x^2 + (k + 1)x + 1 = 0$

Since the given quadratic equation has equal roots, its discriminant should be zero.

∴ D = 0 ⇒ $b^2 – 4ac = 0$ ⇒ ⇒ $(k + 1)^2 – 4 \times (k + 4) \times 1=0$ ⇒k^2-2k-15=0 ⇒k^2-5k+3k-15=0 ⇒k=5 or -3

Thus, the values of k are 5 and −3.

For $k = 5$: $\Rightarrow 9x^2+6x+1=0$ $\Rightarrow 3x2+2x3x+1=0$ $\Rightarrow (3x+1)^2=0$ $\Rightarrow x= -1/3$

For $k = -3$: $x^2-2x+1=0$ $\Rightarrow (x-1)^2=0$ $\Rightarrow x=1$

Thus, the equal root of the given quadratic equation is either 1 or -13.

Q31 In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

Solution:

The sum of terms in an arithmetic progression is
the average of first an last terms added, times the number of terms added.
So, if

a_1 = the first term,

d = the common difference, then

$a_n = a_1 + (n-1) \cdot d$ = the nth term,

$a_1 + a_2 + \ldots + a_9 + a_{10} = \frac{(a_1 + a_{10}) \cdot 10}{2} = 210$ --> $a_1 + a_{10} = \frac{210 \cdot 2}{10}$ --> $a_1 + a_{10} = 42$

Substituting $a_{10} = a_1 + 9d$ we get

$a_1 + a_1 + 9d = 42$ --> $\boxed{2a_1 + 9d = 42}$

The last 15 of the 50 terms are all but the first $50 - 15 = 35$ terms.

That is a_{36} through a_{50}.
Their sum is

$\frac{(a_{36} + a_{50}) \cdot 15}{2} = 2565$ --> $a_{36} + a_{50} = \frac{2565 \cdot 2}{15}$ --> $a_{36} + a_{50} = 342$

Substituting $a_{36} = a_1 + 35d$ and $a_{50} = a_1 + 49d$ we get

$a_1 + 35d + a_1 + 49d = 342$ --> $2a_1 + 84d = 342$ --> $a_1 + 42d = 171$

Now we solve

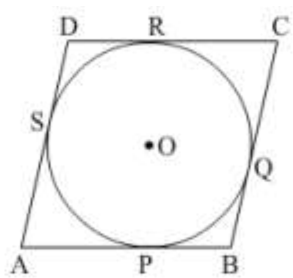
$$\begin{cases} a_1 + 42d = 171 \\ 2a_1 + 9d = 42 \end{cases}$$

to get $\boxed{a_1 = 3}$ and $\boxed{d = 4}$

Q32 Prove that a parallelogram circumscribing a circle is a rhombus.

Given: ABCD be a parallelogram circumscribing a circle with centre O.

To prove: ABCD is a rhombus.



We know that the tangents drawn to a circle from an exterior point are equal in length.

∴ AP = AS, BP = BQ, CR = CQ and DR = DS.

AP + BP + CR + DR = AS + BQ + CQ + DS

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

∴ AB + CD = AD + BC or 2AB = 2BC (since AB = DC and AD = BC)

∴ AB = BC = DC = AD.

Therefore, ABCD is a rhombus.

Q33 Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which 25th of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant?

Height (h) of the conical vessel = 11 cm
Radius (r_1) of the conical vessel = 2.5 cm
Radius (r_2) of the metallic spherical balls = 0.52=0.25 cm
Let n be the number of spherical balls that were dropped in the vessel.

Volume of the water spilled = Volume of the spherical balls dropped

25×Volume of cone = n×Volume of one spherical ball

⇒2.5 x 2.5 ×11=n×10×0.25 x 0.25 ⇒ 68.75=0.15625n

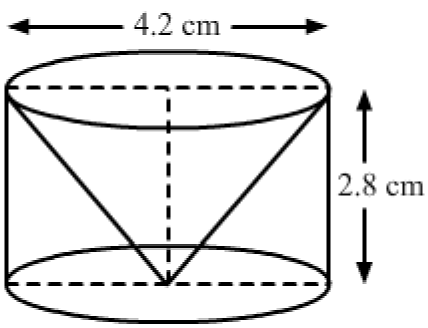
⇒n=440 ; Hence, the number of spherical balls that were dropped in the vessel is 440

Sushant made the arrangement so that the water that flows out, irrigates the flower beds. This shows the conservation of water

Q34 From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. Take π=227

Solution:

The following figure shows the required cylinder and the conical cavity.



Given:

Height (*h*) of the conical part = Height (*h*) of the cylindrical part = 2.8 cm

Diameter of the cylindrical part = Diameter of the conical part = 4.2 cm

∴ Radius (*r*) of the cylindrical part = Radius (*r*) of the conical part = 2.1 cm

Slant height (*l*) of the conical part = r2+h2

=2.12+2.82 cm =4.41+7.84 cm= 12.25 cm= 3.5 cm

Total surface area of the remaining solid = Curved surface area of the cylindrical part + Curved surface area of the conical part + Area of the cylindrical base =73.92 cm2