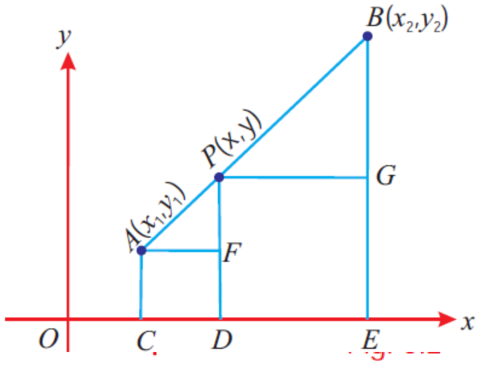
Class X – Chapter: Coordinate Geometry: **Section formula**

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two distinct points such that a point $P(x, y)$ divides AB internally in the ratio $l : m$. That is, $\frac{AP}{PB} = \frac{l}{m}$



From the Fig. 5.2, we get

$$AF = CD = OD - OC = x - x_1$$

$$PG = DE = OE - OD = x_2 - x$$

Also, $PF = PD - FD = y - y_1$

$$BG = BE - GE = y_2 - y$$

Now, ΔAFP and ΔPGB are similar.

Thus, $\frac{AF}{PG} = \frac{PF}{BG} = \frac{AP}{PB} = \frac{l}{m}$

$\therefore \quad \frac{AF}{PG} = \frac{l}{m}$ and $\frac{PF}{BG} = \frac{l}{m}$

$$\Rightarrow \frac{x - x_1}{x_2 - x} = \frac{l}{m} \qquad \Rightarrow \frac{y - y_1}{y_2 - y} = \frac{l}{m}$$

$$\Rightarrow mx - mx_1 = lx_2 - lx \qquad \Rightarrow my - my_1 = ly_2 - ly$$

$$lx + mx = lx_2 + mx_1 \qquad ly + my = ly_2 + my_1$$

$$\Rightarrow x = \frac{lx_2 + mx_1}{l + m} \qquad \Rightarrow y = \frac{ly_2 + my_1}{l + m}$$

Thus, the point P which divides the line segment joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $l : m$ is

$$P\left(\frac{lx_2 + mx_1}{l + m}, \frac{ly_2 + my_1}{l + m}\right)$$

This formula is known as **section formula**.

Results

(i) If P divides a line segment AB joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $l : m$, then the point P is $\left(\frac{lx_2 - mx_1}{l - m}, \frac{ly_2 - my_1}{l - m}\right)$. In this case $\frac{l}{m}$ is negative.

(ii) **Midpoint of AB**

If M is the midpoint of AB, then M divides the line segment AB internally in the ratio 1:1. By substituting $l = 1$ and $m = 1$ in the section formula, we obtain

the midpoint of AB as $M\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$.

The midpoint of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Centroid of a triangle

Consider a ΔABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$. Let AD, BE and CF be the medians of the ΔABC.

We know that the medians of a triangle are concurrent and the point of concurrency is the centroid.

Let $G(x, y)$ be the centroid of ΔABC.

Now the midpoint of BC is $D\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

By the property of triangle, the centroid G divides the median AD internally in the ratio $2 : 1$

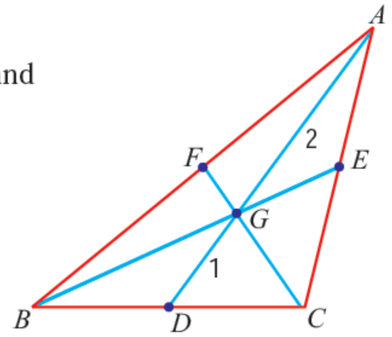


Fig. 5.3

$\therefore$ By section formula, the centroid

$$G(x, y) = G\left(\frac{2\frac{(x_2 + x_3)}{2} + 1(x_1)}{2 + 1}, \frac{2\frac{(y_2 + y_3)}{2} + 1(y_1)}{2 + 1}\right)$$

$$= G\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

The centroid of the triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3), is $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Example

Find the midpoint of the line segment joining the points $(3, 0)$ and $(-1, 4)$.

Solution Midpoint $M(x, y)$ of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$M(x, y) = M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

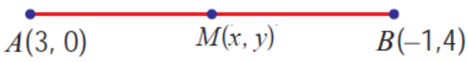


Fig. 5.4

$\therefore$ Midpoint of the line segment joining the points $(3, 0)$ and $(-1, 4)$ is

$$M(x, y) = M\left(\frac{3-1}{2}, \frac{0+4}{2}\right) = M(1, 2).$$

Find the point which divides the line segment joining the points $(3, 5)$ and $(8, 10)$ internally in the ratio $2 : 3$.

Solution Let $A(3, 5)$ and $B(8, 10)$ be the given points.

Let the point $P(x,y)$ divide the line AB internally in the ratio $2 : 3$.

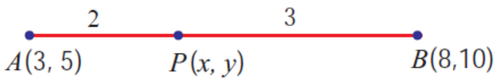


Fig. 5.5

By section formula, $P(x, y) = P\left(\frac{lx_2 + mx_1}{l + m}, \frac{ly_2 + my_1}{l + m}\right)$

Here $x_1 = 3, y_1 = 5, x_2 = 8, y_2 = 10$ and $l = 2, m = 3$

$$\therefore \quad P(x, y) = P\left(\frac{2(8) + 3(3)}{2 + 3}, \frac{2(10) + 3(5)}{2 + 3}\right) = P(5, 7)$$

In what ratio does the point $P(-2, 3)$ divide the line segment joining the points $A(-3, 5)$ and $B(4, -9)$ internally?

Solution Given points are $A(-3, 5)$ and $B(4, -9)$.

Let $P(-2, 3)$ divide AB internally in the ratio $l : m$

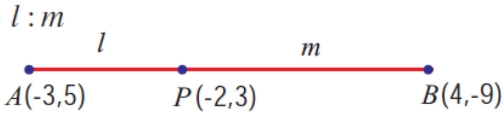


Fig. 5.6

By the section formula,

$$P\left(\frac{lx_2 + mx_1}{l + m}, \frac{ly_2 + my_1}{l + m}\right) = P(-2, 3) \quad (1)$$

Here $x_1 = -3, y_1 = 5, x_2 = 4, y_2 = -9$.

(1) $\Longrightarrow \left(\dfrac{l(4)+m(-3)}{l+m}, \dfrac{l(-9)+m(5)}{l+m}\right) = (-2, 3)$

Equating the x-coordinates, we get

$$\frac{4l-3m}{l+m} = -2$$
$$\Longrightarrow \quad 6l = m$$
$$\frac{l}{m} = \frac{1}{6}$$
i.e., $\quad l : m = 1 : 6$

Hence P divides AB internally in the ratio $1 : 6$

Find the points of trisection of the line segment joining $(4, -1)$ and $(-2, -3)$.

Solution Let $A(4, -1)$ and $B(-2, -3)$ be the given points.

Let $P(x, y)$ and $Q(a, b)$ be the points of trisection of AB so that $AP = PQ = QB$

Fig. 5.7

Hence P divides AB internally in the ratio $1 : 2$ and Q divides AB internally in the ratio $2 : 1$

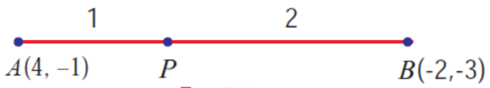


Fig. 5.8

$\therefore$ By the section formula, the required points are

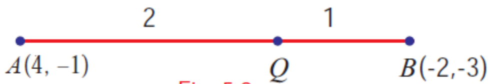


Fig. 5.9

$P\left(\dfrac{1(-2)+2(4)}{1+2}, \dfrac{1(-3)+2(-1)}{1+2}\right)$ and

$Q\left(\dfrac{2(-2)+1(4)}{2+1}, \dfrac{2(-3)+1(-1)}{2+1}\right)$

$\Longrightarrow P(x, y) = P\left(\dfrac{-2+8}{3}, \dfrac{-3-2}{3}\right)$ and $Q(a, b) = Q\left(\dfrac{-4+4}{3}, \dfrac{-6-1}{3}\right)$

$= P\left(2, -\dfrac{5}{3}\right)$ $\qquad = Q\left(0, -\dfrac{7}{3}\right)$.

Note that Q is the midpoint of PB and P is the midpoint of AQ.

Find the centroid of the triangle whose vertices are $A(4, -6)$, $B(3, -2)$ and $C(5, 2)$.

Solution The centroid $G(x, y)$ of a triangle whose vertices are

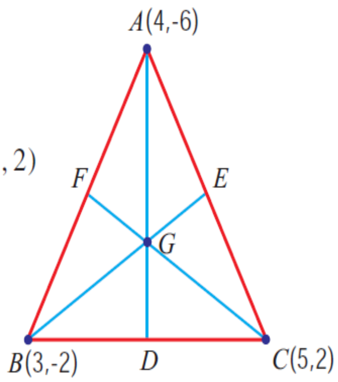
(x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$G(x, y) = G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

We have $(x_1, y_1) = (4, -6)$, $(x_2, y_2) = (3, -2)$, $(x_3, y_3) = (5, 2)$

$\therefore$ The centroid of the triangle whose vertices are $(4, -6), (3, -2)$ and $(5, 2)$ is

$$G(x, y) = G\left(\frac{4+3+5}{3}, \frac{-6-2+2}{3}\right) = G(4, -2).$$



If $(7,3),(6,1),(8,2)$ and $(p,4)$ are the vertices of a parallelogram taken in order, then find the value of p.

Solution Let the vertices of the parallelogram be $A(7,3), B(6,1), C(8,2)$ and $D(p,4)$.

We know that the diagonals of a parallelogram bisect each other.

$\therefore$ The midpoints of the diagonal AC and the diagonal BD coincide.

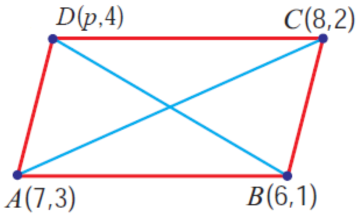


Fig. 5.11

Hence $\left(\frac{7+8}{2}, \frac{3+2}{2}\right) = \left(\frac{6+p}{2}, \frac{1+4}{2}\right)$

$$\Longrightarrow \left(\frac{6+p}{2}, \frac{5}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$$

Equating the x-coordinates, we get,

$$\frac{6+p}{2} = \frac{15}{2}$$

$$\therefore p = 9$$

If C is the midpoint of the line segment joining $A(4\,,\,0)$ and $B(0\,,\,6)$ and if O is the origin, then show that C is equidistant from all the vertices of $\triangle OAB$.

Solution The midpoint of AB is $C\left(\frac{4+0}{2}, \frac{0+6}{2}\right) = C(2,3)$

We know that the distance between $P(x_1,y_1)$ and $Q(x_2,y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$.

Distance between $O(0,0)$ and $C(2,3)$ is

$$OC = \sqrt{(2-0)^2+(3-0)^2} = \sqrt{13}.$$

Distance between $A(4,0)$ and $C(2,3)$,

$$AC = \sqrt{(2-4)^2+(3-0)^2} = \sqrt{4+9} = \sqrt{13}$$

Distance between $B(0,6)$ and $C(2,3)$,

$$BC = \sqrt{(2-0)^2+(3-6)^2} = \sqrt{4+9} = \sqrt{13}$$

$\therefore \quad OC = AC = BC$

$\therefore$ The point C is equidistant from all the vertices of the $\triangle OAB$.

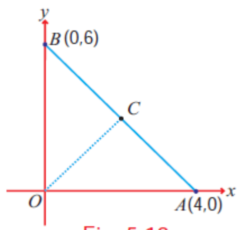


Fig. 5.12

Area of a triangle

Let ABC be a triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.

Draw the lines AD, BE and CF perpendicular to x-axis.

From the figure, $ED = x_1 - x_2$, $DF = x_3 - x_1$ and $EF = x_3 - x_2$.

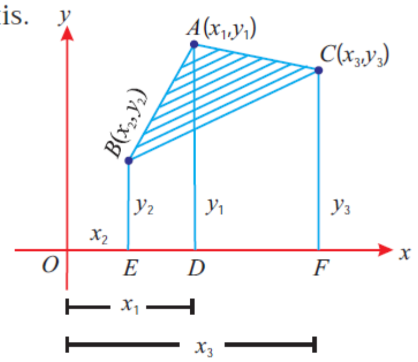


Fig. 5.13

Area of the triangle ABC

= Area of the trapezium $ABED$

+ Area of the trapezium $ADFC$

− Area of the trapezium $BEFC$

$$= \frac{1}{2}(BE + AD)ED + \frac{1}{2}(AD + CF)DF - \frac{1}{2}(BE + CF)EF$$

$$= \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

$$= \frac{1}{2}\{x_1y_2 - x_2y_2 + x_1y_1 - x_2y_1 + x_3y_1 - x_1y_1 + x_3y_3 - x_1y_3 - x_3y_2 + x_2y_2 - x_3y_3 + x_2y_3\}$$

$\therefore$ Area of the ΔABC is $\frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$ sq.units.

Remember:

Suppose that the three points $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are collinear. Then they cannot form a triangle. Hence the area of the ΔABC is zero.

i.e., $\frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\} = 0$

$$\Longrightarrow \quad x_1y_2 + x_2y_3 + x_3y_1 = x_2y_1 + x_3y_2 + x_1y_3$$

One can prove that the converse is also true.

Hence the area of ΔABC is zero if and only if the points A, B and C are collinear.

Area of the Quadrilateral

Let $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ and $D(x_4,y_4)$ be the vertices of a quadrilateral $ABCD$.

Now the area of the quadrilateral $ABCD$ = area of the ΔABD + area of the ΔBCD

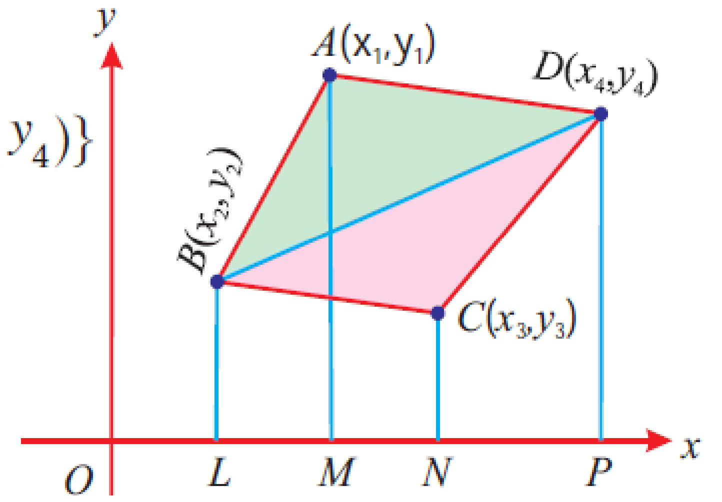
$$= \frac{1}{2}\{(x_1y_2 + x_2y_4 + x_4y_1) - (x_2y_1 + x_4y_2 + x_1y_4)\}$$
$$+ \frac{1}{2}\{(x_2y_3 + x_3y_4 + x_4y_2) - (x_3y_2 + x_4y_3 + x_2y_4)\}$$

$\therefore$ Area of the quadrilateral $ABCD$

$$= \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\}$$

or

$$\frac{1}{2}\{(x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3)\} \text{ sq.units}$$



Find the area of the triangle whose vertices are (1, 2), (−3 , 4), and (−5 ,−6).

Solution

Now the area of $\triangle ABC$ is

$$= \frac{1}{2}\left\{(x_1y_2 + x_2\ y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right\}$$

$$= \frac{1}{2}\{(4 + 18 - 10) - (-6 - 20 - 6)\}$$

$$= \frac{1}{2}\{12 + 32\} = 22.\ \text{sq. units}$$

If the area of the ΔABC is 68 sq.units and the vertices are $A(6\ ,7)$, $B(-4\ ,\ 1)$ and $C(a\ ,-9)$ taken in order, then find the value of a.

Solution Area of $\triangle ABC$ is

$$\frac{1}{2}\{(6 + 36 + 7a) - (-28 + a - 54)\} = 68$$

$$\Longrightarrow (42 + 7a) - (a - 82) = 136$$

$$\Longrightarrow 6a = 12 \qquad \therefore \quad a = 2$$

Show that the points $A(2\ ,\ 3)$, $B(4\ ,\ 0)$ and $C(6,\ -3)$ are collinear.

Solution Area of the ΔABC is

$$= \frac{1}{2}\{(0 - 12 + 18) - (12 + 0 - 6)\} \quad = \frac{1}{2}\{6 - 6\} = 0.$$

$\therefore$ The given points are collinear.

If $P(x, y)$ is any point on the line segment joining the points $(a, 0)$ and $(0, b)$, then, prove that $\frac{x}{a} + \frac{y}{b} = 1$, where $a, b \neq 0$.

Solution Now the points (x, y), $(a, 0)$ and $(0, b)$ are collinear.

$\therefore$ The area of the triangle formed by them is zero.

$\Longrightarrow \quad ab - bx - ay = 0$ use: $\frac{1}{2}\begin{Bmatrix} a & 0 & x & a \\ 0 & b & y & 0 \end{Bmatrix}$

$\therefore \quad bx + ay = ab$

Dividing by ab on both sides, we get,

$$\frac{x}{a} + \frac{y}{b} = 1, \quad \text{where} \quad a, b \neq 0$$

Find the area of the quadrilateral formed by the points (-4, -2), (-3, -5), (3, -2) and (2 , 3).

Solution:

Let the vertices be

$$A(-4, -2), B(-3, -5), C(3, -2) \text{ and } D(2, 3).$$

Area of the quadrilateral $ABCD$

$$= \frac{1}{2}\{(20 + 6 + 9 - 4) - (6 - 15 - 4 - 12)\}$$

$$= \frac{1}{2}\{31 + 25\} = 28 \text{ sq.units.}$$

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