Class 9th **Solved Questions** **Chapter Quadrilateral**

Example 1. Four angles of a quadrilateral are in the ratio 1 : 2 : 3 : 4. Find them.

Solution. Let the four angles be x, $2x$, $3x$ and $4x$.

Since, sum of four angles of a quadrilateral is 360°,

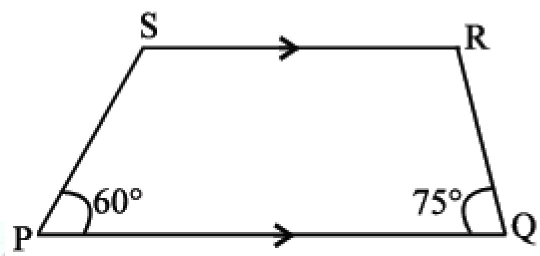
$x + 2x + 3x + 4x = 360°$

$10x = 360°$, or $x = 36°$

four angles are 36°, 2 × 36°, 3 × 36°, 4 × 36°

i.e. **36°, 72°, 108°, 144° Ans.**

Example 2. In the given figure, PQRS is a trapezium in which PQ || SR. If <P = 60° and <Q = 75°, find ,<S
and <R.



Solution. <P +<S = 180° and

<Q +<R = 180° (interior angles on the same side of transversal are supplementary)

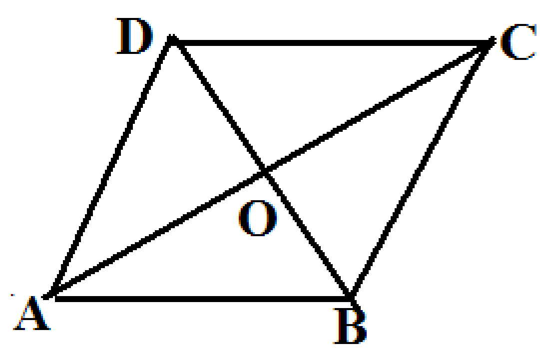
< 60° + <S = 180° and 75° + <R = 180°

<S = 180 – 60° and <R = 180° – 75°

<S = **120°** and <R **= 105° Ans.**

Example 3. Show that the diagonals of a rhombus are perpendicular to each other.

Solution. Let ABCD be a given Rhombus.



here, AB = BC = CD = DA

Now, In ∆AOD and ∆COD,

OA = OC

(∴ diagonals of parallelogram bisect each other)

OD = OD (common side)

AD = CD

∆AOD ≅ ∆COD (SSS congruence condition)

<AOD = <COD (c.p.c.t.)

But, <AOD + <COD = 180° (linear pair)

2<AOD = 180°

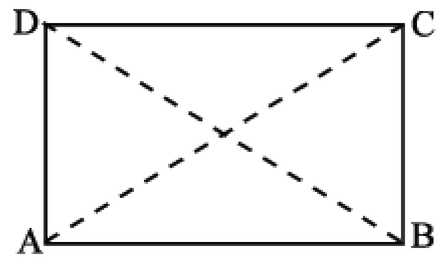
<AOD = 90°

So, the diagonals of a rhombus are perpendicular to each other.

Example 4. If the diagonals of a parallelogram are equal, then show that it is a rectangle

Solution. Given : A parallelogram ABCD in which AC = BD.

To prove : ABCD is a rectangle.



Proof : In ΔABC and ΔDCB, we have

AB = DC (opp. sides of parallelogram)

BC = BC (common side)

AC = DB (given)

ΔABC ≅ Δ<DCB (SSS congruence condition)

<ABC = <DCB ...(1) (cpct)

But AB || DC and BC cuts them.

<ACB + < DCB = 180° (sum of consecutive interior angles is 180°)

2<ABC = 180°

<ABC = 90°

Thus <ABC = <DCB = 90° So, ABCD is a parallelogram one of whose angle is 90°.

Hence, ABCD is a rectangle.

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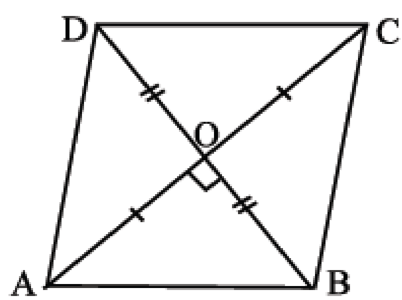
Example 5. Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution. Given : A quadrilateral ABCD in which the diagonals AC and BD intersect at O such that AO = OC,

BO = OD and AC ⊥ BD.

To prove : ABCD is a rhombus.



Proof : In ΔAOD and ΔCOB, we have

AO = OC (given)

OD = OB given)

<AOD = <COB (vertically opp. angles)

ΔAOD ≅ ΔCOB (SAS congruence condition)

<OAD = <OCB ...(*i*) (cpct)

Now, line AC intersects AD and BC at A and C respectively such that <AOD = <OCB (proved in (*i*))

i.e. alternate interior angles are equal.

AD || BC. Similarly, AB || CD

Hence, ABCD is a parallelogram.

Again, In ΔAOD and ΔCOD, we have

OA = OC (given) <AOD =<COD (each = 90°) OD = OD (common side)

<AOD ≅ <COD (SAS congruence condition)

AD = CD ... (2) (cpct)

Now, ABCD is a parallelogram, AB = CD and AD = BC (opp. sides of a parallelogram are equal)

AB = CD = AD = BC (using (2)) Hence, quadrilateral ABCD is a rhombus

Example 6. Show that the diagonals of a square are equal and bisect each other at right angles

Solution. Given : A square ABCD.

To prove : AC = BD, AC $\perp$ BD and OA = OC, OB = OD.

Proof: Since ABCD is square,

AB || DC and AD || BC.

Now, AB || DC and transversal AC intersects them at A and C respectively.

<BAC = <DCA (alternate interior angles are equal) <BAO = <DCO ...(1)

Again AB || DC and BD intersects them at B and D respectively.

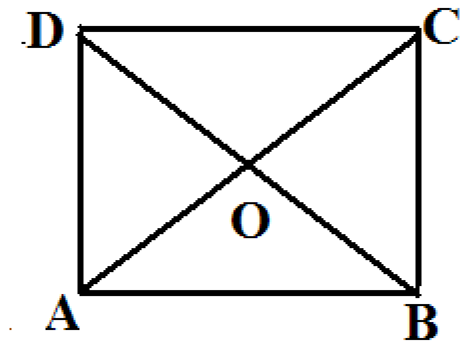
<ABD = <CDB (alternate interior angles are equal) <ABO = < CDO ...(2)

Now, In ΔAOB and Δ COD, we have

< BAO = < DCO (from (1))

AB = CD (opp. sides of a parallelogram are equal)

<ABO = <CDO (from (2))



ΔAOB ≅ΔCOD (ASA congruence condition)

OA = OC and OB = OD (cpct)

Hence, the diagonals bisect each other.

Again, In ΔADB and Δ BCA, we have

AD = BC (sides of a square are equal)

<BAD =<ABC (each 90°)

AB = AB (common side)

ΔADB ≅Δ BCA (SAS congruence condition)

AC = BD (cpct)

Hence, diagonals are equal.

Now, In ΔAOB and ΔAOD, we have,

OB = OD (diagonals of a parallelogram bisect each other)

AB = AD (sides of a square are equal)   AO = AO (common side)

Δ AOB ≅ Δ AOD (SSS congruence condition)   < AOB =< AOD (cpct)

Example 7. Show that if the diagonals of a quadrilateral one equal and bisect each other at righ angles, then it is a square.

Solution. Given : A quadrilateral ABCD in which the diagonals AC = BD, AO = OC, BO = OD and AC ⊥ BD.

To prove : Quadrilateral ABCD is a square.

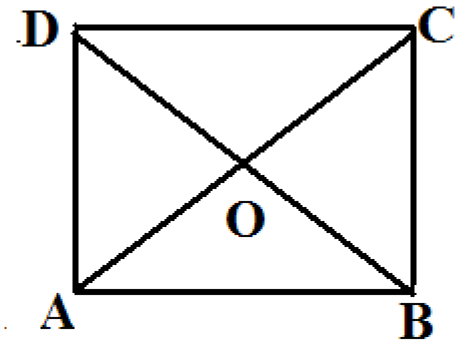
Proof : In Δ AOD and ΔCOB, we have

AO = OC (given)   OD = OB (given)   <AOD = <COB (vertically opp. angles)

ΔAOD ≅ ΔCOB (SAS congruence condition)   <OAD = <OCB ...(1) (cpct)

Now, line AC intersects AD and BC at A and C respectively such that <OAD = <OCB, i.e.

alternate interior angles are equal.



AD || BC   Similarly, AB || CD.

Hence, ABCD is a parallelogram.

Now, In ∆AOB and ∆AOD, we have

AO = AO (common side) <AOB = <AOD (each = 90°)

OB = OD (diagonals of a parallelogram bisect each other)

∆ AOB ≅ ∆AOD (SAS congruence condition)

. AB = AD (cpct)   But, AB = CD and AD = BC (opposite sides of a parallelogram are equal)

AB = BC = CD = AD ...(2)

Now, In∆ABD and ∆BAC, we have

AB = AB (common side)   AD = BC (opp. sides of parallelogram are equal)

 BD = AC (given)

∆ABD ≅ ∆BAC (SSS congruence condition)

<DAB = <CBA (cpct)   But, <DAB + <CBA = 180°

<DAB = <CBA = 180/2 = 90°

Thus, ABCD is a parallelogram whose all the sides are equal and one of the angle is 90°.

∴ ABCD is a square.   Hence proved.

Example 8. ABCD is a trapezium in which AB || CD and AD = BC
 Show that: (*i*) <A = <B   (*ii*) <C = <D   (*iii*) ∆ABC≅ ∆ BAD   (*iv*) diagonal AC = diagonal BD

Solution: ABCD is a trapezium, in which AB || CD and AD = BC.
To prove (*i*) <A = <B (*ii*) <C = <D   (*iii*) ∆ABC≅ ∆ BAD
                (*iv*) Diagonal AC = diagonal BD

Construction : Produce AB and draw a line CE || AD.

Proof : (*i*) Since AD || CE and transversal AE cuts them at A and E respectively.

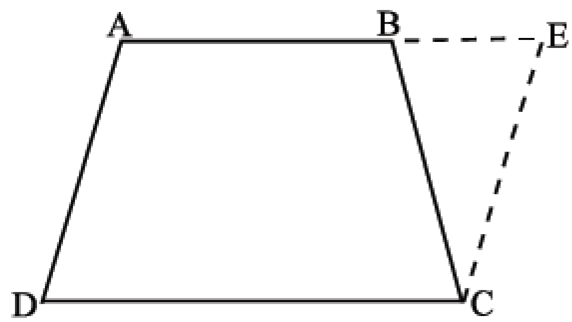
<A + < E = 180° ...(1)

Since AB || CD and AD || CE, AECD is a parallelogram.

AD = CE

BC = CE (AD = BC (given))

Thus, In Δ BCE, we have



BC = CE

<CBE = <CEB

180° – < B = <E

<B + <E = 180° ...(2)

from (1) and (2), we get,

<A + <E = <B + <E

<A = < B

(*ii*) Since <A = <B < BAD = <ABD

180° – <BAD = 180° – <ABD

<ADB = <BCD

< D = < C i.e. < C = < D

(*iii*) In Δ ABC and Δ BAD, we have

BC = AD (given)

AB = AB (common)

< A = < B (proved above)

Δ ABC ≅ Δ BAD (SAS Congruence condition)

(*iv*) Since, Δ ABC ≅ Δ BAD AC = BD (cpct) Hence proved.

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Example 9 : ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that :

(*i*) D is the mid-point of AC (*ii*) MD ⊥ AC (*iii*) CM = MA =1/2 AB

Solution. Given : ΔABC is right angled at C, M is the mid-point of hyp. AB . MD || BC.

To prove : (*i*) D is the mid-point of AC (*ii*) MD ⊥ AC (*iii*) CM = MA =1/2 AB

Proof : (*i*) In ΔABC, M is the mid-point of AB and MD || BC.

D is the mid-point of AC

i.e. AD = DC ...(1)

(*ii*) Since MD || BC <ADM = < ACB (corresponding angles)

<ADM = 90° (<ACB = 90°, MD || BC)

But, <ADM + <CDM = 180° (linear pair)

90° + >CDM = 180°

<CDM = 90°

Thus, <ADM = <CDM = 90° ...(2)

MD ⊥ AC.

(*iii*) Now, In ΔAMD and ΔCMD, we have

AD = CD (from (1))

<ADM = <CDM (from (2))

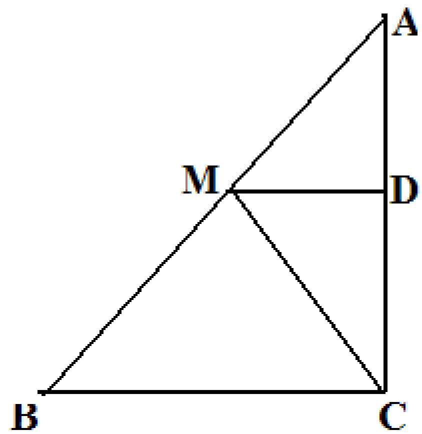
MD = MD (common side)

Δ AMD ≅ Δ CMD (SAS congruence condition)

MA = MC (cpct)

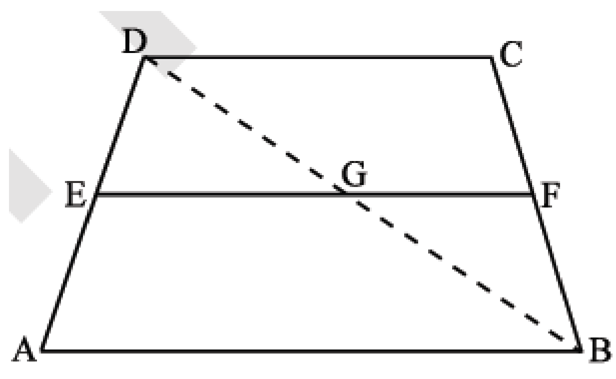
Also, MA = 1/2AB Since M is the mid-point of AC.

Hence, CM =MA=1/2 AB



Example 10. ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the mid-point of BC.

Solution. Given: A trapezium ABCD, in which AD || BC. E is the mid-point of AD and EF || AB.



Construction : Join B to D. Let it intersect EF in G.

Proof : In Δ DAB, E is the mid-point of AD (given)

EG || AB (EF || AB)

By converse of mid-point theorem, G is the mid-point of DB.

Now, In Δ BCD, G is the mid-point of BD (proved above)

GF || DC (AB || DC, EF || AB so, DC || EF)

- By converse of mid-point theorem, F is the mid point of BC.