Question 1:

Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.

(i) Yes. It can be observed that trapezium ABCD and triangle PCD have a common base CD and these are lying between the same parallel lines AB and CD.

(ii) 

(v) 

Answer:

(i)
No. It can be observed that parallelogram PQRS and trapezium MNRS have a common base RS. However, their vertices, (i.e., opposite to the common base) P, Q of parallelogram and M, N of trapezium, are not lying on the same line.

(iii)

Yes. It can be observed that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same parallel lines PS and QR.

(iv)

No. It can be observed that parallelogram ABCD and triangle PQR are lying between same parallel lines AD and BC. However, these do not have any common base.

(v)

Yes. It can be observed that parallelogram ABCD and parallelogram APQD have a common base AD and these are lying between the same parallel lines AD and BQ.
(vi)

No. It can be observed that parallelogram PBCS and PQRS are lying on the same base PS. However, these do not lie between the same parallel lines.
Exercise 9.2

Question 1:

In the given figure, ABCD is parallelogram, \(AE \perp DC\) and \(CF \perp AD\). If \(AB = 16\) cm, \(AE = 8\) cm and \(CF = 10\) cm, find \(AD\).

Answer:

In parallelogram ABCD, \(CD = AB = 16\) cm \([\text{Opposite sides of a parallelogram are equal}]\) We know that
Area of a parallelogram = Base \(\times\) Corresponding altitude
Area of parallelogram ABCD = \(CD \times AE = AD \times CF\)

\[16\,\text{cm} \times 8\,\text{cm} = AD \times 10\,\text{cm}\]

\[AD = \frac{16 \times 8}{10} \,\text{cm} = 12.8\,\text{cm}\]

Thus, the length of \(AD\) is 12.8 cm.

Question 2:

If \(E, F, G\) and \(H\) are respectively the mid-points of the sides of a parallelogram ABCD show that

\[\frac{1}{2} \frac{\text{ar}(EFGH)}{\text{ar}(ABCD)} = 1\]
Answer:

Let us join HF.

In parallelogram ABCD,

\( AD = BC \) and \( AD \parallel BC \) (Opposite sides of a parallelogram are equal and parallel)

\( AB = CD \) (Opposite sides of a parallelogram are equal)

\[ \Rightarrow \frac{1}{2} AD = \frac{1}{2} BC \quad \text{and} \quad AH \parallel BF \]

\( \Rightarrow AH = BF \) and \( AH \parallel BF \) (H and F are the mid-points of AD and BC) Therefore, ABFH is a parallelogram.

Since \( \triangle HEF \) and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

\[ \therefore \text{Area} (\triangle HEF) = \frac{1}{2} \text{Area} (ABFH) \ldots (1) \]

Similarly, it can be proved that

\[ \text{Area} (\triangle HGF) = \frac{1}{2} \text{Area} (HDCF) \ldots (2) \]

On adding equations (1) and (2), we obtain

\[ \text{Area} (\triangle HEF) + \text{Area} (\triangle HGF) = \frac{1}{2} \text{Area} (ABFH) + \frac{1}{2} \text{Area} (HDCF) \]

\[ = \frac{1}{2} \left[ \text{Area} (ABFH) + \text{Area} (HDCF) \right] \]

\[ \Rightarrow \text{Area} (EFGH) = \frac{1}{2} \text{Area} (ABCD) \]
Question 3:

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that \( \text{ar} \ (\text{APB}) = \text{ar} \ (\text{BQC}) \).

Answer:

It can be observed that \( \Delta \text{BQC} \) and parallelogram ABCD lie on the same base BC and these are between the same parallel lines AD and BC.

\[
\frac{1}{2} \text{Area} \ (\Delta \text{BQC}) = \frac{1}{2} \text{Area} \ (\text{ABCD}) \ ... \ (1)
\]

Similarly, \( \Delta \text{APB} \) and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.

\[
\frac{1}{2} \text{Area} \ (\Delta \text{APB}) = \frac{1}{2} \text{Area} \ (\text{ABCD}) \ ... \ (2)
\]

From equation (1) and (2), we obtain Area (\( \Delta \text{BQC} \)) = Area (\( \Delta \text{APB} \))

Question 4:

In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

(i) \( \frac{1}{2} \text{ar} \ (\text{APB}) + \text{ar} \ (\text{PCD}) = \text{ar} \ (\text{ABCD}) \)

(ii) \( \text{ar} \ (\text{APD}) + \text{ar} \ (\text{PBC}) = \text{ar} \ (\text{APB}) + \text{ar} \ (\text{PCD}) \)

[Hint: Through P, draw a line parallel to AB]
Answer:

(i) Let us draw a line segment EF, passing through point P and parallel to line segment AB.

In parallelogram ABCD,

AB \parallel EF (By construction) ...

(1) ABCD is a parallelogram.

\therefore \ AD \parallel BC (Opposite sides of a parallelogram) \Rightarrow \ AE \parallel BF ... (2)

From equations (1) and (2), we obtain AB \parallel EF and AE \parallel BF

Therefore, quadrilateral ABFE is a parallelogram.

It can be observed that \( \Delta \)APB and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

\[
\text{Area (} \Delta \text{APB)} = \frac{1}{2} \text{Area (ABFE)} \quad ... (3)
\]

Similarly, for \( \Delta \)PCD and parallelogram EFCD,

\[
\text{Area (} \Delta \text{PCD)} = \frac{1}{2} \text{Area (EFCD)} \quad ... (4)
\]

Adding equations (3) and (4), we obtain
Let us draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD,

\[ \text{MN} \parallel \text{AD} \quad \text{(By construction)} \ldots \]

(6) ABCD is a parallelogram.

\[ \therefore \text{AB} \parallel \text{DC} \quad \text{(Opposite sides of a parallelogram)} \Rightarrow \text{AM} \parallel \text{DN} \ldots (7) \]

From equations (6) and (7), we obtain MN \parallel AD and AM \parallel DN

Therefore, quadrilateral AMND is a parallelogram.

It can be observed that \( \Delta \text{APD} \) and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

\[ \therefore \frac{1}{2} \text{Area (\Delta APD)} = \frac{1}{2} \text{Area (AMND)} \ldots (8) \]

Similarly, for \( \Delta \text{PCB} \) and parallelogram MNCB,

\[ \frac{1}{2} \text{Area (\Delta PCB)} = \frac{1}{2} \text{Area (MNCB)} \ldots (9) \]

Adding equations (8) and (9), we obtain
On comparing equations (5) and (10), we obtain
\[ \text{Area (ΔAPD) + Area (ΔPBC) = Area (ΔAPB) + Area (ΔPCD)} \]

Question 5:
In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that
(i) \( \text{ar (PQRS) = ar (ABRS)} \)
(ii) \( \text{ar (ΔPXS) = \frac{1}{2} ar (PQRS)} \)

Answer:
(i) It can be observed that parallelogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.
\[ \therefore \text{Area (PQRS) = Area (ABRS)} \ldots (1) \]

(ii) Consider ΔAXS and parallelogram ABRS.
As these lie on the same base and are between the same parallel lines AS and BR,
\[ \therefore \text{Area (ΔAXS) = \frac{1}{2} Area (ABRS)} \ldots (2) \]
From equations (1) and (2), we obtain
\[ \text{Area (ΔAXS) = \frac{1}{2} Area (PQRS)} \]
Question 6:

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Answer:

From the figure, it can be observed that point A divides the field into three parts. These parts are triangular in shape – ΔPSA, ΔPAQ, and ΔQRA.

Area of ΔPSA + Area of ΔPAQ + Area of ΔQRA = Area of PQRS ... (1)

We know that if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

∴ Area (ΔPAQ) = \frac{1}{2} Area (PQRS) ... (2)

From equations (1) and (2), we obtain

\[ \frac{1}{2} \text{ Area (ΔPSA)} + \frac{1}{2} \text{ Area (ΔQRA)} = \frac{1}{2} \text{ Area (PQRS)} \]  ... (3)

Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.
Exercise 9.3

Question 1:

In the given figure, E is any point on median AD of a ΔABC. Show that ar (ABE) = ar (ACE)

Answer:

AD is the median of ΔABC. Therefore, it will divide ΔABC into two triangles of equal areas.

\[ \text{Area (ΔABD)} = \text{Area (ΔACD)} \ldots (1) \]

ED is the median of ΔEBC.

\[ \text{Area (ΔEBD)} = \text{Area (ΔECD)} \ldots (2) \]

On subtracting equation (2) from equation (1), we obtain

\[ \text{Area (ΔABD)} - \text{Area (ΔEBD)} = \text{Area (ΔACD)} - \text{Area (ΔECD)} \]

\[ \text{Area (ΔABE)} = \text{Area (ΔACE)} \]

Question 10:

ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD. (See the given figure). Show that

(i) ΔAPB ≅ ΔCQD

(ii) AP = CQ
Answer:

(i) In ΔAPB and ΔCQD, 
\[ \angle APB = \angle CQD \text{ (Each 90°)} \]
\[ AB = CD \text{ (Opposite sides of parallelogram ABCD)} \]
\[ \angle ABP = \angle CDQ \text{ (Alternate interior angles for AB || CD)} \] 
\[ \therefore \angle APB \cong \angle CQD \text{ (By AAS congruency)} \]

(ii) By using the above result 
\[ \triangle APB \cong \triangle CQD, \text{ we obtain} \]
\[ AP = CQ \text{ (By CPCT)} \]

Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area. Answer:

We know that diagonals of parallelogram bisect each other. Therefore, O is the mid-point of AC and BD.

BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.

\[ \therefore \text{Area (ΔAOB) = Area (ΔBOC)} \ldots \]

(1) In ΔABCD, CO is the median.

\[ \therefore \text{Area (ΔBOC) = Area (ΔCOD)} \ldots (2) \]

Similarly, Area (ΔCOD) = Area (ΔAOD) \ldots (3)

From equations (1), (2), and (3), we obtain

\[ \text{Area (ΔAOB) = Area (ΔBOC) = Area (ΔCOD) = Area (ΔAOD)} \]

Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.
Question 4:

In the given figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).

Answer:
Consider ΔACD.

Line-segment CD is bisected by AB at O. Therefore, AO is the median of ΔACD.

∴ Area (ΔACO) = Area (ΔADO) ... (1)

Considering ΔBCD, BO is the median.

∴ Area (ΔBCO) = Area (ΔBDO) ... (2)

Adding equations (1) and (2), we obtain

Area (ΔACO) + Area (ΔBCO) = Area (ΔADO) + Area (ΔBDO)
⇒ Area (ΔABC) = Area (ΔABD)

Question 6:

In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)
(ii) ar (DCB) = ar (ACB)
(iii) DA || CB or ABCD is a parallelogram.

[Hint: From D and B, draw perpendiculars to AC.]
Let us draw DN $\perp AC$ and BM $\perp AC$.

(i) In $\triangle DON$ and $\triangle BOM$,

$\bot DNO = \bot BMO$ (By construction)

$\bot DON = \bot BOM$ (Vertically opposite angles) $OD = OB$ (Given)

By AAS congruence rule, $\triangle DON \cong \triangle BOM$

$\bot DN = BM$ ... (1)

We know that congruent triangles have equal areas.

Area ($\triangle DON$) = Area ($\triangle BOM$) ... (2)

In $\triangle DNC$ and $\triangle BMA$,

$\bot DNC = \bot BMA$ (By construction) $CD = AB$ (Given)

$\bot DN = BM$ [Using equation (1)]

$\triangle DNC \cong \triangle BMA$ (RHS congruence rule)

Area ($\triangle DNC$) = Area ($\triangle BMA$) ... (3)

On adding equations (2) and (3), we obtain

Area ($\triangle DON$) + Area ($\triangle DNC$) = Area ($\triangle BOM$) + Area ($\triangle BMA$)

Therefore, Area ($\triangle DOC$) = Area ($\triangle AOB$)

(ii) We obtained,

Area ($\triangle DOC$) = Area ($\triangle AOB$)

Area ($\triangle DOC$) + Area ($\triangle OCB$) = Area ($\triangle AOB$) + Area ($\triangle OCB$)

(Adding Area ($\triangle OCB$) to both sides)

Area ($\triangle DCB$) = Area ($\triangle ACB$)

(iii) We obtained,
Area (ΔDCB) = Area (ΔACB)

If two triangles have the same base and equal areas, then these will lie between the same parallels.
⊥ DA || CB ... (4)

In quadrilateral ABCD, one pair of opposite sides is equal (AB = CD) and the other pair of opposite sides is parallel (DA || CB).
Therefore, ABCD is a parallelogram.

**Question 7:**

D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

**Answer:**

Since ΔBCE and ΔBCD are lying on a common base BC and also have equal areas, ΔBCE and ΔBCD will lie between the same parallel lines.
⊥ DE || BC

**Question 8:**

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and E respectively, show that
ar (ABE) = ar (ACF)
Answer:

It is given that

\[ XY \parallel BC \perp EY \parallel BC \]
\[ BE \parallel AC \perp BE \parallel CY \]

Therefore, EBCY is a parallelogram. It is given that

\[ XY \parallel BC \perp XF \parallel BC \]
\[ FC \parallel AB \perp FC \parallel XB \]

Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

\[ \text{Area (EBCY)} = \text{Area (BCFX)} \ldots (1) \]

Consider parallelogram EBCY and \( \triangle AEB \)

These lie on the same base BE and are between the same parallels BE and AC.

\[ \frac{1}{2} \cdot \text{Area (\triangle ABE)} = \frac{1}{2} \cdot \text{Area (EBCY)} \ldots (2) \]

Also, parallelogram BCFX and \( \triangle ACF \) are on the same base CF and between the same parallels CF and AB.

\[ \frac{1}{2} \cdot \text{Area (\triangle ACF)} = \frac{1}{2} \cdot \text{Area (BCFX)} \ldots (3) \]

From equations (1), (2), and (3), we obtain

\[ \text{Area (\triangle ABE)} = \text{Area (\triangle ACF)} \]
Question 9:

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that 

ar (ABCD) = ar (PBQR).

[Hint: Join AC and PQ. Now compare area (ACQ) and area (APQ)]

Answer:

Let us join AC and PQ.

ΔACQ and ΔAQP are on the same base AQ and between the same parallels AQ and CP.

Area (ΔACQ) = Area (ΔAPQ)

Area (ΔACQ) − Area (ΔABQ) = Area (ΔAPQ) − Area (ΔABQ)

Area (ΔABC) = Area (ΔQBP) ... (1)

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

\[ \frac{1}{2} \text{ Area} (\Delta ABC) = \text{ Area} (\Delta QBP) \] ...

(2)
Area ($\triangle QBP$) = $\frac{1}{2}$ Area (PBQR) ... (3)

From equations (1), (2), and (3), we obtain

$$\frac{1}{2} \text{ Area (ABCD)} = \frac{1}{2} \text{ Area (PBQR)}$$

$$\text{Area (ABCD)} = \text{Area (PBQR)}$$

**Question 10:**

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

**Answer:**

It can be observed that $\triangle DAC$ and $\triangle DBC$ lie on the same base DC and between the same parallels AB and CD.

$$\text{Area (DAC)} = \text{Area (DBC)}$$

$$\text{Area (DAC)} - \text{Area (DOC)} = \text{Area (DBC)} - \text{Area (DOC)}$$

$$\text{Area (AOD)} = \text{Area (BOC)}$$

**Question 11:**

In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) $\text{ar (ACB)} = \text{ar (ACF)}$

(ii) $\text{ar (AEDF)} = \text{ar (ABCDE)}$
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Answer:

(i) $\triangle ACB$ and $\triangle ACF$ lie on the same base AC and are between the same parallels AC and BF.

Area ($\triangle ACB$) = Area ($\triangle ACF$)

(ii) It can be observed that Area ($\triangle ACB$) = Area ($\triangle ACF$)

Area ($\triangle ACB$) + Area (ACDE) = Area (ACF) + Area (ACDE)

Area (ABCDE) = Area (AEDF)

Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Answer:

Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.
Join diagonal BD and draw a line parallel to BD through point A. Let it meet the extended side CD of ABCD at point E. Join BE and AD. Let them intersect each other at O. Then, portion $\triangle AOB$ can be cut from the original field so that the new shape of the field will be $\triangle BCE$. (See figure)

We have to prove that the area of $\triangle AOB$ (portion that was cut so as to construct Health Centre) is equal to the area of $\triangle DEO$ (portion added to the field so as to make the area of the new field so formed equal to the area of the original field)

![Diagram showing the relationship between the triangles](image)

It can be observed that $\triangle DEB$ and $\triangle DAB$ lie on the same base BD and are between the same parallels BD and AE.

\[
\text{Area (}\triangle DEB\text{)} = \text{Area (}\triangle DAB\text{)}
\]

\[
\text{Area (}\triangle DEB\text{)} - \text{Area (}\triangle DOB\text{)} = \text{Area (}\triangle DAB\text{)} - \text{Area (}\triangle DOB\text{)}
\]

\[
\text{Area (}\triangle DEO\text{)} = \text{Area (}\triangle AOB\text{)}
\]

**Question 13:**

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

[Hint: Join CX.]

**Answer:**

![Diagram showing the relationship between the triangles](image)

It can be observed that $\triangle ADX$ and $\triangle ACX$ lie on the same base AX and are between the same parallels AB and DC.
Area (ΔADX) = Area (ΔACX) ... (1)

ΔACY and ΔACX lie on the same base AC and are between the same parallels AC and XY.

Area (ΔACY) = Area (ACX) ... (2) From equations (1) and (2), we obtain Area (ΔADX) = Area (ΔACY)

Question 14:
In the given figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR). Answer:

Since ΔABQ and ΔPBQ lie on the same base BQ and are between the same parallels AP and BQ,
Area (ΔABQ) = Area (ΔPBQ) ... (1)

Again, ΔBCQ and ΔBRQ lie on the same base BQ and are between the same parallels BQ and CR.
Area (ΔBCQ) = Area (ΔBRQ) ... (2)

On adding equations (1) and (2), we obtain
Area (ΔABQ) + Area (ΔBCQ) = Area (ΔPBQ) + Area (ΔBRQ) ⊥ Area (ΔAQC) = Area (ΔPBR)

Question 15:
Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Answer:
It is given that

\[ \text{Area (ΔAOD)} = \text{Area (ΔBOC)} \]

\[ \text{Area (ΔAOD) + Area (ΔAOB) = Area (ΔBOC) + Area (ΔAOB)} \]
\[ \text{Area (ΔADB) = Area (ΔACB)} \]

We know that triangles on the same base having areas equal to each other lie between the same parallels.

Therefore, these triangles, ΔADB and ΔACB, are lying between the same parallels. i.e., AB \parallel CD

Therefore, ABCD is a trapezium.

**Question 16:**

In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

**Answer:**

\[ \text{Area (ΔDRC)} = \text{Area (ΔDPC)} \]

As ΔDRC and ΔDPC lie on the same base DC and have equal areas, therefore, they must lie between the same parallel lines.

\[ \perp \text{DC \parallel RP} \]

Therefore, DCPR is a trapezium.
It is also given that

\[ \text{Area (} \Delta BDP \text{)} = \text{Area (} \Delta ARC \text{)} \]

\[ \text{Area (} \text{BDP} \text{)} - \text{Area (} \Delta DPC \text{)} = \text{Area (} \Delta ARC \text{)} - \text{Area (} \Delta DRC \text{)} \]

\[ \text{Area (} \Delta BDC \text{)} = \text{Area (} \Delta ADC \text{)} \]

Since \( \Delta BDC \) and \( \Delta ADC \) are on the same base CD and have equal areas, they must lie between the same parallel lines.

\[ \perp AB \parallel CD \]

Therefore, ABCD is a trapezium.
Question 1:

Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Answer:

As the parallelogram and the rectangle have the same base and equal area, therefore, these will also lie between the same parallels.

Consider the parallelogram ABCD and rectangle ABEF as follows.

Here, it can be observed that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.

We know that opposite sides of a parallelogram or a rectangle are of equal lengths. Therefore,

\[
AB = EF \text{ (For rectangle)} \\
AB = CD \text{ (For parallelogram)} \\
CD = EF \\
AB + CD = AB + EF \quad (1)
\]

Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.

\[
\perp AF < AD \\
\perp AF + BE < AD + BC \quad (2)
\]

From equations (1) and (2), we obtain AB + EF + AF + BE < AD + BC + AB + CD
Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD

Question 2:

In the following figure, D and E are two points on BC such that BD = DE = EC. Show that \( \text{ar} \ (ABD) = \text{ar} \ (ADE) = \text{ar} \ (AEC) \).

Can you answer the question that you have left in the *Introduction* of this chapter, whether the field of *Budhia* has been actually divided into three parts of equal area?

[Remark: Note that by taking \( BD = DE = EC \), the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into \( n \) equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide \( \Delta ABC \) into \( n \) triangles of equal areas.]

Answer:

Let us draw a line segment AM \( \perp \) BC.

We know that,

\[
\text{Area of a triangle} = \frac{1}{2} \times \text{Base} \times \text{Altitude}
\]
It is given that DE = BD = EC

\[ \frac{1}{2} \times DE \times AM = \frac{1}{2} \times BD \times AM = \frac{1}{2} \times EC \times AM \]

\[ \text{Area } \triangle ADE = \text{Area } \triangle ABD = \text{Area } \triangle AEC \]

It can be observed that Budhia has divided her field into 3 equal parts.

**Question 3:**

In the following figure, ABCD, DCFE and ABFE are parallelograms. Show that \text{ar} (ADE) = \text{ar} (BCF).

![Parallelogram Diagram]

**Answer:**

It is given that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.

\[ \text{AD} = \text{BC} \ldots (1) \]

Similarly, for parallelograms DCEF and ABFE, it can be proved that \text{DE} = \text{CF} \ldots (2)

And, \text{EA} = \text{FB} \ldots (3)

In \triangle ADE and \triangle BCF,

\[ \text{AD} = \text{BC} \ [\text{Using equation } (1)] \ 	ext{DE} = \text{CF} \ [\text{Using equation } (2)] \]
Question 4:

In the following figure, ABCD is parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that

\[ \text{ar (BPC)} = \text{ar (DPQ)}. \]

[Hint: Join AC.]

Answer:

It is given that ABCD is a parallelogram.

\[ AD \parallel BC \text{ and } AB \parallel DC \] (Opposite sides of a parallelogram are parallel to each other) Join point A to point C.

Consider \( \triangle APC \) and \( \triangle BPC \)
ΔAPC and ΔBPC are lying on the same base PC and between the same parallels PC and AB. Therefore,

\[ \text{Area (ΔAPC)} = \text{Area (ΔBPC)} \] ... (1)

In quadrilateral ACDQ, it is given that \( AD = CQ \)

Since ABCD is a parallelogram,

\( AD \parallel BC \) (Opposite sides of a parallelogram are parallel)

CQ is a line segment which is obtained when line segment BC is produced.

\( AD \parallel CQ \)

We have,

\( AC = DQ \) and \( AC \parallel DQ \)

Hence, ACQD is a parallelogram.

Consider ΔDCQ and ΔACQ

These are on the same base CQ and between the same parallels CQ and AD. Therefore,

\[ \text{Area (ΔDCQ)} = \text{Area (ΔACQ)} \]

\[ \text{Area (ΔDCQ)} - \text{Area (ΔPQC)} = \text{Area (ΔACQ)} - \text{Area (ΔPQC)} \]

\[ \text{Area (ΔDPQ)} = \text{Area (ΔAPC)} \] ... (2)

From equations (1) and (2), we obtain

\[ \text{Area (ΔBPC)} = \text{Area (ΔDPQ)} \]

**Question 5:**

In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that
(i) \( \text{ar}(\triangle BDE) = \frac{1}{4} \text{ar}(\triangle ABC) \)

(ii) \( \text{ar}(\triangle BDE) = \frac{1}{2} \text{ar}(\triangle BAE) \)

(iii) \( \text{ar}(\triangle ABC) = 2 \text{ar}(\triangle BEC) \)

(iv) \( \text{ar}(\triangle BFE) = \text{ar}(\triangle AFD) \)

(v) \( \text{ar}(\triangle BFE) = 2 \text{ar}(\triangle FED) \)

(vi) \( \text{ar}(\triangle FED) = \frac{1}{8} \text{ar}(\triangle AFC) \)

[Hint: Join EC and AD. Show that BE || AC and DE || AB, etc.] Answer:

(i) Let G and H be the mid-points of side AB and AC respectively.

Line segment GH is joining the mid-points. Therefore, it will be parallel to third side BC and also its length will be half of the length of BC (mid-point theorem).
GH = \frac{1}{2} BC and GH \parallel BD

GH = BD = DC and GH \parallel BD (D is the mid-point of BC)

Consider quadrilateral GHDB.

GH \parallel BD and GH = BD

Two line segments joining two parallel line segments of equal length will also be equal and parallel to each other.

Therefore, BG = DH and BG \parallel DH

Hence, quadrilateral GHDB is a parallelogram.

We know that in a parallelogram, the diagonal bisects it into two triangles of equal area.

Hence, Area (\Delta BDG) = Area (\Delta HGD)

Similarly, it can be proved that quadrilaterals DCHG, GDHA, and BEDG are parallelograms and their respective diagonals are dividing them into two triangles of equal area.

ar (\Delta GDH) = ar (\Delta CHD) (For parallelogram DCHG) ar (\Delta GDH) = ar (\Delta HAG) (For parallelogram GDHA) ar (\Delta BDE) = ar (\Delta DBG) (For parallelogram BEDG)

ar (\Delta ABC) = ar(\Delta BDG) + ar(\Delta GDH) + ar(\Delta DCH) + ar(\Delta AGH) ar (\Delta ABC) = 4 \times ar(\Delta BDE)
Hence,

(ii) Area (ΔBDE) = Area (ΔAED) (Common base DE and DE||AB) 

Area (ΔBDE) = Area (ΔAED) – Area (ΔFED) 

Area (ΔFED) Area (ΔBEF) = Area (ΔAFD) 

Area (ΔABD) = Area (ΔABF) + Area (ΔAFD) 

Area (ΔABD) = Area (ΔABF) + Area (ΔBEF) [From equation (1)] 

Area (ΔABD) = Area (ΔABE) (2) 
AD is the median in ΔABC.

\[
\text{ar} (\triangle ABD) = \frac{1}{2} \text{ar} (\triangle ABC) \\
= \frac{4}{2} \text{ar} (\triangle BDE) \\
\text{ar} (\triangle ABD) = 2 \text{ar} (\triangle BDE) 
\]

(As proved earlier) 

(3)

From (2) and (3), we obtain

\[2 \text{ar} (\triangle BDE) = \text{ar} (\triangle ABE)\]

Or,

(iii)

\[\text{ar} (\triangle ABE) = \frac{1}{2} \text{ar} (\triangle ABE)\]

\[\text{ar} (\triangle ABE) = \frac{1}{2} \text{ar} (\triangle BEC)\]

(Common base BE and BE||AC) 

\[\text{ar} (\triangle ABE) + \text{ar} (\triangle BEF) = \text{ar} (\triangle BEC)\]
Using equation (1), we obtain

\[ \text{ar} (\triangle ABF) + \text{ar} (\triangle AFD) = \text{ar} (\triangle BEC) \]
\[ \text{ar} (\triangle ABD) = \text{ar} (\triangle BEC) \]
\[ \frac{1}{2} \text{ar} (\triangle ABC) = \text{ar} (\triangle BEC) \]
\[ \text{ar} (\triangle ABC) = 2 \text{ar} (\triangle BEC) \]

(iv) It is seen that \( \triangle BDE \) and \( \triangle AED \) lie on the same base (DE) and between the parallels DE and AB.

\[ \text{ar} (\triangle BDE) = \text{ar} (\triangle AED) \]

\[ \text{ar} (\triangle BDE) - \text{ar} (\triangle FED) = \text{ar} (\triangle AED) - \text{ar} (\triangle FED) \]

\[ \text{ar} (\triangle BFE) = \text{ar} (\triangle AFD) \]

(v) Let \( h \) be the height of vertex E, corresponding to the side BD in \( \triangle BDE \). Let \( H \) be the height of vertex A, corresponding to the side BC in \( \triangle ABC \).

In (i), it was shown that

\[ \frac{1}{2} \times BD \times h = \frac{1}{4} \left( \frac{1}{2} \times BC \times H \right) \]
\[ \Rightarrow BD \times h = \frac{1}{4} (2BD \times H) \]
\[ \Rightarrow h = \frac{1}{2} H \]

In (iv), it was shown that \( \text{ar} (\triangle BFE) = \text{ar} (\triangle AFD) \).

\[ \frac{1}{2} \times FD \times H = \frac{1}{2} \times FD \times 2h = 2 \left( \frac{1}{2} \times FD \times h \right) \]
\[ = 2 \text{ar} (\triangle FED) \]

Hence,

\[ \text{ar} (\triangle BFE) = 2 \text{ar} (\triangle FED) \]

(vi) Area (AFC) = area (AFD) + area (ADC)
= \text{ar}(BFE) + \frac{1}{2}\text{ar}(ABC) \quad \text{[In (iv), \text{ar}(BFE) = \text{ar}(AFD) ; AD is median of \triangle ABC]}

= \text{ar}(BFE) + \frac{1}{2} \times 4 \text{ar}(BDE) \quad \text{[In (i), \text{ar}(BDE) = \frac{1}{4}\text{ar}(ABC)]}

= \text{ar}(BFE) + 2 \text{ar}(BDE) \quad \text{...(5)}

Now, by (v), \quad \text{ar}(BFE) = 2 \text{ar}(FED). \quad \text{...(6)}

\text{ar}(BDE) = \text{ar}(BFE) + \text{ar}(FED) = 2 \text{ar}(FED) + \text{ar}(FED) = 3 \text{ar}(FED) \quad \text{...(7)}

Therefore, from equations (5), (6), and (7), we get:

\text{ar}(AFC) = 2 \text{ar}(FED) + 2 \times 3 \text{ar}(FED) = 8 \text{ar}(FED)

\therefore \text{ar}(AFC) = 8 \text{ar}(FED)

Hence, \text{ar}(FED) = \frac{1}{8} \text{ar}(AFC)

Question 6:

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that

\text{ar}(APB) \times \text{ar}(CPD) = \text{ar}(APD) \times \text{ar}(BPC)

[Hint: From A and C, draw perpendiculars to BD]

Answer:

Let us draw AM \perp BD and CN \perp BD

Area of a triangle

\[= \frac{1}{2} \times \text{Base} \times \text{Altitude}\]
\begin{align*}
ar (APB) \times ar (CPD) &= \left[\frac{1}{2} \times BP \times AM\right] \times \left[\frac{1}{2} \times PD \times CN\right] \\
&= \frac{1}{4} \times BP \times AM \times PD \times CN \\
ar(APD) \times ar(BPC) &= \left[\frac{1}{2} \times PD \times AM\right] \times \left[\frac{1}{2} \times CN \times BP\right] \\
&= \frac{1}{4} \times PD \times AM \times CN \times BP \\
&= \frac{1}{4} \times BP \times AM \times PD \times CN \\
\end{align*}

\[ \therefore ar (APB) \times ar (CPD) = ar (APD) \times ar (BPC) \]

Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

\begin{align*}
(i) \quad ar(PRQ) &= \frac{1}{2} ar(ARC) \\
(ii) \quad ar(RQC) &= \frac{3}{8} ar(ABC) \\
(iii) \quad ar(PBQ) &= ar(ARC) \\
\end{align*}

Answer:

Take a point S on AC such that S is the mid-point of AC.

Extend PQ to T such that PQ = QT.

Join TC, QS, PS, and AQ.
In \( \triangle ABC \), \( P \) and \( Q \) are the mid-points of \( AB \) and \( BC \) respectively. Hence, by using mid-point theorem, we obtain

\[
PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC
\]

\[
PQ \parallel AS \text{ and } PQ = AS \text{ (As } S \text{ is the mid-point of } AC)
\]

\( \triangle PQSA \) is a parallelogram. We know that diagonals of a parallelogram bisect it into equal areas of triangles.

\[
\text{ar (} \triangle PAS \text{)} = \text{ar (} \triangle SQP \text{)} = \text{ar (} \triangle PAQ \text{)} = \text{ar (} \triangle SQA \text{)}
\]

Similarly, it can also be proved that quadrilaterals \( PSCQ, QSCT, \) and \( PSQB \) are also parallelograms and therefore,

\[
\text{ar (} \triangle PSQ \text{)} = \text{ar (} \triangle CQS \text{)} \quad (\text{For parallelogram } PSCQ)
\]

\[
\text{ar (} \triangle QSC \text{)} = \text{ar (} \triangle CTQ \text{)} \quad (\text{For parallelogram } QSCT)
\]

\[
\text{ar (} \triangle PSQ \text{)} = \text{ar (} \triangle QBP \text{)} \quad (\text{For parallelogram } PSQB)
\]

Thus,

\[
\text{ar (} \triangle PAS \text{)} = \text{ar (} \triangle SQP \text{)} = \text{ar (} \triangle PAQ \text{)} = \text{ar (} \triangle SQA \text{)} = \text{ar (} \triangle QSC \text{)} = \text{ar (} \triangle CTQ \text{)} = \text{ar (} \triangle QBP \text{)} ... (1)
\]

Also,

\[
\text{ar (} \triangle ABC \text{)} = \text{ar (} \triangle PBQ \text{)} + \text{ar (} \triangle PAS \text{)} + \text{ar (} \triangle PQS \text{)} + \text{ar (} \triangle QSC \text{)} \quad (\text{For parallelogram } PSCQ)
\]

\[
\text{ar (} \triangle ABC \text{)} = \text{ar (} \triangle PBQ \text{)} + \text{ar (} \triangle PBQ \text{)} + \text{ar (} \triangle PBQ \text{)} + \text{ar (} \triangle PBQ \text{)}
\]

\[
= 4 \times \text{ar (} \triangle PBQ \text{)}
\]

\[
= \frac{1}{2} \times \text{ar (} \triangle ABC \text{)} \quad (2)
\]

(i) Join point \( P \) to \( C \).
In $\triangle PAQ$, $QR$ is the median.

\[ \therefore \text{ar}(\triangle PRQ) = \frac{1}{2} \text{ar}(\triangle PAQ) = \frac{1}{2} \times \frac{1}{4} \text{ar}(\triangle ABC) = \frac{1}{8} \text{ar}(\triangle ABC) \quad \ldots (3) \]

In $\triangle ABC$, $P$ and $Q$ are the mid-points of $AB$ and $BC$ respectively. Hence, by using mid-point theorem, we obtain

\[ \text{PQ} = \frac{1}{2} \text{AC} \]

Also, $PQ \parallel AC$  \: \: $PT \parallel AC$

Hence, $PACT$ is a parallelogram.

\[ \text{ar} (PACT) = \text{ar} (PACQ) + \text{ar} (\triangle QTC) \]

\[ = \text{ar} (PACQ) + \text{ar} (\triangle PBQ) \quad [\text{Using equation (1)}] \perp \text{ar} \]

\[ \text{ar} (PACT) = \text{ar} (\triangle ABC) \quad \ldots (4) \]
ar(ΔARC) = \frac{1}{2} ar(ΔPAC) \quad (CR \text{ is the median of } ΔPAC)

= \frac{1}{2} \times \frac{1}{2} ar(ΔPACT) \quad (PC \text{ is the diagonal of parallelogram } PACT)

= \frac{1}{4} ar(ΔPACT) = \frac{1}{4} ar(ΔABC)

⇒ \frac{1}{2} ar(ΔARC) = \frac{1}{8} ar(ΔABC)

⇒ \frac{1}{2} ar(ΔARC) = ar(ΔPRQ) \quad [\text{Using equation (3)}] \quad \ldots (5)

(ii)

![Diagram showing parallelogram PACT with points R, Q, and T]

ar(PACT) = ar(ΔPRQ) + ar(ΔARC) + ar(ΔQTC) + ar(ΔRQC)

Putting the values from equations (1), (2), (3), (4), and (5), we obtain

ar(ΔABC) = \frac{1}{8} ar(ΔABC) + \frac{1}{4} ar(ΔABC) + \frac{1}{4} ar(ΔABC) + ar(ΔRQC)

ar(ΔABC) = \frac{5}{8} ar(ΔABC) + ar(ΔRQC)

ar(ΔRQC) = \left(1 - \frac{5}{8}\right) ar(ΔABC)

ar(ΔRQC) = \frac{3}{8} ar(ΔABC)

(iii) In parallelogram PACT,
Question 8:

In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX \perp DE meets BC at Y. Show that:

(i) $\Delta MBC \perp \Delta ABD$

(ii) $\text{ar}(\triangle BYXD) = 2\text{ar}(\triangle MBC)$

(iii) $\text{ar}(\triangle BYXD) = 2\text{ar}(\triangle ABMN)$

(iv) $\Delta FCB \perp \Delta ACE$
(v) \[
\text{ar}(\text{CYXE}) = 2\text{ar}(\text{FCB})
\]

(vi) \[
\text{ar}(\text{CYXE}) = \text{ar}(\text{ACFG})
\]

(vii) \[
\text{ar}(\text{BCED}) = \text{ar}(\text{ABMN}) + \text{ar}(\text{ACFG})
\]

**Note:** Result (vii) is the famous **Theorem of Pythagoras.** You shall learn a simpler proof of this theorem in class X.

**Answer:**

(i) We know that each angle of a square is 90°.

Hence, \(\perp ABM = \perp DBC = 90°\)

\[\perp ABM + \perp ABC = \perp DBC + \perp ABC\]

\[\perp MBC = \perp ABD\]

In \(\triangle MBC\) and \(\triangle ABD\),

\[\perp MBC = \perp ABD\] (Proved above)

\[MB = AB\] (Sides of square \(ABMN\))

\[BC = BD\] (Sides of square \(BCED\))

\[\triangle MBC \perp \triangle ABD\] (SAS congruence rule) (ii)

We have

\[\triangle MBC \perp \triangle ABD\]

\[\text{ar}(\triangle MBC) = \text{ar}(\triangle ABD)\] ... (1)

It is given that \(AX \perp DE\) and \(BD \perp DE\) (Adjacent sides of square \(BDEC\))

\(\perp BD \parallel AX\) (Two lines perpendicular to same line are parallel to each other)

\(\triangle ABD\) and parallelogram \(BYXD\) are on the same base \(BD\) and between the same parallels \(BD\) and \(AX\).

\[\therefore \text{ar}(\triangle ABD) = \frac{1}{2} \text{ar}(\triangle BYXD)\]

\[\text{ar}(\triangle BYXD) = 2 \text{ar}(\triangle ABD)\]

\[\text{Area}(\triangle BYXD) = 2 \text{area}(\triangle MBC)\] [Using equation (1)] ... (2)
(iii) \(\Delta \text{MBC} \) and parallelogram \(\text{ABMN} \) are lying on the same base MB and between same parallels MB and NC.

\[ \therefore \text{ar (\Delta MBC)} = \frac{1}{2} \text{ar (ABMN)} \]

\[ 2 \text{ar (\Delta MBC)} = \text{ar (ABMN)} \]

\[ \text{ar (BYXD)} = \text{ar (ABMN)} \quad \text{[Using equation (2)]} \quad \text{(3)} \]

(iv) We know that each angle of a square is 90°. ⊥ ⊥

\[ \text{FCA} = \perp \text{BCE} = 90^\circ \]

\[ \perp \text{FCA} + \perp \text{ACB} = \perp \text{BCE} + \perp \text{ACB} \]

\[ \text{FCB} = \perp \text{ACE} \]

In \(\Delta \text{FCB} \) and \(\Delta \text{ACE}, \)

\[ \perp \text{FCB} = \perp \text{ACE} \]

\[ \text{FC} = \text{AC} \quad (\text{Sides of square \text{ACFG}}) \]

\[ \text{CB} = \text{CE} \quad (\text{Sides of square \text{BCED}}) \]

\[ \Delta \text{FCB} \perp \Delta \text{ACE} \quad (\text{SAS congruence rule}) \]

(v) It is given that \(\text{AX} \perp \text{DE} \) and \(\text{CE} \perp \text{DE} \) (Adjacent sides of square \text{BDEC})

Hence, \(\text{CE} \parallel \text{AX} \) (Two lines perpendicular to the same line are parallel to each other) Consider \(\Delta \text{ACE} \) and parallelogram \(\text{CYXE} \)

\(\Delta \text{ACE} \) and parallelogram \(\text{CYXE} \) are on the same base \(\text{CE} \) and between the same parallels \(\text{CE} \) and \(\text{AX} \).

\[ \therefore \text{ar (\Delta ACE)} = \frac{1}{2} \text{ar (CYXE)} \]

\[ \text{ar (CYXE)} = 2 \text{ar (\Delta ACE)} \quad \text{(4)} \]

We had proved that

\[ \Delta \text{FCB} \perp \Delta \text{ACE} \]

\[ \text{ar (\Delta FCB)} \perp \text{ar (\Delta ACE)} \quad \text{(5)} \]

On comparing equations (4) and (5), we obtain ar

\[ \text{(CYXE)} = 2 \text{ar (\Delta FCB)} \quad \text{(6)} \]

(vi) Consider \(\Delta \text{FCB} \) and parallelogram \(\text{ACFG} \)
ΔFCB and parallelogram ACFG are lying on the same base CF and between the same parallels CF and BG.

\[ \therefore \text{ar (ΔFCB)} = \frac{1}{2}\text{ar (ACFG)} \]

\[ \text{ar (ACFG)} = 2 \text{ar (ΔFCB)} \]

\[ \text{ar (ACFG)} = \text{ar (CYXE)} \] [Using equation (6)] ...(7) (vii)

From the figure, it is evident that

\[ \text{ar (BCED)} = \text{ar (BYXD)} + \text{ar (CYXE)} \]

\[ \text{ar (BCED)} = \text{ar (ABMN)} + \text{ar (ACFG)} \] [Using equations (3) and (7)]