Solved Paper–2

Class 10th, Mathematics, SA–2

Time: 3hours **Max. Marks 90**

General Instructions

1. All questions are compulsory.
2. Draw neat labeled diagram wherever necessary to explain your answer.
3. Q.No. 1 to 8 are of objective type questions, carrying 1 mark each.
4. Q.No.9 to 14 are of short answer type questions, carrying 2 marks each.
5. Q. No. 15 to 24 carry 3 marks each. Q. No. 25 to 34 carry 4 marks each.

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1. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(A) 0 only (B) 4

(C) 8 only (D) 0, 8

2. The list of numbers – 10, – 6, – 2, 2,... is

(A) an AP with d = – 16 (B) an AP with d = 4

(C) an AP with d = – 4 (D) not an AP

3. If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is

(A) 0 (B) 5

(C) 6 (D) 15

4. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

(A) 4 cm (B) 5 cm

(C) 6 cm (D) 8 cm

5. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(A) 22 : 7 (B) 14 : 11

(C) 7 : 22 (D) 11: 14

6. The points A (9, 0), B (9, 6), C (–9, 6) and D (–9, 0) are the vertices of a

(A) square (B) rectangle

(C) rhombus (D) trapezium

7. A shuttle cock used for playing badminton has the shape of the combination of

(A) a cylinder and a sphere (B) a cylinder and a hemisphere

(C) a sphere and a cone (D) frustum of a cone and a hemisphere

8. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the plane is called

(A) a frustum of a cone (B) cone

(C) cylinder (D) sphere

9. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30 ° to the ground, where as for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60 ° to the ground. What should be the length of the slide in each case?

10. A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

(i) an orange flavoured candy? (ii) a lemon flavoured candy?

11. John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

12. Which term of the A.P. 3, 8, 13, 18, … is 78?

13. Draw a triangle ABC with side BC = 7 cm, ∠B = 45°, ∠A = 105°. Then, construct a triangle whose sides are 4/3 times the corresponding side of ΔABC. Give the justification of the construction.

14. Find the area of a quadrant of a circle whose circumference is 22 cm. $\left[\text{Use } \pi = \frac{22}{7}\right]$

15. 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboids.

16. Determine if the points (1, 5), (2, 3) and (− 2, − 11) are collinear.

17. Check whether (5, − 2), (6, 4) and (7, − 2) are the vertices of an isosceles triangle.

18. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it? (ii) She will not buy it?

19. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

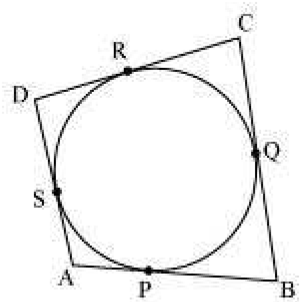
(I) $2x^2 + kx + 3 = 0$ (II) $kx(x - 2) + 6 = 0$

20. Show that $a_1, a_2 \ldots, a_n, \ldots$ form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$ (ii) $a_n = 9 - 5n$

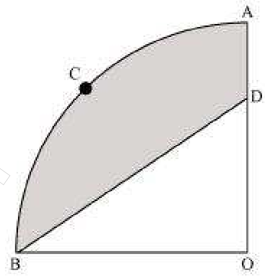
Also find the sum of the first 15 terms in each case.

21. A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that AB + CD = AD + BC



22. In the given figure, OACB is a quadrant of circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the
(i) Quadrant OACB
(ii) Shaded region

$\left[\text{Use } \pi = \frac{22}{7}\right]$



23. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

24. Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are 7/5 of the corresponding sides of the first triangle.
Give the justification of the construction.

25. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

26. Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.
(i) The median from A meets BC at D. Find the coordinates of point D.
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1
(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

27. Find a relation between *x* and *y* if the points (*x*, *y*), (1, 2) and (7, 0) are collinear.

28. Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
(i) the same day? (ii) consecutive days? (iii) different days?

29. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

30. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

31. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

32. A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.)

33. A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass. $\left[\text{Use } \pi = \frac{22}{7}\right]$

34. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

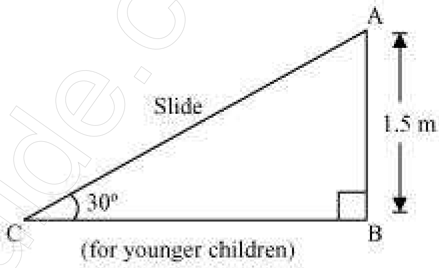
Solved Paper–2

Class 10th, Mathematics, SA–2

Solutions

1. D	2. B	3. A	4. D
5. B	6. B	7. D	8. A

9. It can be observed that AC and PR are the slides for younger and elder children respectively.

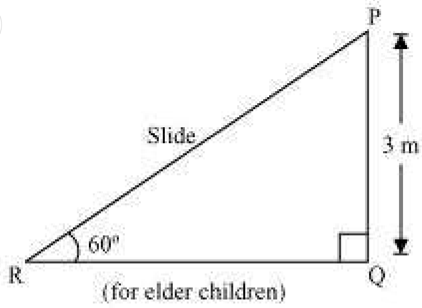


In ΔABC,

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$



In ΔPQR,

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$ m.

10. (i) The bag contains lemon flavoured candies only. It does not contain any orange flavoured candies. This implies that every time, she will take out only lemon flavoured candies. Therefore, event that Malini will take out an orange flavoured candy is an impossible event.

Hence, P (an orange flavoured candy) = 0

(ii) As the bag has lemon flavoured candies, Malini will take out only lemon flavoured candies. Therefore, event that Malini will take out a lemon flavoured candy is a sure event. P (a lemon flavoured candy) = 1

11. Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = $45 - x$

After losing 5 marbles,

Number of John's marbles = $x - 5$

Number of Jivanti's marbles = $45 - x - 5 = 40 - x$

It is given that the product of their marbles is 124.

$$\therefore (x-5)(40-x) = 124$$
$$\Rightarrow x^2 - 45x + 324 = 0$$
$$\Rightarrow x^2 - 36x - 9x + 324 = 0$$
$$\Rightarrow x(x-36) - 9(x-36) = 0$$
$$\Rightarrow (x-36)(x-9) = 0$$

Either $x - 36 = 0$ or $x - 9 = 0$

i.e., $x = 36$ or $x = 9$

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = $45 - 36 = 9$

If number of John's marbles = 9,

Then, number of Jivanti's marbles = $45 - 9 = 36$

12. 3, 8, 13, 18, …

For this A.P.,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be 78.

$a_n = a + (n - 1) d$

$78 = 3 + (n - 1) 5$

$75 = (n - 1) 5$

$(n - 1) = 15$

$n = 16$

Hence, 16^{th} term of this A.P. is 78.

13. $\angle B = 45°$, $\angle A = 105°$

Sum of all interior angles in a triangle is 180°.

$\angle A + \angle B + \angle C = 180°$

$105° + 45° + \angle C = 180°$

$\angle C = 180° - 150°$

$\angle C = 30°$

The required triangle can be drawn as follows.

Step 1

Draw a ΔABC with side BC = 7 cm, $\angle B = 45°$, $\angle C = 30°$.

Step 2

Draw a ray BX making an acute angle with BC on the opposite side of vertex A.

Step 3

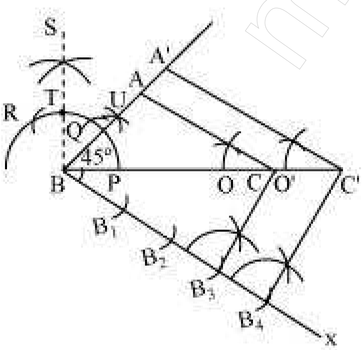
Locate 4 points (as 4 is greater in 4 and 3), B_1, B_2, B_3, B_4, on BX.

Step 4

Join B_3C. Draw a line through B_4 parallel to B_3C intersecting extended BC at C'.

Step 5

Through C', draw a line parallel to AC intersecting extended line segment at C'. $\Delta A'BC'$ is the required triangle.



Justification

The construction can be justified by proving that

$A'B = \frac{4}{3}AB,\ BC' = \frac{4}{3}BC,\ A'C' = \frac{4}{3}AC$

In ΔABC and ΔA'BC',

∠ABC = ∠A'BC' (Common)

∠ACB = ∠A'C'B (Corresponding angles)

∴ ΔABC ~ ΔA'BC' (AA similarity criterion)

$\Rightarrow \frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'}$ … (1)

In ΔBB_3C and $\Delta BB_4C'$,

$\angle B_3BC = \angle B_4BC'$ (Common)

$\angle BB_3C = \angle BB_4C'$ (Corresponding angles)

∴ $\Delta BB_3C \sim \Delta BB_4C'$ (AA similarity criterion)

$\Rightarrow \frac{BC}{BC'} = \frac{BB_3}{BB_4}$

$\Rightarrow \frac{BC}{BC'} = \frac{3}{4}$ ...(2)

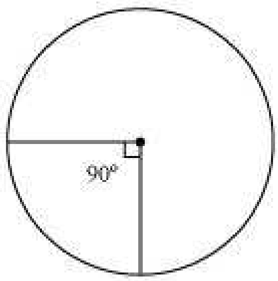
On comparing equations (1) and (2), we obtain

$\frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} = \frac{3}{4}$

$\Rightarrow A'B = \frac{4}{3}AB,\ BC' = \frac{4}{3}BC,\ A'C' = \frac{4}{3}AC$

This justifies the construction.

14. Let the radius of the circle be *r*.



Circumference = 22 cm

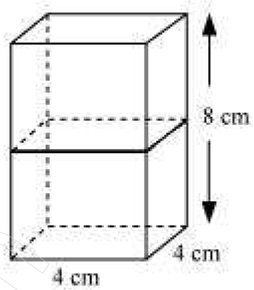
$2\pi r = 22$

$r = \frac{22}{2\pi} = \frac{11}{\pi}$

Quadrant of circle will subtend 90° angle at the centre of the circle.

Area of such quadrant of the circle $= \frac{90°}{360°} \times \pi \times r^2$

$$=\frac{1}{4\pi}\times\pi\times\left(\frac{11}{}\right)^2$$
$$=\frac{121}{4\pi}=\frac{121\times7}{4\times22}$$
$$=\frac{77}{8}\text{ cm}^2$$



15. Given that,

Volume of cubes = 64 cm^3

$(\text{Edge})^3 = 64$

Edge = 4 cm

If cubes are joined end to end, the dimensions of the resulting cuboid will be 4 cm, 4 cm, 8 cm.

$$\therefore \text{Surface area of cuboids} = 2(lb+bh+lh)$$
$$=2(4\times4+4\times8+4\times8)$$
$$=2(16+32+32)$$
$$=2(16+64)$$
$$=2\times80=160\text{ cm}^2$$

16. Let the points (1, 5), (2, 3), and (−2, −11) be representing the vertices A, B, and C of the given triangle respectively.

Let $A=(1,5), B=(2,3), C=(-2,-11)$

$$\therefore AB=\sqrt{(1-2)^2+(5-3)^2}=\sqrt{5}$$
$$BC=\sqrt{(2-(-2))^2+(3-(-11))^2}=\sqrt{4^2+14^2}=\sqrt{16+196}=\sqrt{212}$$
$$CA=\sqrt{(1-(-2))^2+(5-(-11))^2}=\sqrt{3^2+16^2}=\sqrt{9+256}=\sqrt{265}$$

Since $AB+BC\neq CA$,

Therefore, the points (1, 5), (2, 3), and (−2, −11) are not collinear.

17. Let the points (5, −2), (6, 4), and (7, −2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$
$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$
$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

Therefore, AB = BC

As two sides are equal in length, therefore, ABCis an isosceles triangle.

18. Total number of pens = 144

Total number of defective pens = 20

Total number of good pens = 144 − 20 = 124

(i) Probability of getting a good pen $= \frac{124}{144} = \frac{31}{36}$

P (Nuri buys a pen) $= \frac{31}{36}$

(ii) P (Nuri will not buy a pen) $= 1 - \frac{31}{36} = \frac{5}{36}$

19. We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant ($b^2 - 4ac$) will be 0.

(I) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$, we obtain

$a = 2$, $b = k$, $c = 3$

Discriminant = $b^2 - 4ac = (k)^2 - 4(2)(3)$

$= k^2 - 24$

For equal roots,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(II) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = k$, $b = -2k$, $c = 6$

Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6)$

$= 4k^2 - 24k$

For equal roots,

$b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

20. (i) $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

It can be observed that

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(7) + (15-1)4\right]$$

$$= \frac{15}{2}\left[(14) + 56\right]$$

$$= \frac{15}{2}(70)$$

$= 15 \times 35$

$= 525$

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

It can be observed that

$a_2 - a_1 = -1 - 4 = -5$

$a_3 - a_2 = -6 - (-1) = -5$

$a_4 - a_3 = -11 - (-6) = -5$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as -5 and first term as 4.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(4) + (15-1)(-5)\right]$$

$$= \frac{15}{2}\left[8 + 14(-5)\right]$$

$$= \frac{15}{2}(8 - 70)$$

$$= \frac{15}{2}(-62) = 15(-31)$$

$= -465$

21. It can be observed that

DR = DS (Tangents on the circle from point D) … (1)

CR = CQ (Tangents on the circle from point C) … (2)

BP = BQ (Tangents on the circle from point B) … (3)

AP = AS (Tangents on the circle from point A) … (4)

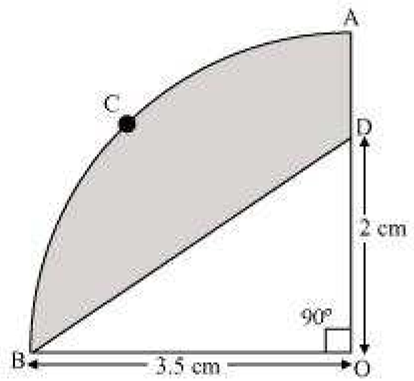
Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

22. (i) Since OACB is a quadrant, it will subtend 90° angle at O.



Area of quadrant OACB $= \frac{90^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times (3.5)^2 = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= \frac{11 \times 7 \times 7}{2 \times 7 \times 2 \times 2} = \frac{77}{8}$ cm^2

(ii) Area of ΔOBD $= \frac{1}{2} \times \text{OB} \times \text{OD}$

$= \frac{1}{2} \times 3.5 \times 2$

$= \frac{1}{2} \times \frac{7}{2} \times 2$

$= \frac{7}{2}$ cm^2

Area of the shaded region = Area of quadrant OACB − Area of ΔOBD

$= \frac{77}{8} - \frac{7}{2}$

$= \frac{77 - 28}{8}$

$= \frac{49}{8}$ cm^2

23. Height (h_1) of cylindrical container = 15 cm

Radius (r_1) of circular end of container = $\frac{12}{2} = 6$ cm

Radius (r_2) of circular end of ice-cream cone = $\frac{6}{2} = 3$ cm

Height (h_2) of conical part of ice-cream cone = 12 cm

Let n ice-cream cones be filled with ice-cream of the container.

Volume of ice-cream in cylinder = n × (Volume of 1 ice-cream cone + Volume of hemispherical shape on the top)

$$\pi r_1^2 h_1 = n\left(\frac{1}{3}\pi r_2^2 h_2 + \frac{2}{3}\pi r_2^3\right)$$

$$n = \frac{6^2 \times 15}{\frac{1}{3}\times 9\times 12 + \frac{2}{3}\times (3)^3}$$

$$n = \frac{36\times 15\times 3}{108+54}$$

$$n = 10$$

Therefore, 10 ice-cream cones can be filled with the ice-cream in the container.

24. **Step 1**

Draw a line segment AB of 5 cm. Taking A and B as centre, draw arcs of 6 cm and 5 cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having length of sides as 5 cm, 6 cm, and 7 cm respectively.

Step 2

Draw a ray AX making acute angle with line AB on the opposite side of vertex C.

Step 3

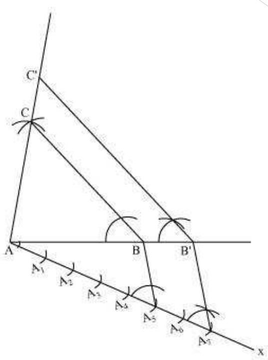
Locate 7 points, A_1, A_2, A_3, A_4 A_5, A_6, A_7 (as 7 is greater between 5and 7), on line AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

Step 4

Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect extended line segment AB at point B'.

Step 5

Draw a line through B' parallel to BC intersecting the extended line segment AC at C'. ΔAB'C' is the required triangle.



Justification

The construction can be justified by proving that

$$AB' = \frac{7}{5}AB,\ B'C' = \frac{7}{5}BC,\ AC' = \frac{7}{5}AC$$

In ΔABC and $\Delta AB'C'$,

$\angle ABC = \angle AB'C'$ (Corresponding angles)

$\angle BAC = \angle B'AC'$ (Common)

$\therefore \Delta ABC \sim \Delta AB'C'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} \quad \ldots (1)$$

In ΔAA_5B and $\Delta AA_7B'$,

$\angle A_5AB = \angle A_7AB'$ (Common)

$\angle AA_5B = \angle AA_7B'$ (Corresponding angles)

$\therefore \Delta AA_5B \sim \Delta AA_7B'$ (AA similarity criterion)

$$\Rightarrow \frac{AB}{AB'} = \frac{AA_5}{AA_7}$$

$$\Rightarrow \frac{AB}{AB'} = \frac{5}{7} \quad \ldots (2)$$

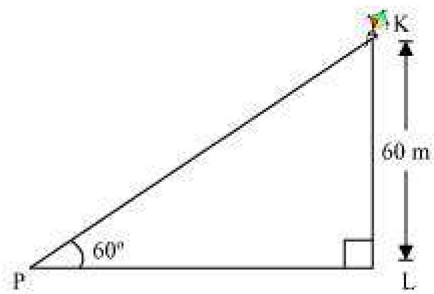
On comparing equations (1) and (2), we obtain

$$\frac{AB}{AB'} = \frac{BC}{B'C'} = \frac{AC}{AC'} = \frac{5}{7}$$

$$\Rightarrow AB' = \frac{7}{5}AB,\ B'C' = \frac{7}{5}BC,\ AC' = \frac{7}{5}AC$$

This justifies the construction.

25. Let K be the kite and the string is tied to point P on the ground.



In ΔKLP,

$$\frac{KL}{KP} = \sin 60°$$

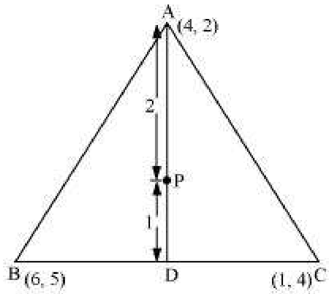
$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3}\ \text{m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

26. (i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.



$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2\times\frac{7}{2}+1\times 4}{2+1}, \frac{2\times\frac{9}{2}+1\times 2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left(\frac{2\times\frac{5}{2}+1\times 6}{2+1}, \frac{2\times 3+1\times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC, having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3,y_3).

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.

Coordinates of D $= \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

Let the centroid of this triangle be O.

Point O divides the side AD in a ratio 2:1.

$$\text{Coordinates of O} = \left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$$

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

27. If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

$$\text{Area} = \frac{1}{2}[x(2-0)+1(0-y)+7(y-2)]$$

$$0 = \frac{1}{2}[2x-y+7y-14]$$

$$0 = \frac{1}{2}[2x+6y-14]$$

$$2x+6y-14=0$$

$$x+3y-7=0$$

This is the required relation between *x* and *y*.

28. There are a total of 5 days. Shyam can go to the shop in 5 ways and Ekta can go to the shop in 5 ways.

Therefore, total number of outcomes = 5 × 5 = 25

(i) They can reach on the same day in 5 ways.

i.e., (t, t), (w, w), (th, th), (f, f), (s, s)

P (both will reach on same day) $= \frac{5}{25} = \frac{1}{5}$

(ii) They can reach on consecutive days in these 8 ways - (t, w), (w, th), (th, f), (f, s), (w, t), (th, w), (f, th), (s, f).

Therefore, P (both will reach on consecutive days) $= \frac{8}{25}$

(iii) P (both will reach on same day) $=\frac{1}{5}$ [(From (i)]

P (both will reach on different days) $=1-\frac{1}{5}=\frac{4}{5}$

29. Let the speed of the train be x km/hr.

Time taken to cover 360 km $=\frac{360}{x}$ hr

According to the given question,

$$(x+5)\left(\frac{360}{x}-1\right)=360$$
$$\Rightarrow (x+5)\left(\frac{360}{x}-1\right)=360$$
$$\Rightarrow 360-x+\frac{1800}{x}-5=360$$
$$\Rightarrow x^2+5x-1800=0$$
$$\Rightarrow x^2+45x-40x-1800=0$$
$$\Rightarrow x(x+45)-40(x+45)=0$$
$$\Rightarrow (x+45)(x-40)=0$$
$$\Rightarrow \ x=40,-45$$

However, speed cannot be negative.

Therefore, the speed of train is 40 km/h

30. Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = $(x - 10)$ hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8}=\frac{75}{8}$ hours by both the pipes together.

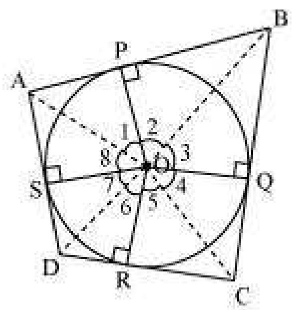
Therefore,

$$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$$
$$\frac{x-10+x}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow 75(2x-10)=8x^2-80x$$
$$\Rightarrow 150x-750=8x^2-80x$$
$$\Rightarrow 8x^2-230x+750=0$$
$$\Rightarrow 8x^2-200x-30x+750=0$$
$$\Rightarrow 8x(x-25)-30(x-25)=0$$
$$\Rightarrow (x-25)(8x-30)=0$$
$$i.e., x=25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 =15 hours respectively.

31. Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.



Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence criterion)

Therefore, A ↔ A, P ↔ S, O ↔ O

And thus, ∠POA = ∠AOS

∠1 = ∠8

Similarly,

∠2 = ∠3

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^{o}$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^{o}$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^{o}$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^{o}$

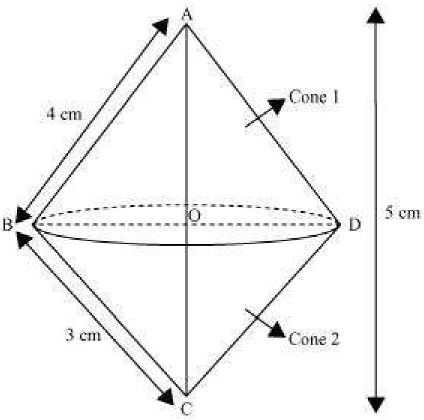
$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^{o}$

$\angle AOB + \angle COD = 180^{o}$

Similarly, we can prove that $\angle BOC + \angle DOA = 180^{o}$

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

32. The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.



Hypotenuse $AC = \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$ cm

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$\frac{1}{2} \times AC \times OB = \frac{1}{2} \times 4 \times 3$

$\frac{1}{2} \times 5 \times OB = 6$

$OB = \frac{12}{5} = 2.4$ cm

Volume of double cone = Volume of cone 1 + Volume of cone 2

$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3}\pi r^2 (h_1 + h_2) = \frac{1}{3}\pi r^2 (OA + OC)$

$= \frac{1}{3} \times 3.14 \times (2.4)^2 (5)$

$= 30.14$ cm^3

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

$= \pi r l_1 + \pi r l_2$

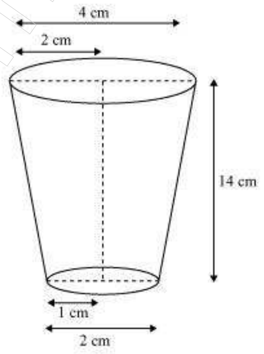
$= \pi r[4+3] = 3.14 \times 2.4 \times 7$

$= 52.75 \text{ cm}^2$

33. Radius (r_1) of upper base of glass $= \frac{4}{2} = 2$ cm

Radius (r_2) of lower base of glass $= \frac{2}{2} = 1$ cm

Capacity of glass = Volume of frustum of cone



$$= \frac{1}{3}\pi h\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{1}{3}\pi h\left[(2)^2 + (1)^2 + (2)(1)\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 14[4+1+2]$$

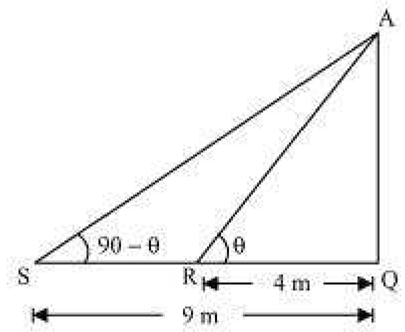
$$= \frac{308}{3} = 102\frac{2}{3} \text{ cm}^3$$

Therefore, the capacity of the glass is $102\frac{2}{3} \text{ cm}^3$.

34. Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.

The angles are complementary. Therefore, if one angle is θ, the other will be 90 − θ.

In ΔAQR,



$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \qquad \ldots(i)$$

In ΔAQS,

$$\frac{AQ}{SQ} = \tan(90 - \theta)$$

$$\frac{AQ}{9} = \cot\theta \qquad \ldots(ii)$$

On multiplying equations (*i*) and (*ii*), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right)=(\tan\theta).(\cot\theta)$$

$$\frac{AQ^2}{36}=1$$

$$AQ^2=36$$

$$AQ=\sqrt{36}=\pm 6$$

However, height cannot be negative.

Therefore, the height of the tower is 6 m.