Properties Of Arithmetic Mean

1. The sum of the deviations, of all the values of x, from their arithmetic mean, is zero.

   Justification : $\sum f_i (x_i - \bar{x}) = \sum f_i x_i - \bar{x} \sum f_i = 0$

   Since $\bar{x}$ is a constant, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} \therefore \sum f_i x_i = \bar{x} \sum f_i$

2. The product of the arithmetic mean and the number of items gives the total of all items.

   Justification : $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} \Rightarrow \sum f_i x_i = \bar{x} \sum f_i$

   or $\bar{x} = \frac{\sum x_i}{N} \Rightarrow \bar{x} \cdot N = \sum x_i$

3. If $\bar{x}_1$ and $\bar{x}_2$ are the arithmetic mean of two samples of sizes n_1 and n_2 respectively then, the arithmetic mean $\bar{x}$ of the distribution combining the two can be calculated as

   $$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

   This formula can be extended for still more groups or samples.

   $$\bar{x}_1 = \frac{\sum x_{1i}}{n_1} \Rightarrow \sum x_{1i} = n_1 \bar{x}_1$$

   Justification : $\bar{x}_1 = \frac{\sum x_{1i}}{n_1} \Rightarrow \sum x_{1i} = n_1 \bar{x}_1$ = total of the observations of the first sample

   Similarly $\sum x_{2i} = n_2 \bar{x}_2$ = total of the observations of the first sample

   The combined mean of the two samples

   $$= \frac{\text{combined total}}{n_1 + n_2}$$

   $$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$