## PROBABILITY

## Life is a school of probability.

1. An integer is chosen at random from the first two hundreds digit. What is the probability that the integer chosen is divisible by 6 or 8.

Ans: Multiples of 6 first 200 integers $6,12,18, \underline{24}, 30,36,42, \underline{48}, 54,60,66, \underline{72}, 78,84,90, \underline{96}, 102,108,114$, $\underline{120}, 126,132,138, \underline{144}, 150,156,162, \underline{168}, 174,180,186, \underline{192}, 198$

Multiples of 8 first 200 integers $8,16, \underline{24}, 32,40, \underline{48}, 56,64, \underline{72}, 80,88, \underline{96}, 104,112, \underline{120}, 128,136, \underline{44}, 152,160$, 168, 176,184,192,200

Number of Multiples of 6 or $8=50$
$\mathrm{P}($ Multiples of 6 or 8$)=50 / 200=1 / 4$
2. A box contains 12 balls out of whith $x$ are black .if one ball is drawn at random from the box what is the probability that it will be a black ball? If 6 more black balls are put in the box ,the probability of drawing a black ball is now double of what it was before. Find x.
(Ans: $x=3$ )
Ans: Random drawing of balls ensures equally likely outcomes
Total number of balls $=12$
Total number of possible outcomes $=12$
Number of black balls $=x$
(1) out of total 12 outcomes, favourable outcomes $=x$


According to the question
Probability of drawing black ball is second case

$$
\begin{aligned}
& =2 \mathrm{X} \text { probability drawing of black ball in first case } \\
& \frac{x+6}{18}=2\left(\frac{x}{12}\right) \\
& \frac{x+6}{18}=\frac{x}{6} \\
& 6 \mathrm{x}+36=18 x \\
& x=3
\end{aligned}
$$

hence number of black balls $=3$
3. A bag contains 8 red balls and $x$ blue balls, the odd against drawing a blue ball are $2: 5$. What is the value of $x$ ?

Ans: $\quad$ No. of blue balls be $x$
No. of red balls be 8
Total no. of balls $=x+8$

$\frac{8}{8+x}: \frac{x}{8+x}$

$2 \mathrm{x}=40$

4. A card is drawn from a well shuffled deck of cards


Ans: Total cards 52
Spade $=13$
Remaining cards 39
i) The odds in favour of getting spade 13

The odds is not in favour of getting spade 39

$$
=\frac{13}{52}: \frac{39}{52}=1: 3
$$

ii) The odds against getting a spade 39

The odds not against getting a spade 13

$$
=\quad \frac{39}{52}: \frac{13}{52}=3: 1
$$

iii) The odds in favour of getting a face card 12

The odds not in favour of getting a face card 40

$$
=\frac{12}{52}: \frac{40}{52}=3: 10
$$

iv) The odds in favour of getting a red king 2

The odds not in favour of getting a red king 50

$$
=\quad \frac{2}{52}: \frac{50}{52}=1: 25
$$

5 A die is thrown repeatedly until a six comes up. What is the sample space for this experiment? HINT $; \mathrm{A}=\{6\} \mathrm{B}=\{1,2,3,4,5$,

Ans: The sample space is $=\{\mathrm{A}, \mathrm{BA}, \mathrm{BBA}, \mathrm{BBBA}, \mathrm{BBBBA} \ldots \ldots .\}$.
6. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a foot ball match?

Ans: equally likely because they are mutually exclusive events .
7. A bag contain 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag. (Ans:10)

Ans: Let the number of blue balls is the bag be x
Then total number of balls is the bag $=5+x$

- Number of all possible outcomes $=5+x$

Number of outcomes favourable to the event of drawing a blue ball $=x$ ( $\because$ there are x blue balls)
$\therefore$ Probability of drawing a blue ball $\frac{x}{5+x}$
Similarly, probability of drawing a red ball $=\frac{5}{5+x}$

According to the answer
$\frac{x}{5+x}=2\left(\frac{5}{5+x}\right)$
$x=10$
8. A box contains 12 balls out of which $x$ are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box the probability of drawing a black ball is now double of what it was before. Find $x$ ?

Ans: Number of all possible outcomes $=12$
Number of outcomes favourable to the event of drawing black ball=x
Required probability $=\frac{x}{12}$
Now when 6 more black balls are put in the box


Number of all possible outcomes $=12+6=18$
Number of outcomes favourable to the/event of drawing a black ball $=x+6$
$\therefore$ Probability of drawing a black bal $=\frac{x+6}{18}$
According to the question,

$$
\frac{x+6}{18}=2\left(\frac{x}{12}\right)
$$

$\therefore$
9. If $65 \%$ of the populations have black eyes, $25 \%$ have brown eyes and the remaining have blue eyes. What is the probability that a person selected at random has (i) Blue eyes (ii) Brown or black eyes (iii) Blue or black eyes
(iv) neither blue nor brown eyes
(Ans: $\frac{1}{10}, \frac{9}{10}, \frac{3}{4}, \frac{13}{20}$ )

Ans:
No. of black eyes $=65$
No. of Brown eyes $=25$
No. of blue eyes $=10$
Total no. of eyes $=180$
i) $\quad P($ Blue eyes $)=\frac{10}{100}=\frac{1}{10}$
ii) $\quad \mathrm{P}($ Brown or black eyes $)=\frac{90}{100}=\frac{9}{10}$
iii) $\quad \mathrm{P}($ Blue or black eyes $)=\frac{75}{100}=\frac{3}{4}$
iv) $\quad \mathrm{P}($ neither blue nor brown eyes $)=\frac{65}{100}=\frac{13}{20}$
10. Find the probability of having 53 Sundays in
(i) a leap year
(ii) a non leap year

Ans: An ordinary year has 365 days i.e. 52 weeks and 1 day This day can be any one of the 7 days of the week.
$\therefore \mathrm{P}($ that this day is Sunday $)=\frac{1}{7}$
Hence, $\mathrm{P}($ an ordinary year has 53 Sunday $)=\frac{1}{7}$
A leap year 366 days i.e. 52 weeks and 2 days
This day can be any one of the 7 days of the week
$\therefore \mathrm{P}$ (that this day is Sunday) $=\frac{2}{7}$
Hence, $\mathrm{P}\left(\right.$ a leap year has $53{ }^{\circ}$ Sanday $)=\frac{2}{7}$
11. Find the probability that the nonth June may have 5 Mondays in
(i) a leap year
(iii) a non leap year
(Ans: $\frac{2}{7}, \frac{2}{7}$ )

## Self Practice

12. Find the prebability that the month February may have 5 Wednesdays in
(i) a leap year
(ii) a non leap year
(Ans: $\frac{1}{7}, 0$ )
Self Practice
13. Five cards - the ten, jack, queen, king and ace, are well shuffled with their face downwards. One card is then picked up at random.
(i) What is the probability that the card is a queen?
(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is a
(a) an ace
(b) a queen
(Ans: $\frac{1}{5}, \frac{1}{4}, 0$ )

Ans : $\quad$ Here, the total number of elementary events $=5$
(i) Since, there is only one queen
$\therefore$ Favourable number of elementary events $=1$
$\therefore$ Probability of getting the card of queen $=\frac{1}{5}$
(ii) Now, the total number of elementary events $=4$
(a) Since, there is only one ace
$\therefore$ Favourable number of elementary events $=1$
$\therefore$ Probability of getting an ace card $=\frac{1}{4}$
(b) Since, there is no queen (as queen is put aside)
$\therefore$ Favourable number of elementary events $=0$
$\therefore$ Probability of getting a queen $=\frac{0}{4}=0$
14. A number $x$ is chosen at random from the numbers $-3,-2,-1,0,2,3$. What is the probability that $|x|<2$

Ans: $x$ can take 7 values
To get $|\mathrm{x}|<2$ take $-1,0,1$
Probability $(|x|<2)=\frac{3}{7}$
15. A number x is selected from the numbers $1,2,3$ and then a second number y is randomly selected from the numbers $1,4,9$. What is the probability that the product xy of the two numbers will be less than 9 ?
(Ans: $\frac{5}{9}$ )
Ans: Number X can be selected in three ways and corresponding to each such way there are three ways of selecting number y . Therefore, two numbers can be selected in 9 ways as listed below:
$(1,1),(1,4),(2,1),(2,4),(3,1)$
$\therefore$ Favourable number of elementary events $=5$

- Hence, required probability $=\frac{5}{9}$

16. In the adjoining figure a dart is thrown at the dart board and lands in the interior of the circle. What is the probability that the dart will land in the shaded region.


$$
\left(\text { Ans: } \frac{25 \pi-48}{25 \pi}\right)
$$

Ans: We have
$\mathrm{AB}=\mathrm{CD}=8$ and $\mathrm{AD}=\mathrm{BC}=6$
using Pythagoras Theorem is $\triangle \mathrm{ABC}$, we have
$\mathrm{AC}^{2}=\mathrm{AB}^{2}+\mathrm{BC}^{2}$
$A C^{2}=8^{2}+6^{2}=100$
$\mathrm{AC}=10$
$\mathrm{OA}=\mathrm{OC}=5 \quad[\because \mathrm{O}$ is the midpoint of AC$]$
$\therefore \quad$ Area of the circle $=\pi(\mathrm{OA})^{2}=25 \pi$ sq units $\left[\because\right.$ Area $\left.=\pi \mathrm{r}^{2}\right]$ Area of rectangle $\mathrm{ABCD}=\mathrm{AB} \times \mathrm{BC}=8 \times 6=48$ sq units
Area of shaded region $=$ Area of the circle - Area of rectangle ABCD Area of shaded region $=25 \pi-48$ sq unit. Hence

17. In the fig points $A, B, C$ and $D$ are the centres of fout circles, each having a radius of 1 unit. If a point is chosen at random from the interior of a square ABCD ,what is the probability that the point will be chosen from the shaded region.

Ans: Radius of the circle is Munit
Area of the circle $=$ Area of 4 sector
$\pi \mathrm{r}^{2}=\pi \times \mathrm{l}^{2}=\pi$
Side of the square $\mathrm{ABCD}=2$ units
Area of square $=2 \times 2=4$ units
Area shaded region is

-     - Area of square $-4 x$ Area of sectors
$=4-\pi$
Probability $=\left(\frac{4-\Pi}{4}\right)$

18. In the adjoining figure ABCD is a square with sides of length 6 units points P \& $Q$ are the mid points of the sides $B C \& C D$ respectively. If a point is selected at random from the interior of the square what is the probability that the point will be chosen from the interior of the triangle APQ.

Ans: Area of triangle $\mathrm{PQC}=\frac{1}{2} \times 3 \times 3=\frac{9}{2}=4.5$ units
Area of triangle $\mathrm{ABP}=\frac{1}{2} \times 6 \times 3=9$
Area of triangle $\mathrm{ADQ}=\frac{1}{2} \times 6 \times 3=9$
Area of triangle APQ $=$ Area of a square $-($ Area of a trianglg $\mathrm{PQC}+$ Area of triangle

$$
\begin{aligned}
& =36-(18+4.5) \\
& =36-22.5 \\
& =13.5
\end{aligned}
$$

$\mathrm{ABP}+$ Atea of triangle ABP )

Probability that the point will be choserf from the interior of the triangle APQ $=\frac{13.5}{36}$

$$
=\frac{135}{360}=\frac{3}{8}
$$

19. In a musical chair game the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the half minute after starting.

Ans: Here the possible outcomes are all the numbers between 0 and 2 .
This is the portion of the number line from 0 to 2 as shown in figure.
Let A bethe event that 'the music is stopped within the first half minute.' Then, cutcomes favorable to event A are all points on the number line from O to Q i.e., - from 0 to $1 / 2$.


The total number of outcomes are the points on the number line from O to P i.e., 0 to 2 .

$$
\therefore \mathrm{P}(\mathrm{~A})=\underline{\text { Length of } \mathrm{OQ}=\underline{1 / 2}=\underline{1}}
$$

Length of OP 24

20. A jar contains 54 marbles each of which is blue, green or white. The probability of selecting a blue marble at random from the jar is $\frac{1}{3}$ and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does the jar contain?

Ans: Let there be $b$ blue, $g$ green and $w$ white marbles in the marbles in the jat. Then,

$$
b+g+w=54
$$

$\therefore \mathrm{P}($ Selecting a blue marble $)=\frac{b}{54}$
It is given that the probability of selecting a blue marble
$\therefore \frac{1}{3}=\frac{b}{54} \Rightarrow \mathrm{~b}=18$
We have,
$P($ Selecting a green marble $)=\frac{4}{9}$
$\Rightarrow \frac{g}{54}=\frac{4}{9} \quad\left[\because \mathrm{P}(\right.$ Selecting a green marble $)=\frac{4}{9}$ (Given $\left.)\right]$
$\Rightarrow g=24$
Substituting the values, of $b$ and $g$ in (i), we get

$$
18+24+w=54 \Rightarrow w=12
$$

