Class X – Chapter: Coordinate Geometry Explanation

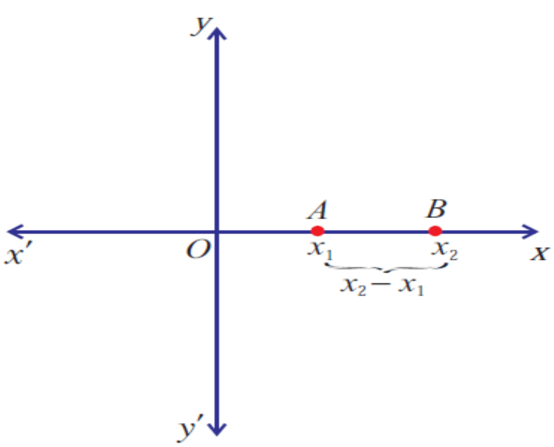
Distance between two points on coordinate axes:

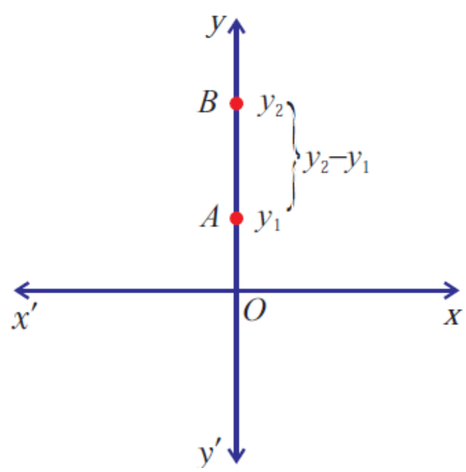
If two points lie on the x-axis, the distance is equal to the difference between x coordinates the two points A (x_1 , 0) and B (x_2,0) on the x-axis.

The distance of B from A = AB = OB – OA = x_2 - x_1

if two points lie on y axis, then the distance between them is equal to the difference between the y coordinates. Consider two points A(0, y_1) and B(0, y_2)

The distance of B from A = AB = OB - OA= $y_2 - y_1$

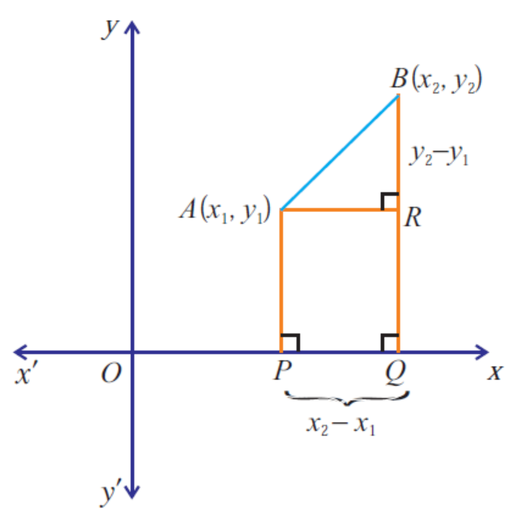




Distance between two points on a line parallel to coordinate axes

Consider the points A (x_1 , y_1) and B(x_2 , y_2) . Since the y ordinates are equal, the two points lie on a line parallel to x-axis.
Draw AP and BQ perpendicular to x-axis. Distance between A and B is equal to distance between P and Q.
Hence
Distance AB = Distance PQ = $|x_1 - x_2|$

Remember: The distance between two points on a line parallel to the coordinate axes is the absolute value of the difference between respective coordinates.



Distance between two points

Let A (x_1, y_1) and B(x_2, y_2) be any two points in the plane. We shall now find the distance between these two points.

Let *P* and *Q* be the foot of the perpendiculars from *A* and *B* to the *x*-axis respectively. *AR* is drawn perpendicular to *BQ*.

$AR = PQ = OQ - OP = x_2 - x_1$ and

$BR = BQ - RQ = y_2 - y_1$

From right triangle ARB

$$AB^2 = AR^2 + RB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

(By Pythagoras theorem)

i. e., $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Hence the distance between the points *A* and *B* is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Remark

Given the two points (x_1, y_1) and (x_2, y_2), the distance between these points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance of the point $P(x_1, y_1)$ from the origin O is $OP = \sqrt{x_1^2 + y_1^2}$

Find the distance between the points $(-4, 0)$ and $(3, 0)$

Solution The points $(-4, 0)$ and $(3, 0)$ lie on the *x*-axis. Hence

$$d = |x_1 - x_2| = |3 - (-4)| = |3 + 4| = 7$$

Aliter :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 + 4)^2 + 0^2} = \sqrt{49} = 7$$

Find the distance between the points $(-7, 2)$ and $(5, 2)$

Solution The line joining $(5, 2)$ and $(-7, 2)$ is parallel to *x* axis. Hence, the distance

$$d = |x_1 - x_2| = |-7 - 5| = |-12| = 12$$

Aliter :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 + 7)^2 + (2 - 2)^2} = \sqrt{12^2} = \sqrt{144} = 12$$

Find the distance between the points (0, 8) and (6, 0)

Solution The distance between the points (0, 8) and (6, 0) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6-0)^2 + (0-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$$

Show that the three points (4, 2), (7, 5) and (9, 7) lie on a straight line.

Solution Let the points be A (4, 2), B (7, 5) and C (9, 7). By the distance formula

$$AB^2 = (4-7)^2 + (2-5)^2 = (-3)^2 + (-3)^2 = 9 + 9 = 18$$

$$BC^2 = (9-7)^2 + (7-5)^2 = 2^2 + 2^2 = 4 + 4 = 8$$

$$CA^2 = (9-4)^2 + (7-2)^2 = 5^2 + 5^2 = 25 + 25 = 50$$

So, $AB = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$; $BC = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$;

$CA = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$.

This gives $AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC$. Hence the points A, B, and C are collinear.

Determine whether the points are vertices of a right triangle A $(-3, -4)$ B (2, 6) and C $(-6, 10)$

Solution Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$$AB^2 = (2+3)^2 + (6+4)^2 = 5^2 + 10^2 = 25 + 100 = 125$$

$$BC^2 = (-6-2)^2 + (10-6)^2 = (-8)^2 + 4^2 = 64 + 16 = 80$$

$$CA^2 = (-6+3)^2 + (10+4)^2 = (-3)^2 + (14)^2 = 9 + 196 = 205$$

i. e., $AB^2 + BC^2 = 125 + 80 = 205 = CA^2$

Hence ABC is a right angled triangle since the square of one side is equal to sum of the squares of the other two sides.

Show that the points (a, a), $(-a, -a)$ and $(-a\sqrt{3}, a\sqrt{3})$ form an equilateral triangle.

Solution Let the points be represented by $A(a, a)$, $B(-a, -a)$ and $C(-a\sqrt{3}, a\sqrt{3})$. Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we have

$$AB = \sqrt{(a+a)^2 + (a+a)^2}$$

$$= \sqrt{(2a)^2 + (2a)^2} = \sqrt{4a^2 + 4a^2} = \sqrt{8a^2} = 2\sqrt{2}\,a$$

$$BC = \sqrt{(-a\sqrt{3}+a)^2 + (a\sqrt{3}+a)^2} = \sqrt{3a^2 + a^2 - 2a^2\sqrt{3} + 3a^2 + a^2 + 2a^2\sqrt{3}}$$

$$= \sqrt{8a^2} = \sqrt{4 \times 2a^2} = 2\sqrt{2}\,a$$

$$CA = \sqrt{(a+a\sqrt{3})^2 + (a-a\sqrt{3})^2} = \sqrt{a^2 + 2a^2\sqrt{3} + 3a^2 + a^2 - 2a^2\sqrt{3} + 3a^2}$$

$$= \sqrt{8a^2} = 2\sqrt{2}\,a$$

$$\therefore AB = BC = CA = 2\sqrt{2}\,a.$$

Since all the sides are equal the points form an equilateral triangle.

Prove that the points $(-7, -3)$, $(5, 10)$, $(15, 8)$ and $(3, -5)$ taken in order are the corners of a parallelogram.

Solution Let A, B, C and D represent the points $(-7, -3)$, $(5, 10)$, $(15, 8)$ and $(3, -5)$ respectively. Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we find

$$AB^2 = (5+7)^2 + (10+3)^2 = 12^2 + 13^2 = 144 + 169 = 313$$

$$BC^2 = (15-5)^2 + (8-10)^2 = 10^2 + (-2)^2 = 100 + 4 = 104$$

$$CD^2 = (3-15)^2 + (-5-8)^2 = (-12)^2 + (-13)^2 = 144 + 169 = 313$$

$$DA^2 = (3+7)^2 + (-5+3)^2 = 10^2 + (-2)^2 = 100 + 4 = 104$$

So, $AB = CD = \sqrt{313}$ and $BC = DA = \sqrt{104}$

i.e,. The opposite sides are equal. Hence $ABCD$ is a parallelogram.

Show that the following points (3, −2), (3, 2), (−1, 2) and (−1, −2) taken in order are vertices of a square.

Solution Let the vertices be taken as A (3, −2), B (3, 2), C (−1, 2) and D (−1, −2)

$$AB^2 = (3-3)^2 + (2+2)^2 = 4^2 = 16$$

$$BC^2 = (3+1)^2 + (2-2)^2 = 4^2 = 16$$

$$CD^2 = (-1+1)^2 + (2+2)^2 = 4^2 = 16$$

$$DA^2 = (-1-3)^2 + (-2+2)^2 = (-4)^2 = 16$$

$$AC^2 = (3+1)^2 + (-2-2)^2 = 4^2 + (-4)^2 = 16 + 16 = 32$$

$$BD^2 = (3+1)^2 + (2+2)^2 = 4^2 + 4^2 = 16 + 16 = 32$$

$AB = BC = CD = DA = \sqrt{16} = 4$. (That is, all the sides are equal.)

$AC = BD = \sqrt{32} = 4\sqrt{2}$. (That is, the diagonals are equal.)

Hence the points A, B, C and D form a square.

Let P be a point on the perpendicular bisector of the segment joining (2, 3) and (6, 5). If the abscissa and the ordinate of P are equal, find the coordinates of P.

Solution Let the point be $P(x, y)$. Since the abscissa of P is equal to its ordinate, we have $y = x$. Therefore, the coordinates of P are (x, x). Let A and B denote the points (2, 3) and (6, 5). Since P is equidistant from A and B, we get $PA = PB$. Squaring on both sides, we get $PA^2 = PB^2$.

i.e., $$(x-2)^2 + (x-3)^2 = (x-6)^2 + (x-5)^2$$

$$x^2 - 4x + 4 + x^2 - 6x + 9 = x^2 - 12x + 36 + x^2 - 10x + 25$$

$$2x^2 - 10x + 13 = 2x^2 - 22x + 61$$

$$22x - 10x = 61 - 13$$

$$12x = 48$$

$$x = \frac{48}{12} = 4$$

Therefore, the coordinates of P are (4, 4)

Show that (4, 3) is the centre of the circle which passes through the points (9, 3), (7, − 1) and (1, − 1). Find also its radius.

Solution Suppose C represents the point (4, 3). Let P, Q and R denote the points (9, 3), (7, − 1) and (1, − 1) respectively. Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$$CP^2 = (9 - 4)^2 + (3 - 3)^2 = 5^2 = 25$$

$$CQ^2 = (7 - 4)^2 + (-1 - 3)^2 = 3^2 + (-4)^2 = 9 + 16 = 25$$

$$CR^2 = (4 - 1)^2 + (3 + 1)^2 = 3^2 + 4^2 = 9 + 16 = 25$$

So, $CP^2 = CQ^2 = CR^2 = 25$ or $CP = CQ = CR = 5$. Hence the points P, Q, R are on the circle with centre at (4, 3) and its radius is 5 units.

If the point (α, β) is equidistant from (3, − 4) and (8, − 5), show that $5\alpha - \beta - 32 = 0$.

Solution Let P deonte the point (α, β). Let A and B represent the points (3, − 4) and (8, − 5) respectively. Since P is equidistant from A and B, we have $PA = PB$ and hence $PA^2 = PB^2$.

Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we have

$$(\alpha - 3)^2 + (\beta + 4)^2 = (\alpha - 8)^2 + (\beta + 5)^2$$

$$\alpha^2 - 6\alpha + 9 + \beta^2 + 8\beta + 16 = \alpha^2 - 16\alpha + 64 + \beta^2 + 10\beta + 25$$

$$-6\alpha + 8\beta + 25 + 16\alpha - 10\beta - 89 = 0$$

$$10\alpha - 2\beta - 64 = 0$$

Dividing throughout by 2, we get $5\alpha - \beta - 32 = 0$

Show that $S(4, 3)$ is the circum-centre of the triangle joining the points $A(9, 3)$, $B(7, -1)$ and $C(1, -1)$

Solution $SA = \sqrt{(9-4)^2 + (3-3)^2} = \sqrt{25} = 5$

$$SB = \sqrt{(7-4)^2 + (-1-3)^2} = \sqrt{25} = 5$$

$$SC = \sqrt{(1-4)^2 + (-1-3)^2} = \sqrt{25} = 5$$

$\therefore SA = SB = SC$.

It is known that the circum-center is equidistant from all the vertices of a triangle. Since S is equidistant from all the three vertices, it is the circum-centre of the triangle ABC.

Self practice

1.Q. Three vertices of a rectangle are (3, 2), (– 4, 2) and (– 4, 5). Plot these points and find the coordinates of the fourth vertex.

2. Q. A point lies on the *x*-axis at a distance of 7 units from the *y*-axis. What are its coordinates? What will be the coordinates if it lies on *y*-axis at a distance of –7 units from *x*-axis?

3. Q. In which quadrant or on which axis each of the following points lie?

(– 3, 5), (4, – 1), (2, 0), (2, 2), (– 3, – 6)

4.Q. If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) taken in order are the vertices of a parallelogram, find the value of p using distance formula.

5.Q. The radius of the circle with centre at the origin is 10 units. Write the coordinates of the point where the circle intersects the axes. Find the distance between any two of such points.

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