

# Introduction to Trigonometry

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1. Prove the following identities :  $1 + \sec A / \sec A = \sin^2 A / 1 - \cos A$
2. Prove that :  $1 / \sec \theta - \tan \theta = 1 / \cos \theta = 1 / \cos \theta - 1 / \sec \theta + \tan \theta$
3. Prove the following identity:  
 $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A.$
4. If  $x/a \cos = y/b \sin$  and  
 $ax/\cos = by/\sin = a^2 - b^2$  prove that  $x^2/a^2 + y^2/b^2$
5. If  $\cot A = 4/3$  check  $(1 - \tan^2 A) / (1 + \tan^2 A) = \cot^2 A - \sin^2 A$
6.  $\sin(A - B) = 1/2$ ,  $\cos(A + B) = 1/2$  find A and B
7. Evaluate  $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ$
8. Verify  $4(\sin^4 30^\circ + \cos^4 60^\circ) - 3(\cos^2 45^\circ - \sin^2 90^\circ) = 2$
9. Show that  $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$
10.  $\sec 4A = \operatorname{cosec}(A - 20)$  find A
11.  $\tan A = \cot B$  prove  $A + B = 90$
12. A, B, and C are the interior angles of  $\triangle ABC$  show that  $\sin(B + C)/2 = \cos A/2$
13. In  $\triangle ABC$ , if  $\sin(A + B - C) = \sqrt{3}/2$  and  $\cos(B + C - A) = 1/\sqrt{2}$ , find A, B and C.
14. If  $\cos \theta =$  and  $\theta + \varphi = 900$ , find the value of  $\sin \varphi$ .
15. If  $\tan 2A = \cot(A - 18^\circ)$ , where  $2A$  is an acute angle, find the value of A.
16. If  $2\sin(x/2) = 1$ , then find the value of x.
17. If  $\tan A = \frac{1}{2}$  and  $\tan B = 1/3$ ,

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by using  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$  prove that  $A + B = 45^\circ$

18. Express  $\sin 76^\circ + \cos 63^\circ$  in terms of trigonometric ratios of angles between  $0^\circ$  and  $45^\circ$ .
19. Prove that:  $2 \sec^2 \theta - \sec^4 \theta - 2 \operatorname{cosec}^2 \theta + \operatorname{cosec}^4 \theta = \cot^4 \theta - \tan^4 \theta$
20. Find the value of  $\theta$  for which  $\sin \theta - \cos \theta = 0$
21. Given that  $\sin 2A + \cos 2A = 1$ , prove that  $\cot 2A = \operatorname{cosec} 2A - 1$
22. If  $\sin(A + B) = 1$  and  $\sin(A - B) = 1/2$ ,  $0^\circ < A + B \leq 90^\circ$ ;  $A > B$ , find  $A$  and  $B$ .
23. Show that  $\tan 62^\circ / \cot 28^\circ = 1$
24. If  $\sin A + \sin^2 A = 1$ , prove that  $\cos^2 A + \cos^4 A = 1$ .
25. If  $\sec 4A = \operatorname{cosec}(A - 20^\circ)$ , where  $4A$  is an acute angle, find the value of  $A$ .
26. Prove that  $(\operatorname{cosec} \theta - \sec \theta)(\cot \theta - \tan \theta) = (\operatorname{cosec} \theta + \sec \theta)(\sec \theta \cdot \operatorname{cosec} \theta - 2)$
27. Given that  $A = 60^\circ$ , verify that  $1 + \sin A = (\cos A/2 + \sin A/2)^2$
28. If  $\sin \theta + \cos \theta = x$  and  $\sin \theta - \cos \theta = y$ , show that  $x^2 + y^2 = 2$
29. Show that  $\sin^4 \theta - \cos^4 \theta = 1 - 2 \cos^2 \theta$
30. If  $\theta = 45^\circ$ . Find the value of  $\sec 2\theta$
31. Evaluate:  $\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$
32. Find the value of  $\tan 15^\circ \cdot \tan 25^\circ \cdot \tan 30^\circ \cdot \tan 65^\circ \cdot \tan 85^\circ$
33. If  $\theta$  is a positive acute angle such that  $\sec \theta = \operatorname{cosec} 60^\circ$ , then find the value of  $2 \cos^2 \theta - 1$
34. Find the value of  $\sin 65^\circ - \cos 25^\circ$  without using tables.
35. If  $\sec 5A = \operatorname{cosec}(A - 36^\circ)$ . Find the value of  $A$ .
36. If  $2 \sin x/2 - 1 = 0$ , find the value of  $x$ .
37. If  $A$ ,  $B$  and  $C$  are interior angles of  $\triangle ABC$ , then prove that  $\cos(B+C)/2 = \sin A/2$
38. Find the value of  $9 \sec^2 A - 9 \tan 2A$ .

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39. Prove that  $\sin 6\theta + \cos 6\theta = 1 - 3\sin 2\theta \cos 2\theta$ .
40. If  $5\tan\theta - 4 = 0$ , then find the value of  $(5\sin\theta - 4\cos\theta)(5\sin\theta + 4\cos\theta)$
41. In  $\triangle ABC$ ,  $\angle C = 90^\circ$ ,  $\tan A = 1$  and  $\tan B = \frac{1}{3}$ . Prove that  $\sin A \cdot \cos B + \cos A \cdot \sin B = 1$ .
42. In  $\triangle ABC$ , right angled at B, if  $\tan A = \frac{1}{\sqrt{3}}$  find the value of  $\sin A \cos C + \cos A \sin C$ .
43. Show that  $2(\cos^4 60 + \sin^4 30) - (\tan^2 60 + \cot^2 45) + 3\sec^2 30 = \frac{1}{4}$
44.  $\sin(50 + q) - \cos(40 - q) + \tan 1 \tan 10 \tan 20 \tan 70 \tan 80 \tan 89 = 1$
45. Given  $\tan A = \frac{4}{3}$ , find the other trigonometric ratios of the angle A.
46. In a right triangle ABC, right-angled at B, if  $\tan A = 1$ , then verify that  $2 \sin A \cos A = 1$ .
47. In  $\triangle OPQ$ , right-angled at P,  $OP = 7$  cm and  $OQ - PQ = 1$  cm. Determine the values of  $\sin Q$  and  $\cos Q$ .
48. In  $\triangle ABC$ , right-angled at B,  $AB = 24$  cm,  $BC = 7$  cm. Determine: (i)  $\sin A$ ,  $\cos A$  (ii)  $\sin C$ ,  $\cos C$
49. If  $\angle A$  and  $\angle B$  are acute angles such that  $\cos A = \cos B$ , then show that  $\angle A = \angle B$ .
50. If  $\cot A = \frac{7}{8}$  evaluate:  $\frac{(1 + \sin A)(1 - \sin A)}{(1 + \cos A)(1 - \cos A)}$
51. In triangle ABC, right-angled at B, if  $\tan A = \frac{1}{\sqrt{3}}$   
find the value of: (i)  $\sin A \cos C + \cos A \sin C$  (ii)  $\cos A \cos C - \sin A \sin C$
52. In  $\triangle ABC$ , right angled at B,  $AB = 5$  cm and  $\angle ACB = 30^\circ$  Determine the lengths of the sides BC and AC.
53. In  $\triangle PQR$ , right - angled at Q,  $PQ = 3$  cm and  $PR = 6$  cm. Determine  $\angle QPR$  and  $\angle PRQ$
54. If  $\sin(A-B) = \frac{1}{2}$ ,  $\cos(A+B) = \frac{1}{2}$ ,  $A+B = 90^\circ$ ,  $0 < A+B \leq 90^\circ$ ,  $A > B$  find A and B
55. Evaluate the following:  $(5\cos^2 60 + 4\sec^2 30 - \tan^2 45) / (\sin^2 30 + \cos^2 30)$
56. If  $\sin 3A = \cos(A - 26)$ , where  $3A$  is an acute angle, find the value of A.
57. Prove the trigonometric identities  $(1 - \cos A) / (1 + \cos A) = (\operatorname{cosec} A - \cot A)^2$
58. Prove the trigonometric identities  $(1 + \frac{1}{\tan^2 A})(1 + \frac{1}{\cot^2 A}) = \frac{1}{(\sin^2 A - \cos^4 A)}$

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59. Prove the trigonometric identities  $(\sec^4 A - \sec^2 A) = \tan^4 A + \tan^2 A = \sec^2 A \tan^2 A$
60. Prove the trigonometric identities  $\cot A - \tan A = (2\cos^2 A - 1) / (\sin A \cos A)$
61. Prove the trigonometric identities.  $(1 - \sin A + \cos A)^2 = 2(1 + \cos A)(1 - \sin A)$
62. If  $\tan A + \sin A = m$  and  $\tan A - \sin A = n$  show that  $m^2 - n^2 = 4\sqrt{mn}$
63. If  $x = p \sec A + q \tan A$  and  $y = p \tan A + q \sec A$  prove that  $x^2 - y^2 = p^2 - q^2$
64. If  $\sin A + \sin^2 A = 1$  prove that  $\cos^2 A + \cos^4 A = 1$
65. Express the following in terms of t-ratios of angles between  $0^\circ$  and  $45^\circ$ .
- 1)  $\sin 85^\circ + \operatorname{cosec} 85^\circ$
  - 2)  $\operatorname{cosec} 69^\circ + \cot 69^\circ$
  - 3)  $\sin 81^\circ + \tan 81^\circ$
  - 4)  $\cos 56^\circ + \cot 56^\circ$
66.  $[\sin(90^\circ - A) \sin A] / \tan A - 1 = -\sin^2 A$
67.  $\cos \cos(90^\circ - ) - \sin \sin(90^\circ - ) = 0$
68.  $\sin(90^\circ - ) \cos(90^\circ - ) = \tan / (1 + \tan^2 )$
69.  $\operatorname{cosec}^2(90^\circ - ) - \tan^2 = \cos^2(90^\circ - ) + \cot^2$
70. If  $\cos / \cos = m$  and  $\cos / \sin = n$ , show that  $(m^2 + n^2) \cos^2 = n^2$ . If  $x = r \cos \sin$ ,  $y = r \cos \cos$  and  $z = r \sin$ , show that  $x^2 + y^2 + z^2 = r^2$ .