1. $\sin^6 A+\cos^6 A=1-3\sin^2 A\cdot\cos^2 A$

$\sin^6 A+\cos^6 A$

$=\sin^6 A+\left(\cos^2 A\right)^3$

$=\sin^6 A+\left(1-\sin^2 A\right)^3 \qquad \left[\sin^2 A+\cos^2 A=1\Rightarrow\cos^2 A=1-\sin^2 A\right]$

$=\sin^6 A+1-\sin^6 A-3\sin^2 A\left(1-\sin^2 A\right) \qquad \left[\because (a-b)^3=a^3-b^3-3ab(a-b)\right]$

$=1-3\sin^2 A\cdot\cos^2 A$

2. $\sin^8 A-\cos^8 A=\left(\sin^2 A-\cos^2 A\right)\left(1-2\sin^2 A\cdot\cos^2 A\right)$

$\sin^8 A-\cos^8 A$

$=\left(\sin^4 A\right)^2-\left(\cos^4 A\right)^2$

$=\left(\sin^4 A-\cos^4 A\right)\left(\sin^4 A+\cos^4 A\right) \qquad \left[a^2-b^2=(a-b)(a+b)\right]$

$=\left[\left(\sin^2 A\right)^2-\left(\cos^2 A\right)^2\right]\left[\left(\sin^2 A\right)^2+\left(\cos^2 A\right)^2+2\cdot\sin^2 A\cdot\cos^2 A-2\sin^2 A\cdot\cos^2 A\right]$

$\left\{\text{Add and subtracting } 2\sin^2 A\cos^2 A\right\}$

$=\left(\sin^2 A+\cos^2 A\right)\left(\sin^2 A-\cos^2 A\right)\left[\left(\sin^2 A+\cos^2 A\right)^2-2\cdot\sin^2 A\cdot\cos^2 A\right]$

$\left[\because a^2+b^2+2ab=(a+b)^2 \text{ and} \sin^2 A\cdot\cos^2 A=1\right]$

$=\left(\sin^2 A-\cos^2 A\right)\left(1-2\cdot\sin^2 A\cdot\cos^2 A\right)$

3. $\sec^6 A=1+\tan^6 A+3\tan^2 A\sec^2 A$

$\sec^6 A$

$=\left(\sec^2 A\right)^3$

$=\left(1+\tan^2 A\right)^3 \qquad \left[\because 1+\tan^2\theta=\sec^2\theta\right]$

$=1+\tan^6 A+3\cdot\tan^2 A\left(1+\tan^2 A\right) \qquad \left[(a+b)^3=a^3+b^3+3ab(a+b)\right]$

$=1+\tan^6 A+3\tan^2 A\cdot\sec^2 A$

4.If sin x +sin^{2}x=1, prove that cos^2+cos^4=1

Given, $\sin x+\sin^2 x=1 \;\Rightarrow \sin x=1-\sin^2 x \;\Rightarrow \sin x=\cos^2 x \qquad \ldots(1)$

$\cos^2 x+\cos^4 x=\cos^2 x+\left(\cos^2 x\right)^2=\sin x+\sin^2 x=1 \;\left(\text{Using }(1)\right)$

5. Cosec A=2x and Cot=2/x then find the value of $2(x^2 - 1/x)$

Given, $\operatorname{cosec} A = 2x$ and $\cot A = \frac{2}{x}$ We know that, $\operatorname{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow (2x)^2 - \left(\frac{2}{x}\right)^2 = 1$ $\quad \Rightarrow 4x^2 - \frac{4}{x^2} = 1$ $\quad \Rightarrow 4\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow 2\left(x^2 - \frac{1}{x^2}\right) = \frac{1}{2}$ Thus, the value of $2\left(x^2 - \frac{1}{x^2}\right)$ is $\frac{1}{2}$.

6. sin A+ cos A=√3.prove that tan A +cot A=1

Ans:

Given, $\sin A + \cos A = \sqrt{3}$ Squaring on both sides, we get $(\sin A + \cos A)^2 = (\sqrt{3})^2$

$\Rightarrow \sin^2 A + \cos^2 A + 2\sin A\cos A = 3 \Rightarrow 2\sin A\cos A = 3 - 1 = 2$ $\quad (\because \sin^2\theta + \cos^2\theta = 1)$

$\Rightarrow \sin A\cos A = 1 \quad \ldots(1)$

$\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\cos A\sin A} = \frac{1}{1}$ (Using (1)) $= 1$

7. If cosec A – sin A = a^3 and sec A - cos A = b^3 Find value of : $a^2 b^2 [a^2 + b^2]$

Ans:

$\Rightarrow \frac{1}{\sin A} - \sin A = a^3$

$\Rightarrow \frac{1 - \sin^2 A}{\sin A} = a^3$

$\Rightarrow \frac{\cos^2 A}{\sin A} = a^3$

$\Rightarrow a = \frac{\cos^{\frac{2}{3}} A}{\sin^{\frac{1}{3}} A}$

and $\sec A - \cos A = b^3$

$\Rightarrow \frac{1}{\cos A} - \cos A = b^3$

$\Rightarrow \frac{1 - \cos^2 A}{\cos A} = b^3$

$\Rightarrow \frac{\sin^2 A}{\cos A} = b^3$

$\Rightarrow b = \frac{\sin^{\frac{2}{3}} A}{\cos^{\frac{1}{3}} A}$

$\therefore a^2 b^2 (a^2 + b^2)$

$= \left(\frac{\cos^{\frac{2}{3}} A}{\sin^{\frac{1}{3}} A}\right)^2 \left(\frac{\sin^{\frac{2}{3}} A}{\cos^{\frac{1}{3}} A}\right)^2 \left[\left(\frac{\cos^{\frac{2}{3}} A}{\sin^{\frac{1}{3}} A}\right)^2 + \left(\frac{\sin^{\frac{2}{3}} A}{\cos^{\frac{1}{3}} A}\right)^2\right]$

$= \frac{\cos^{\frac{4}{3}} A}{\sin^{\frac{2}{3}} A} \times \frac{\sin^{\frac{4}{3}} A}{\cos^{\frac{2}{3}} A} \left(\frac{\cos^{\frac{4}{3}} A}{\sin^{\frac{2}{3}} A} + \frac{\sin^{\frac{4}{3}} A}{\cos^{\frac{2}{3}} A}\right)$

$= \cos^{\frac{2}{3}} A \sin^{\frac{2}{3}} A \left(\frac{\cos^{\frac{4}{3}} A \cos^{\frac{2}{3}} A + \sin^{\frac{4}{3}} A \sin^{\frac{2}{3}} A}{\cos^{\frac{2}{3}} A \sin^{\frac{2}{3}} A}\right)$

$= \cos^2 A + \sin^2 A = 1$

Q. If sin θ + cos θ = 1, then what is the value of $\cos^2\theta + \cos^4\theta$

Ans: Given, $\sin\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin\theta$, Squaring on both sides, we have

$(1 - \cos^2\theta)^2 = \sin^2\theta \therefore 1 + \cos^4\theta - 2\cos^2\theta = 1 - \cos^2\theta \Rightarrow \cos^4\theta - \cos^2\theta = 0$

$\Rightarrow \cos^4\theta + \cos^2\theta = 2\cos^2\theta$ Thus, the value of $\cos^4\theta + \cos^2\theta$ is $2\cos^2\theta$

Q. Find value of: 12 cos A + 5 sin A , If 5 cos A – 12 sin A = 13

Solution: $(5\cos A - 12\sin A)^2 + (12\cos A + 5\sin A)^2$

$= 25\cos^2 A + 144\sin^2 A - 120\sin A\cos A + 144\cos^2 A + 25\sin^2 A + 120\sin A\cos A$

$= 169\cos^2 A + 169\sin^2 A = 169(\sin^2 A + \cos^2 A) = 169$

$\therefore (12\cos A + 5\sin A)^2 = 169 - (5\cos A - 12\sin A)^2 = 169 - 169 = 0$

$\Rightarrow (12\cos A + 5\sin A)^2 = 0 \Rightarrow 12\cos A + 5\sin A = 0$

Q. if sin x + cos x - √2 sin x =0 find the value of $\tan^2 x$,$\cot^2 x$ and $\tan^2 x + \cot^2 x$

Ans:

$$\sin x + \cos x - \sqrt{2}\sin x = 0 \quad \ldots \quad (1)$$

$$\Rightarrow \cos x = \left(\sqrt{2}-1\right)\sin x \quad \Rightarrow \frac{\sin x}{\cos x} = \frac{1}{\sqrt{2}-1} \quad \Rightarrow \tan x = \frac{1}{\sqrt{2}-1} \quad \Rightarrow \tan^2 x = \frac{1}{\left(\sqrt{2}-1\right)^2}$$

$$\Rightarrow \tan^2 x = \frac{1}{2+1-2\sqrt{2}} \Rightarrow \tan^2 x = \frac{1}{3-2\sqrt{2}} \quad \text{and} \quad \cot^2 x = \frac{1}{\tan^2 x} = 3-2\sqrt{2}$$

$$\therefore \tan^2 x + \cot^2 x = \frac{1}{3-2\sqrt{2}} + \left(3-2\sqrt{2}\right) = \frac{1+\left(3-2\sqrt{2}\right)^2}{\left(3-2\sqrt{2}\right)} = \frac{1+9+8-12\sqrt{2}}{\left(3-2\sqrt{2}\right)}$$

$$= \frac{18-12\sqrt{2}}{\left(3-2\sqrt{2}\right)} = \frac{6\left(3-2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)} = 6$$

Q. if $x^{x+y} = y^3$ and $y^{x+y} = x^6 y^3$, x and y r natural numbers find x^y and y^x

Q. if $\sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \mathrm{cosec}^2 x = 7$ find the value of $\sin^2 x$, $\cos^2 x$ and more

$$\sin^2 x + \cos^2 x + \tan^2 x + \cot^2 x + \sec^2 x + \mathrm{cosec}^2 x = 7$$

$$\Rightarrow 1 + \tan^2 x + \cot^2 x + \sec^2 x + \mathrm{cosec}^2 x = 7 \qquad \left[\because \left(\sin^2 x + \cos^2 x\right) = 1\right]$$

$$\Rightarrow \sec^2 x + \cot^2 x + \sec^2 x + \mathrm{cosec}^2 x = 7 \qquad \left[\because 1 + \tan^2 x = \sec^2 x\right] \Rightarrow 2\sec^2 x + \cot^2 x + \mathrm{cosec}^2 x = 7$$

$$\Rightarrow 2\sec^2 x + \left(\mathrm{cosec}^2 x - 1\right) + \mathrm{cosec}^2 x = 7 \qquad \left[\because \mathrm{cosec}^2 x - \cot^2 x = 1\right] \Rightarrow 2\sec^2 x + 2\mathrm{cosec}^2 x = 8$$

$$\Rightarrow 2\left(\sec^2 x + \mathrm{cosec}^2 x\right) = 8 \quad \Rightarrow \sec^2 x + \mathrm{cosec}^2 x = 4 \quad \Rightarrow \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = 4 \quad \Rightarrow \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} = 4$$

$\Rightarrow 4\left(\sin^2 x \cdot \cos^2 x\right) = 1 \Rightarrow 4\left(\sin^2 x\left(1 - \sin^2 x\right)\right) = 1 \quad \Rightarrow 4\left(\sin^2 x - \left(\sin^2 x\right)^2\right) = 1 \Rightarrow 4\sin^2 x - 4\left(\sin^2 x\right)^2 = 1$

$\Rightarrow 4\left(\sin^2 x\right)^2 - 4\sin^2 x + 1 = 0$ **Put $\sin^2 x = t$, we have a quadratic equation**

$4t^2 - 4t + 1 = 0 \quad \Rightarrow (2t - 1)^2 = 0 \quad \Rightarrow 2t - 1 = 0 \quad \Rightarrow t = \frac{1}{2}$

i.e. $\sin^2 x = \frac{1}{2}$ and $\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{2} = \frac{1}{2}$

Q. If a cot A + b cosec A = p and b cot A = a cosec A = q then Find value of : $p^2 - q^2$

Ans: Given, *a* cot A + *b* cosec A= *p* and *b* cot A + *a* cosec A = *q*

$p^2 - q^2 = (a \cot A + b \operatorname{cosec} A)^2 - (b \cot A + a \operatorname{cosec} A)^2$

$= (a \cot^2 A + b^2 \operatorname{cosec}^2 A + 2ab \cot A \operatorname{cosec} A) - (b^2 \cot^2 A + a^2 \operatorname{cosec}^2 A + 2ab \cot A \operatorname{cosec} A)$

$= a^2 \cot^2 A + b^2 \operatorname{cosec}^2 A + 2ab \cot A \operatorname{cosec} A - b^2 \cot^2 A - a^2 \operatorname{cosec}^2 A - 2ab \cot A \operatorname{cosec} A$

$= a^2 (\cot^2 A - \operatorname{cosec}^2 A) + b^2 (\operatorname{cosec}^2 A - \cot^2 A) \quad = b^2 (\operatorname{cosec}^2 A - \cot^2 A) - a^2 (\operatorname{cosec}^2 A - \cot^2 A)$

$= b^2 - a^2 \ (\operatorname{cosec}^2 \theta - \cot^2 \theta = 1)$

Q. if sin A +cos A=x, prove that $\sin^6 A + \cos^6 A = 4 - [3(x^2-1)^2]/4$

Ans:

Given, $\sin A + \cos A = x$ Squaring on both sides, we get $(\sin A + \cos A)^2 = x^2$

$\Rightarrow \sin^2 A + \cos^2 A + 2\sin A \cos A = x^2 \quad \Rightarrow 1 + 2\sin A \cos A = x^2$

$\Rightarrow 2\sin A \cos A = x^2 - 1 \quad \Rightarrow \sin A \cos A = \frac{x^2 - 1}{2} \quad \ldots(1)$

Q. if sin A = 1/3, find value of 2 $\cot^2$ A + 2

Ans:

Given, $\sin A = \frac{1}{3} \Rightarrow \operatorname{cosec} A = 3$ we know that, $1 + \cot^2 A = \operatorname{cosec}^2 A$

$\Rightarrow \cot^2 A = \operatorname{cosec}^2 A - 1 = (3)^2 - 1 = 9 - 1 = 8 \therefore 2\cot^2 A + 2 = 2 \times (8) + 2 = 16 + 2 = 18$