Physics _ Archimedes' Principle Class 9(Detail Study)

A1. Density (ρ)

The density of a body is defined as mass per unit volume.

i.e. $\rho = \frac{M}{V}$

ρ = density (kgm^{-3})

M = mass of the body (kg)

V = volume of the body (m^3)

**NOTE:

1. The mass of a body is a constant quantity while the volume varies with temperature (and even with pressure for gas). So the density depends on temperature too. However the variation of density with temperature is generally small enough to be neglected.
2. The density of water at 4^0C is 1000kgm^{-3} or 1 gcm^{-3}.

A.2 Relative Density (R.D.)

The relative density of a body is defined as the ratio of the density of the body to that of water.

i.e. $\text{Relative Density of a body} = \boxed{\frac{\text{Density of the body}}{\text{Density of water}}}$

→ It can easily be seen that the relative density of a body can be defined alternatively as:

$$RD = \frac{\text{mass of a body}}{\text{mass of equal volume of water}} = \frac{\text{weight of the body}}{\text{weight of equal volume of water}}$$

**NOTE:
1. R.D. is only a pure number; it has no unit.
2. R.D. of water is obviously 1.
3. density of a body = R.D. of the body × density of water

   e.g.Let RD of a body = 5

   So, density of body = 5 ×1000 kgm^{-3} = 5000 kgm^{-3}

EXAMPLES:

1. Find the mass of air inside a room measuring 10m × 8m × 3m, if the density of air is 1.28kgm^{-3}.

2. A container of volume 0.05m^3 is full of ice. When the ice melts into water, how many kg of water should be added to fill it up? (density of ice = 900kgm^{-3}; density of water = 1000kgm^{-3})

Mass of ice = ρ_{ice}×0.05 = 45kg

Mass of water = ρ_{water}×0.05 = 50kg

5 kg of water should be added to fill it up

3. If 1m^3 of water is mixed with 3m^3 of a liquid to form a mixture of density 850kgm^{-3}, find the density of the liquid, assuming that there is no contraction of volume. (Density of water = 1000kgm^{-3})

Mass of water = d x V = 1 x 1000 = 1000 kg/m^3

Mass of liquid = d x 3 = 3d kg/m^3

Density of mixture = 850kgm^{-3} = $\frac{1000+3d}{1+3}$

⇒ The density of the liquid $\frac{3400-1000}{3} = 800\text{kg/m3}$

4. An alloy consists of 60% aluminium and 40% tin by weight.

(Density of aluminium = 2700kgm^{-3}; density of tin = 7300kgm^{-3}.)

(i) What is the density of the alloy?

$$\ell = \frac{M}{V_a + V_\ell} = \frac{M}{\frac{0.6M}{\ell_a} + \frac{0.4M}{\ell_t}} = 3610kgm^{-3}$$

(Ans: 3610 kgm^{-3})

(ii) What is the mass of tin in 0.02m^3 of the alloy?

$$\ell = \frac{M}{V} \Rightarrow 3610kgm^{-3} = \frac{M}{0.02} = 72.2kg$$

$$M_a = 0.6M = 0.6 \times 72.2kg = 28.9kg$$

(Ans: 3610 kgm^{-3})

5. Find the density of 14K gold if the densities of gold and copper are respectively 19300 and 8900 kgm^{-3}.

(14K is a mixture of $\frac{10}{24}$ of copper by mass.)

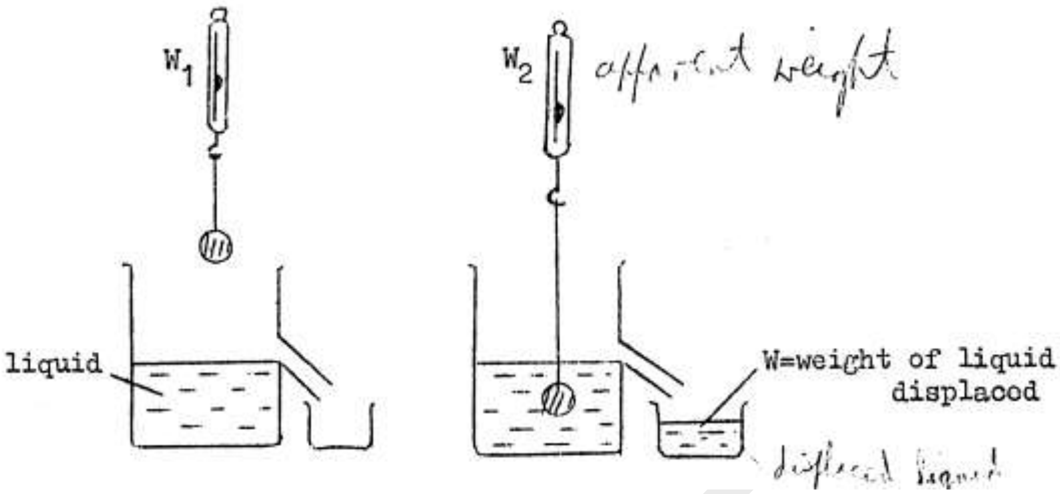
$$\ell = \frac{M}{V_g + V_c} = \frac{M}{\frac{14}{24}\frac{M}{19300} + \frac{10}{24}\frac{M}{2900}} = 12980kgm^{-3}$$

(Ans: 12980 kgm^{-3})

A. 3 : Archimedes' Principle

Introduction

- When an object is lowered into a liquid, it is found to lose weight. It is because the liquid exerts an up thrust on the object.



(i) up thrust = apparent lost in weight of the object = W1 – W2

(ii) Experimentally, W1 – W2 = W = weight of liquid displaced

(ii) Hence, up thrust acting on a body = weight of liquid displaced by the body

(a) Principle

- Archimedes' Principle states that when a body is totally or partially immersed in a fluid, it experiences an up thrust equal to the weight of the fluid displaced.

**NOTE:
1. When a body is totally immersed, it displaced its own volume of fluid.
2. The term fluid covers liquids and gases.
3. The up thrust acts vertically upwards through the centre of gravity of the fluid displaced.

EXAMPLES: (Take g = 10ms-2 whenever necessary.)

1. A block of aluminium has a volume of $0.1m^3$ and a density of $2700\ kgm^{-3}$.

Calculate:

(a) the weight of the aluminum block,

$$m = dxv$$

$$W = mg = 2700N$$

(b) The apparent weight of the block when immersed in water of density $1000kgm^{-3}$.

Apparent weight = 2700 – g(1000)(0.1) = 1700N

2. A rock of mass 114kg is lifted from the bottom of a river 10m deep. Densities of water and rock are 1000 and 5700kgm-3 respectively. Find

(a) the force of buoyancy (i.e. up thrust) on the rock.

U = wℓ. of water displaced

$$= \quad mg = 1000\times V_{H20}g \quad = 1000\times V_{rock}g \quad = 1000\times\frac{M_{rock}}{\ell_{rock}}g$$

(b) its apparent weight, 1140N – 200N = 940N

(c)the work done in lifting the rock. 940×10 = 9400J

3. A block of load weight 5.5N in air and 5N in water. (Density of water = 1000kgm^{-3})

(a) Find the volume of water displaced by the load block.

$5.5 - 5 = mg$

$= 1000vg$

$V = 5\times10^{-5}m^3$

a) Find the volume of the load block.

V of load block×Volume weight displaced

$= 5\times10^{-5}m^3$

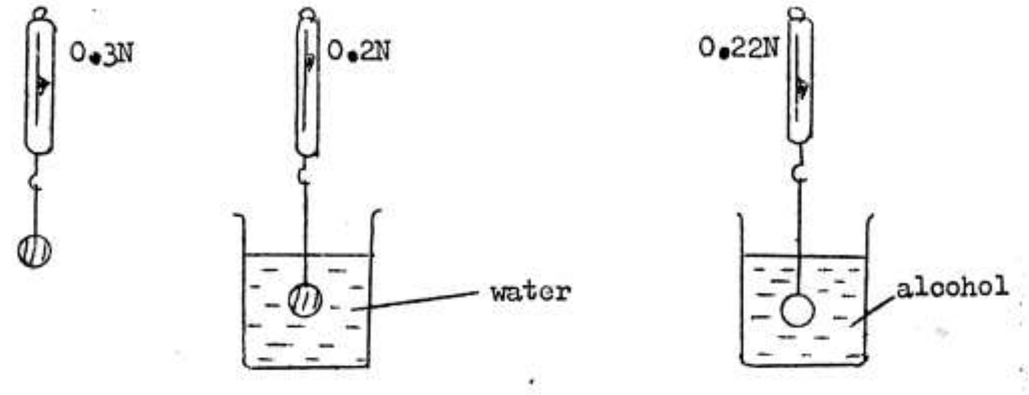
b) What is the density of the load block?

$$m = \frac{5.5}{g}$$

$$\ell = \frac{m}{V}$$

$$= \frac{0.55}{5\times10^{-5}} = 11000kgm^{-3}$$

.4. The following diagrams show that a marble weight 0.3N in air, 0.2N in water, 0.22N in alcohol. (Density of water = 1000kgm^{-3}.)



Find

(a) the up thrust of water on the marble,
0.3 – 0.2 = 0.1N

(b) The weight of water displaced,

up thrust – weight of water displaced

Weight = 0.1N

c) the volume of the marble, (Ans: $1\times10^{-5}m^3$)

d) the density of the marble, (Ans: $3\times10^{3}kgm^{-3}$)

e) the up thrust of alcohol on the marble, (Ans: 0.08N)

(f) the density of alcohol. (Ans: $800kgm^{-3}$)

5. A fish of mass 1kg and density 2000kgm-3 is supported from a light spring of a spring balance. It is to be lowered into a tank of water placed over a compression balance. Before the fish in put into the water, the compression balance reads 60N. When the fish is completely immersed in water, find

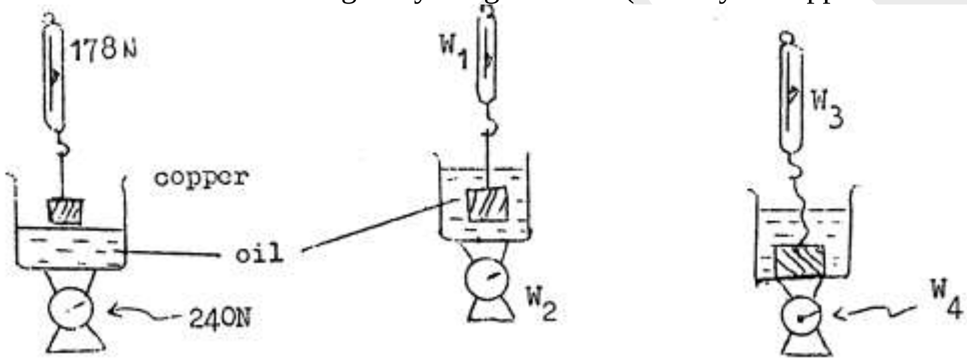
the upthrust acting on the fish, (Ans: 5N)

the reading of the spring balance, (Ans: 5N)

the thrust exerted on the bottom of the tank, (Ans: 65N)

the reading of the compression balance. (Ans: 65N)

6 A spring balance roads 178N as a piece of copper if suspended. The copper is then totally immersed into a beaker of oil which originally weighs 240N. (Density of copper and oil are 8900 and 900 kgm^{-3}.)



a) Find the new readings W_1, W_2 of the spring balance and the compression machine on which the beaker is put. (Ans: W_1 = 160N, W_2 = 258N)

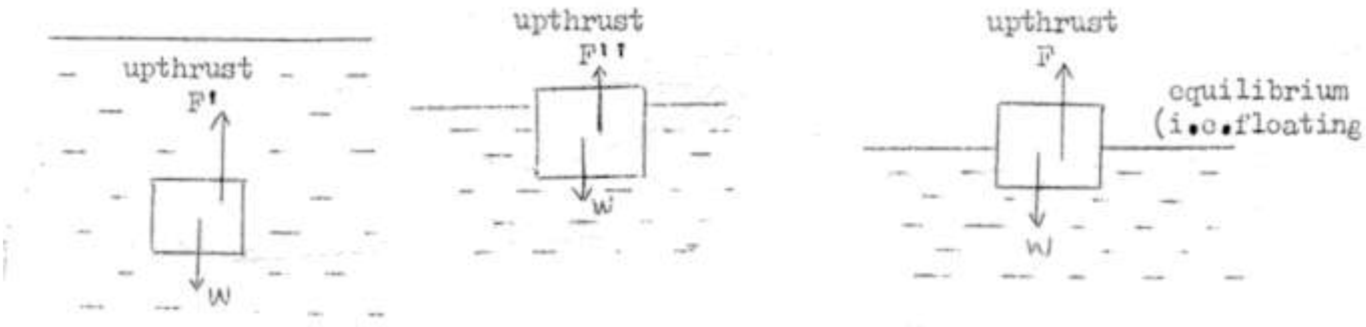
b) If the copper is put on the beaker so that the string becomes slack, what are the readings W_3, W_4 then? Ans: W_3 = 0, W_4 = 418N)

A.4 Law of the floating / floatation

(a) Introduction

Consider a body is completely immersed in a liquid and $\ell < \ell'$.

ℓ' = density of the liquid ℓ = density of the solid W = weight of the body in air



W < upthrust F'	W < upthrust F''	W = upthrust F
< weight of liquid displaced	< weight of liquid displaced	= weight of liquid displaced

a) Law

- Law of the floatation states that a floating body displaces its own weight of fluid in which it floats.

  i.e. weight of floating body = weight of liquid displaced by the body

- Actually, law of floatation is a special case of Archimedes' Principle.

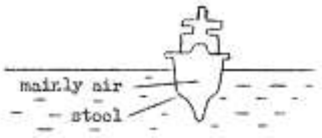
b) NOTE:

(i) Floating or Sinking

- A body (average density ℓ) can float in a liquid (density ℓ') if;

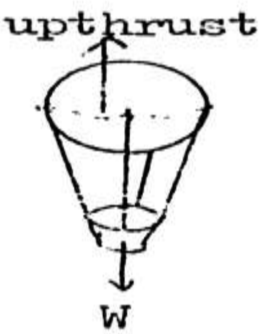
  $\ell < \ell'$

e.g. a ship can float in water.



Average density < density of water of the ship

e.g. a balloon can move upwards in air.



average density of balloon < density of air

⇒ upthrust = weight of air displaced

> weight of the balloon

⇒there is a lifting force acting on the balloon.

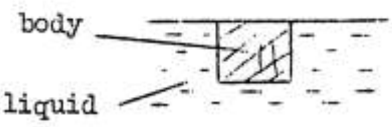
e.g. a submarine sinks by taking water into its buoyancy tanks so that its average density is increased.

(ii) Just Float

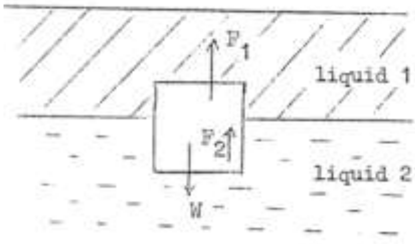
- A body (density ℓ) is said to be just floating in a liquid (density ℓ')

  if: $\ell = \ell'$

  e.g.



(iii)Floating in two liquids



$$W = F_1 + F_2$$

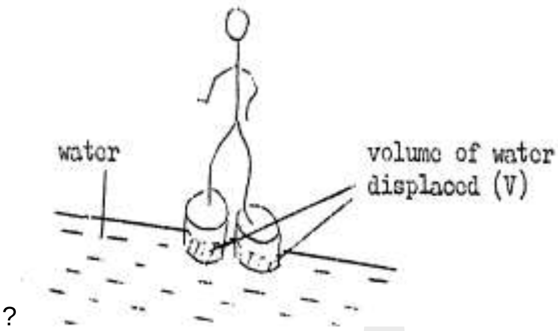
W = weight of the body

F_1 = weight of displaced liquid 1

F_2 = weight of displaced liquid 2

(iv) Question

Can a man easily walk on water surface by fastening 2 pieces of wood under his feet ?(mass of the man = 50kg)



?

Given: weight of the man = 500N and density of water = $1000kgm^{-3}$

weight of the man = weight of water displaced (by law of floatation

500 = (ℓV)g ⇒ 500 = (1000V)10

volume of water displaced V = 0.05 m3 = 50 litres

It is obvious that only very large pieces of wood can displace such volume of water.

EXAMPLES: (Take = g = ms^{-2}) whenever necessary.)

1. A block of wood of volume of 0.1m3 and density of $700kgm^{-3}$ floats in water of density 1000 kgm^{-3}. Calculate

a) the weight of the block, (Ans: 700N)

b) the mass of water displaced by the block,(Ans: 70kg)

c) the volume of the block immersed in water.(Ans: $0.07m^3$)

2.A cylinder is weighted with some lead shot so that it floats upright in water with 6cm of its length under water. If the cylinders is floated in brine (salt water), what length of the rod is then below the surface? (Densities of water and brine are 1000 and $1200kgm^{-3}$ respectively.)

(Ans: 5cm)

3. A balloon of volume $2000m^3$ is filled with hot air of density $0.95kgm^{-3}$. If the total mass of the fabric, basket and pilot is 480kg, calculate the lifting force on the balloon when the surrounding air has a density of $1.2kgm^{-3}$.

Upthrust = weight of cold air displaced

Weight of hot air in the balloon = Weight of fabric, basket and pilot =Lifting force on the balloon = 200N

4.A cubical block of wood of side 0.5m floats in water. Given that the densities of wood and water are 800 and $1000kgm^{-3}$ respectively, calculate

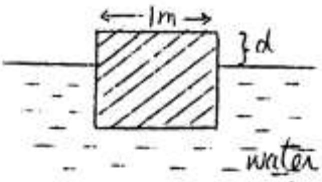
a) the weight of the block, (Ans: 1000N)

b) the weight of water displaced by the block, (Ans: 1000N)

c) the depth of the block immersed in water. (Ans: 0.4m)

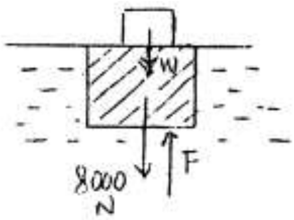
5. A wooden cube of side 1m and density $800kgm^{-3}$ floats in water horizontally. (density of water = $1000kgm^{-3}$)

a) Find the length of the vertical side which does not submerge.



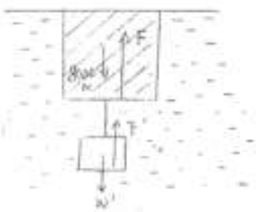
(Ans: 0.2m)

b) If the cube is just totally submerged by placing a piece of tin (density = $7200kgm^{-3}$) on top of the cube, what is the weight of the tin?



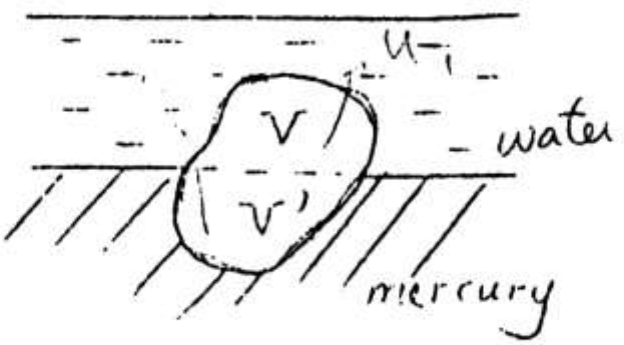
(Ans: 2000N)

c) If the cube is just totally submerged by suspended the tin form its bottom, determine the weight of the tin required.

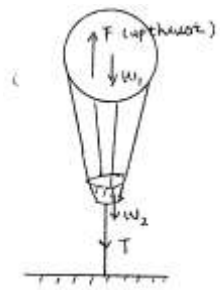


(Ans: 2320N)

e.g.6 A metal floats between some water and some water and some mercury. Find the ratio of the volume which are submerged in water to that in mercury, if the densities of the metal, mercury and water are 7900, 13600 and 1000kgm^{-3} respectively. (Ans: $\frac{19}{23}$)



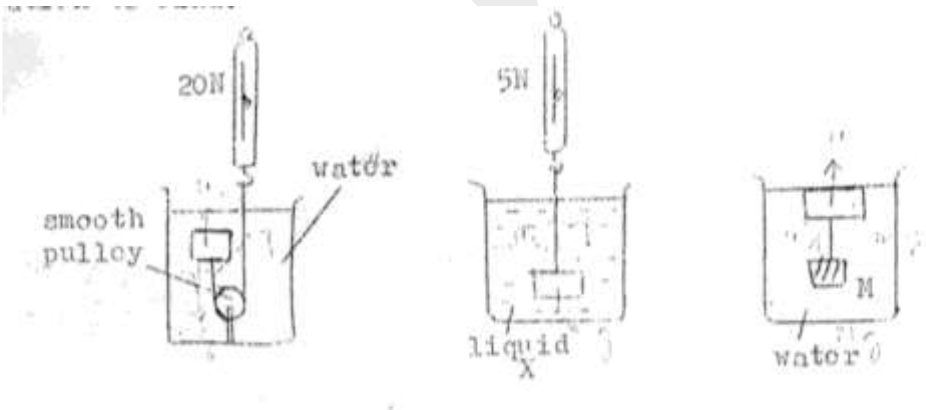
7. The fabric of a large balloon and its basket weigh 8000N. The balloon is inflated with 2500m^3 of helium and moored in still air by a cable. If the temperature and pressure of the helium gas and air are standard, find the tension in the cable. If the cable is cut, find the acceleration with it rises.
(Densities of air and helium are 1.3 and 0.18 kgm^{-3} respectively.)



(Ans: T = 20000N, a = 16 ms^{-2})

e.g.8 A wooden block is weighed in air, water, and liquid X, as shown below. Finally a lump of metal M with density 10000kgm^{-3} is attached to the wooden block and causes it to float in water.

Use this information to find:



the upthrust on the block in water. (Ans: 1000N)
the volume of the block, (Ans: 0.01m^3)

the density of the block, (Ans: 800kgm^{-3})
the density of liquid X , and (Ans: 750kgm^{-3})
the mass of the metal lump M. (Ans: 2kg)

A.5 Hydrometer

- A hydrometer is an instrument for determining the relative density of a liquid, basing on the law of floatation.
- It usually consists of a long graduated thin stem A, a large bulb B to provide buoyancy, and a small bulb C with some lead shots to enable the hydrometer to float upright.
- It should be noted that the readings of the graduations increase downwards and the graduations are not equally apart.