COORDINATEGEOMETRY

Section formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points such that a point $P(x, y)$ divides AB internally in the ratio $l : m$. That is, $\frac{AP}{PB} = \frac{l}{m}$

From the Fig. 5.2, we get

$$AF = CD = OD - OC = x - x_1$$

$$PG = DE = OE - OD = x_2 - x$$

Also, $$PF = PD - FD = y - y_1$$

$$BG = BE - GE = y_2 - y$$

Now, ΔAFP and ΔPGB are similar.

(Refer chapter 6, section 6.3)

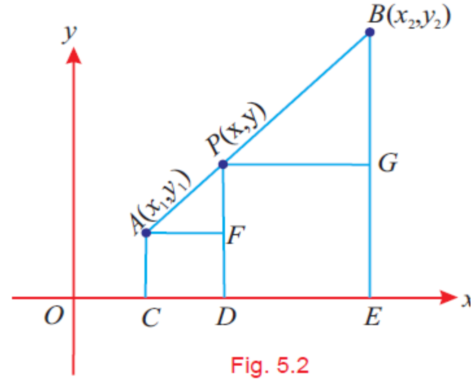

Fig. 5.2

Thus, $$\frac{AF}{PG} = \frac{PF}{BG} = \frac{AP}{PB} = \frac{l}{m}$$

$\therefore$ $\frac{AF}{PG} = \frac{l}{m}$ and $\frac{PF}{BG} = \frac{l}{m}$

$\Longrightarrow \frac{x - x_1}{x_2 - x} = \frac{l}{m}$	$\Longrightarrow \frac{y - y_1}{y_2 - y} = \frac{l}{m}$
$\Longrightarrow mx - mx_1 = lx_2 - lx$	$\Longrightarrow my - my_1 = ly_2 - ly$
$lx + mx = lx_2 + mx_1$	$ly + my = ly_2 + my_1$
$\Longrightarrow x = \frac{lx_2 + mx_1}{l + m}$	$\Longrightarrow y = \frac{ly_2 + my_1}{l + m}$

Thus, the point P which divides the line segment joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $l : m$ is

$$P\left(\frac{lx_2 + mx_1}{l + m}, \frac{ly_2 + my_1}{l + m}\right)$$

This formula is known as **section formula**.

It is clear that the section formula can be used only when the related three points are collinear.

Results

(i) If P divides a line segment AB joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $l : m$, then the point P is $\left(\frac{lx_2 - mx_1}{l - m}, \frac{ly_2 - my_1}{l - m}\right)$. In this case $\frac{l}{m}$ is negative.

(ii) **Midpoint of AB**

If M is the midpoint of AB, then M divides the line segment AB internally in the ratio 1:1. By substituting $l = 1$ and $m = 1$ in the section formula, we obtain

the midpoint of AB as $M\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$.

The midpoint of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.