Class 8^{th} Chapter: Set [Study material by JSUNIL [Central public school, Samastipur]

What is set (in mathematics)?

The collection of well-defined distinct objects is known as a set. The word well-defined refers to a specific property which makes it easy to identify whether the given object belongs to the set or not. The word 'distinct' means that the objects of a set must be all different. Sets are denoted by Capital l letters

Examples:

A = {Color of rainbow] ⇒ express set of the Color of rainbow

A = [1,2,3,……..} ⇒ Represent the set of natural numbers

B = [a, e, i, o, u] ⇒ Represent the set of vowels

M = {1/2, 2/3,3/4…………99/100}

Elements of Set:

The different objects that form a set are called the elements of a set. The elements of the set are written in any order under curly bracket. Elements are denoted by small letters.

B = [a, e, i, o, u] ⇒ Represent the set of vowels

Here element of set B are a, e, i, o, u

Notation of a Set:

A set is usually denoted by capital letters and elements are denoted by small letters

If x is an element of set A, then we say $x \in A$. [x belongs to A]

If x is not an element of set A, then we say $x \notin A$. [x does not belong to A]

x |x ⇒ is read as x such that x

How to state that whether the objects form a set or not?

⇒ Are "All problems of this book, which are difficult to solve" form a set?

No. Some problems may be difficult for one person but may not be difficult for some other persons i.e the given objects are not well-defined.

⇒ A collection of 'lovely flowers' of your garden

No, as the objects (flowers) to be included are not well-defined because flower may appear lovely to one person may not be so to the other person.

⇒ "Young singers" of a competition

A group of "Young singers" is not a set, as the range of the ages of young singers is not given and so it can't be decided that which singer is to be considered young i.e., the objects are not well-defined.

⇒ All problems of math's book, which are difficult you to solve .

The given objects form a set. It can easily be found that which problem are difficult to solve to you and which are not difficult to solve.

We representation a set by following three methods:

(i) Descriptive form or Description method

In this method full detail or description of the elements of the set is given in curly brackets.

For example:

(a) The set of even numbers less than 7 is written as: A = { even numbers less than 7}.

(b) A set of vowel in word BEAUTIFUL is written as: A = { vowel in word BEAUTIFUL }.

(c) A set of numbers multiple of 11 and smaller than 55. is written as: A = { multiple of 11 smaller than 55 }

(ii) Roster method or tabular form :

In this method elements of set are written in form of letters or number without description

For example:

(a) Let N denote the set of first five natural numbers.
Therefore, N = {1, 2, 3, 4, 5} → Roster Form

(b) The set of all vowels of the English alphabet.
Therefore, V = {a, e, i, o, u} → Roster Form

(c) The set of all odd numbers less than 9.
Therefore, X = {1, 3, 5, 7} → Roster Form

(d) The set of all natural number which divide 12.
Therefore, Y = {1, 2, 3, 4, 6, 12} → Roster Form

(e) The set of all letters in the word MATHEMATICS.
Therefore, Z = {M, A, T, H, E, I, C, S} → Roster Form

(f) W is the set of last four months of the year.
Therefore, W = {September, October, November, December} → Roster Form

(iii) Rule method or set builder form:

In this method a rule, or the formula or the statement is written within the pair of brackets so that the set is well defined

Example:

⇒ Let P is a set of counting numbers greater than 12

P = {x | x is a counting number and greater than 12}

⇒ If X = {4, 5, 6, 7} . This is expressed in roster form.

Let us express in set builder form.

X = {x : x is a natural number and $3 < x < 8$}

⇒ The set A of all odd natural numbers can be written as

A = {x : x is a natural number and $x = 2n + 1$ for $n \in W$}

⇒ The set A of all even natural numbers can be written as
A = {x : x is a natural number and $x = 2n$ for $n \in W$}

⇒ Statement form: {I is a set of integers lying between -2 and 3}

Roster form: I = {-1, 0, 1, 2} and Set builder form: I = {x : $x \in I$, $-2 < x < 3$}

Practice

(i)Write in set builder form

a. A = {1/2, 2/3,3/4…………99/100}

A = { x|x = n/(n+1) , n x $\in$ N , n >100}

b. A = { 0, 1/2, 2/3,3/4,4/5,5/6}

A = { x|x = n/(n+1) , n x $\in$ W , $0 \le n \ge 5$}

c. A = { 1,8,27, 64,81}

A = { x|x = n^3 , n x $\in$ N , n $\ge$ 4}

d. { -15,-10,-5,0,5,10,,15}

A = { x|x = 5n , n x $\in$ I , $-3 \le n \ge 3$}

e. A = = {2, 4, 6, 8, ……….}

{x :x is a natural number, which are divisible by 2}

Different types of sets

Finite Set:

A set which contains a definite number of elements is called a finite set. Empty set is also called a finite set.

For example:

- The set of all colors in the rainbow.
- N = {x : x ∈ N, x < 7}
- P = {2, 3, 5, 7, 11, 13, 17, ... 97}

Infinite Set:

The set whose elements cannot be listed, i.e., set containing never-ending elements is called an infinite set.

For example:

- Set of all points in a plane
- A = {x : x ∈ N, x > 1}
- Set of all prime numbers
- B = {x : x ∈ W, x = 2n}

Note:

All infinite sets cannot be expressed in roster form.

For example:

The set of real numbers since the elements of this set do not follow any particular pattern.

Cardinal Number of a Set:

The number of distinct elements in a given set A is called the cardinal number of A. It is denoted by n(A).

For example:

- A {x : x ∈ N, x < 5}

A = {1, 2, 3, 4}

Therefore, n(A) = 4

● B = set of letters in the word ALGEBRA

B = {A, L, G, E, B, R}

Therefore, n(B) = 6

Empty Set or Null Set:

A set which does not contain any element is called an empty set, or the null set or the void set and it is denoted by ∅ and is read as phi. In roster form, ∅ is denoted by {}. An empty set is a finite set, since the number of elements in an empty set is finite, i.e., 0.

For example: (a) The set of whole numbers less than 0.

(b) Clearly there is no whole number less than 0.

Therefore, it is an empty set.

(c) N = {x : x ∈ N, 3 < x < 4}

● Let A = {x : 2 < x < 3, x is a natural number}

Here A is an empty set because there is no natural number between
2 and 3.

● Let B = {x : x is a composite number less than 4}.

Here B is an empty set because there is no composite number less than 4.

Note:

∅ ≠ {0} ∴ has no element.

{0} is a set which has one element 0.

The cardinal number of an empty set, i.e., n(∅) = 0

Singleton Set:

A set which contains only one element is called a singleton set.

For example:

● A = {x : x is neither prime nor composite}

It is a singleton set containing one element, i.e., 1.

● B = {x : x is a whole number, x < 1}

This set contains only one element 0 and is a singleton set.

● Let A = {x : x $\in$ N and $x^2 = 4$}

Here A is a singleton set because there is only one element 2 whose square is 4.

● Let B = {x : x is a even prime number}

Here B is a singleton set because there is only one prime number which is even, i.e., 2.

Equivalent Sets:

Two sets A and B are said to be equivalent if their cardinal number is same, i.e., n(A) = n(B). The symbol for denoting an equivalent set is '$\leftrightarrow$'.

For example:

A = {1, 2, 3} Here n(A) = 3

B = {p, q, r} Here n(B) = 3

Therefore, A $\leftrightarrow$ B

Equal sets:

Two sets A and B are said to be equal if they contain the same elements. Every element of A is an element of B and every element of B is an element of A.

For example:

A = {p, q, r, s}

B = {p, s, r, q}

Therefore, A = B

Disjoint Sets:

Two sets A and B are said to be disjoint, if they do not have any element in common.

For example;

A = {x : x is a prime number}

B = {x : x is a composite number}.

Clearly, A and B do not have any element in common and are disjoint sets.

Overlapping sets:

Two sets A and B are said to be overlapping if they contain at least one element in common.

For example;

- A = {a, b, c, d}

B = {a, e, i, o, u}

- X = {x : x ∈ N, x < 4}

Y = {x : x ∈ I, -1 < x < 4}

Here, the two sets contain three elements in common, i.e., (1, 2, 3)

Subset:

If A and B are two sets, and every element of set A is also an element of set B, then A is called a subset of B and we write it as **A ⊆ B** or **B ⊇ A**

The symbol ⊂ stands for 'is a subset of' or 'is contained in'

For example;

1. Let A = {2, 4, 6}

B = {6, 4, 8, 2}

Here A is a subset of B ⇒ **A ⊆ B**

But B is not the subset of A ⇒ B ⊄ A [⊄ denotes 'not a subset of']

Super Set:

Whenever a set A is a subset of set B, we say the B is a superset of A and we write, B ⊇ A.

Symbol ⊇ is used to denote 'is a super set of'

For example;

A = {a, e, i, o, u}

B = {a, b, c, ..., z}

Here A ⊆ B i.e., A is a subset of B but B ⊇ A i.e., B is a super set of A

Proper Subset:

If A and B are two sets, then A is called the proper subset of B if $A \subseteq B$ but $B \supseteq A$ i.e., $A \neq B$. The symbol '$\subset$' is used to denote proper subset. Symbolically, we write $A \subset B$.

For example;

1. A = {1, 2, 3, 4}   Here n(A) = 4

B = {1, 2, 3, 4, 5}   Here n(B) = 5

We observe that, all the elements of A are present in B but the element '5' of B is not present in A.

So, we say that A is a proper subset of B. Symbolically, we write it as $A \subset B$

Power Set:

The collection of all subsets of set A is called the power set of A. It is denoted by P(A). In P(A), every element is a set.

For example;

If A = {p, q} then all the subsets of A will be

P(A) = {∅, {p}, {q}, {p, q}}

Number of elements of P(A) = n[P(A)] = 4 = 22

In general, n[P(A)] = 2m where m is the number of elements in set A.

Universal Set

A set which contains all the elements of other given sets is called a **universal set**. The symbol for denoting a universal set is ∪ or **ξ**.

For example;

1. If A = {1, 2, 3}   B = {2, 3, 4}   C = {3, 5, 7}

then U = {1, 2, 3, 4, 5, 7}

[Here $A \subseteq U$, $B \subseteq U$, $C \subseteq U$ and $U \supseteq A$, $U \supseteq B$, $U \supseteq C$]

2 . If A = {a, b, c}   B = {d, e}   C = {f, g, h, i}

then U = {a, b, c, d, e, f, g, h, i} can be taken as universal set.

Courtesy: math-only-math