Topic - 01 Relation and Functions

Topic	Concepts	Degree of importance	References NCERT Text Book XII Ed. 2007
Relations & Functions	(i).Domain , Co domain & Range of a relation	*	(Previous Knowledge)
	(ii).Types of relations	***	Ex 1.1 Q.No- 5,9,12
	(iii).One-one , onto & inverse of a function	***	Ex 1.2 Q.No- 7,9
	(iv).Composition of function	*	Ex 1.3 QNo- 7,9,13
	(v).Binary Operations	***	Example 45 Ex 1.4 QNo- 5,11

SOME IMPORTANT RESULTS/CONCEPTS

** A **relation** R in a set A is called

(i) *reflexive*, if $(a, a) \in R$, for every $a \in A$,

(ii) *symmetric*, if $(a_1, a_2) \in R$ implies that $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$.

(iii)*transitive*, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.

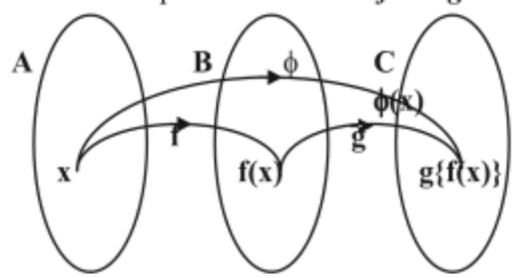
** Equivalence Relation : R is an equivalence relation if it is reflexive, symmetric and transitive.

** **Function** :A relation $f : A \to B$ is said to be a function if every element of A is correlated to unique element in B.

* A is domain
* B is codomain
* For any element $X \in A$, function f correlates it to an element in B, which is denoted by f(x)and is called image of x under f . Again if y= f(x), then x is called as pre-image of y.
* Range = $\{f(x) \mid x \in A\}$. Range $\subseteq$ Codomain
* The largest possible domain of a function is called domain of definition.

****Composite function** :

Let two functions be defined as $f : A \to B$ and $g : B \to C$. Then we can define a function $\phi : A \to C$ by setting $\phi(x) = g\{f(x)\}$ where $x \in A$, $f(x) \in B$, $g\{f(x)\} \in C$. This function $\phi : A \to C$ is called the composite function of f and g in that order and we write. $\phi = gof$.



** **Different type of functions** : Let ***f*** : A →B be a function.

* ***f*** is **one to one (injective) mapping**, if any two different elements in A is always correlated to different elements in B, i.e. $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$or, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

* ***f*** is **many one mapping,** if ∃ at least two elements in A such that their images are same.

* ***f*** is **onto mapping** (subjective), if each element in B is having at least one preimage.

****f*** is **into mapping** if range ⊂ codomain.

* ***f*** is **bijective mapping** if it is both *one to one and onto*.

** **Binary operation :** A binary operation * on a set A is a function * : A × A → A. We denote *(*a*, *b*) by *a* *b

* A binary operation '*' on A is a rule that associates with every ordered pair (a, b) of A x A a unique element a *b.

* An operation '*' on a is said to be commutative iff a * b = b * a ∀ a, b ∈ A.

* An operation '*' on a is said to be associative iff (a * b) * c = a * (b * c) ∀ a, b, c ∈ A.

* Given a binary operation * : A × A → A, an element *e* ∈ A, if it exists, is called ***identity*** for the operation *, if $a * e = a = e * a$, $\forall a \in A$.

* Given a binary operation * : A × A → A with the identity element *e* in A, an element *a* ∈ A is said to be ***invertible*** with respect to the operation*, if there exists an element *b* in A such that $a * b = e = b * a$ and *b* is called the *inverse of a* and is denoted by a^{-1}.

ASSIGNMENTS

(i) Domain , Co domain & Range of a relation

LEVEL I

1. If A = {1,2,3,4,5}, write the relation a R b such that a + b = 8, a ,b ∈ A. Write the domain, range & co-domain.
2. Define a relation R on the set **N** of natural numbers by

   R={(*x* , *y*) : *y* = *x* +7, *x* is a natural number lesst han 4 ; *x*, *y* ∈ **N**}.

   Write down the domain and the range.

2. *Types of relations*

LEVEL II

1. Let R be the relation in the set N given by R = {(a , b)| a = b – 2 , b > 6}
   Whether the relation is reflexive or not ?justify your answer.
2. Show that the relation R in the set N given by R = {(a , b)| a is divisible by b , a , b ∈ N} is reflexive and transitive but not symmetric.
3. Let R be the relation in the set N given by R = {(a ,b)| a > b} Show that the relation is neither reflexive nor symmetric but transitive.
4. Let R be the relation on R defined as (a , b) ∈R iff 1+ ab> 0 ∀ a,b∈R.
   (a) Show that R is symmetric.
   (b) Show that R is reflexive.
   (c) Show that R is not transitive.
5. Check whether the relation R is reflexive, symmetric and transitive.
   R = { (x , y)| x – 3y = 0} on A ={1, 2, 3……….13, 14}.

LEVEL III

1. Show that the relation R on A ,A = { x| x $\in Z$, $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

2.Let N be the set of all natural numbers & R be the relation on N × N defined by
{ (a , b) R (c , d) iff a + d = b + c}. Show that R is an equivalence relation.

3. Show that the relation R in the set A of all polygons as:
   R ={(P_1,P_2), P_1& P_2 have the same number of sides} is an equivalence relation. What is the set of all elements in A related to the right triangle T with sides 3,4 & 5 ?
4. Show that the relation R on A ,A = { x| x $\in Z$, $0 \le x \le 12$ } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.
5. Let N be the set of all natural numbers & R be the relation on N × N defined by
   { (a , b) R (c ,d) iff a + d = b + c}. Show that R is an equivalence relation. **[CBSE 2010]**
6. Let A = Set of all triangles in a plane and R is defined by R={(T_1,T_2) : T_1,$T_2 \in$ A & T_1~T_2 }
   Show that the R is equivalence relation. Consider the right angled Δs, T_1 with size 3,4,5;
   T_2 with size 5,12,13; T_3 with side 6,8,10; Which of the pairs are related?

(iii)One-one , onto & inverse of a function

LEVEL I

1. If f(x) = $x^2 - x^{-2}$, then find f(1/x).
2 Show that the function f: R→R defined by f(x)=x^2 is neither one-one nor onto.

3 Show that the function f: N→N given by f(x)=2x is one-one but not onto.

4 Show that the signum function f: R→R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$
is neither one-one nor onto.

5 Let A = {-1,0,1} and B = {0,1}. State whether the function f : A → B defined by f(x) = x^2 is bijective .

6. Let f(x) = $\frac{x-1}{x+1}$, x≠ -1,then find $f^{-1}(x)$

LEVEL II

1. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1,4),(2,5), (3,6)} be a function from A to B. State whether f is one-one or not. **[CBSE2011]**
2. If f : R→R defined as f(x) = $\frac{2x-7}{4}$ is an invertible function . Find $f^{-1}(x)$.
3. Write the number of all one-one functions on the set A={a, b, c} to itself.
4. Show that function f :R→R defined by f(x)= 7– $2x^3$ for all x $\in$ R is bijective.
5. If f: R→R is defined by f(x)= $\frac{3x+5}{2}$. Find f^{-1}.

LEVEL III

1.Show that the function f: R→R defined by $f(x) = \frac{2x-1}{3}$ .x∈ R is one- one & onto function. Also find the f^{-1}.

2.Consider a function f :R_+→[-5, ∞) defined $f(x) = 9x^2 + 6x - 5$. Show that f is invertible & $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$, where $R_+ = (0,\infty)$.

3.Consider a function f: R→R given by f(x) = 4x + 3. Show that f is invertible & f^{-1}: R→R with $f^{-1}(y) = \frac{y-3}{4}$.

4. Show that f: R→R defined by $f(x)= x^3+4$ is one-one, onto. Show that $f^{-1}(x)=(x-4)^{1/3}$.

5. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Show that f is one one onto and hence find f^{-1}. **[CBSE2012]**

6. Show that $f : N \to N$ defined by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$ is both one one onto.

(iv) *Composition of functions*

[CBSE2012]

LEVEL I

1. If $f(x) = e^{2x}$ and $g(x) = \log\sqrt{x}$, x > 0, find
   (a) (f + g)(x) (b) (f .g)(x) (c) f o g (x) (d) g o f (x).

2. If $f(x) = \frac{x-1}{x+1}$, then show that (a) $f\left(\frac{1}{x}\right) = -f(x)$ (b) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

LEVEL II

1. Let f, g : R→R be defined by f(x)=|x| & g(x) = [x] where [x] denotes the greatest integer function. Find f o g (5/2) & g o f (-√2).

2. Let $f(x) = \frac{x-1}{x+1}$. Then find f(f(x))

3. If $y = f(x) = \frac{3x+4}{5x-3}$, then find (fof)(x)

4. Let f : **R** →**R** be defined as f(x) = 10x +7.Find the function g : **R**→**R**such that $g \circ f(x) = f \circ g(x) = I_R$ **[CBSE2011]**

5. If f : **R** →**R** be defined as $f(x) = \left(3-x^3\right)^{\frac{1}{3}}$, then find $f \circ f(x)$.

[CBSE2010]

6. Let f :R→R& g : R→R be defined as $f(x) = x^2$, g(x) = 2x – 3 . Find fog(x).

(v)Binary Operations

LEVEL I

1. Let * be the binary operation on N given by a*b = LCM of a &b . Find 3*5.
2. Let *be the binary operation on N given by a*b =HCF of {a ,b} , a,b$\in$N. Find 20*16.
3. Let * be a binary operation on the set Q of rational numbers defined as $a * b = \frac{ab}{5}$.

   Write the identity of *, if any.
4. If a binary operation '*' on the set of integer Z , is defined by $a * b = a + 3b^2$

   Then find the value of 2 * 4.

LEVEL 2

1. Let A= N×N & * be the binary operation on A defined by (a ,b) * (c ,d) = (a+c, b+d)

   Show that * is (a) Commutative (b) Associative (c) Find identity for * on A, if any.

2. Let A = Q×Q. Let * be a binary operation on A defined by (a,b)*(c,d)= (ac , ad+b).

   Find: (i) the identity element of A (ii) the invertible element of A.
3. Examine which of the following is a binary operation

   (i) $a * b = \frac{a+b}{2}$; a, b$\in$ N (ii) $a*b = \frac{a+b}{2}$ a, b$\in$ Q

   For binary operation check commutative & associative law.

LEVEL 3

1.Let A= N×N & * be a binary operation on A defined by (a , b) $*$ (c , d) = (ac , bd)

$\forall$ (a , b),(c , d) $\in$N×N (i) Find (2,3) * (4,1)

(ii) Find [(2,3)*(4,1)]*(3,5) and (2,3)*[(4,1)* (3,5)] & show they are equal

(iii) Show that * is commutative & associative on A.

2. Define a binary operation * on the set {0,1,2,3,4,5} as $a * b = \begin{cases} a+b, & \text{if } a+b<6 \\ a+b-6, & a+b \geq 6 \end{cases}$

Show that zero in the identity for this operation & each element of the set is invertible with 6 – a being the inverse of a. **[CBSE2011]**

3. Consider the binary operations $*$:**R** × **R** → **R**and o : **R** × **R** → **R** defined as $a * b = |a - b|$ and a o b = a, $\forall$a, b $\in$**R**. Show that $*$is commutative but not associative, o is associative but not commutative. **[CBSE2012]**

Questions for self evaluation

1. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

2. Show that each of the relation R in the set $A = \{x \in \mathbf{Z} : 0 \leq x \leq 12\}$, given by R = {(a, b) : |a – b| is a multiple of 4} is an equivalence relation. Find the set of all elements related to 1.

3. Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2) : T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

4. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

5. Let A = **R** – {3} and B = **R** – {1}. Consider the function f : A $\to$B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$.

Is f one-one and onto? Justify your answer.

6. Consider f :**R+**$\to$[– 5, ∞) given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and findf^{-1}.

7. On R – {1} a binary operation '$*$' is defined as $a * b = a + b - ab$. Prove that '$*$' is commutative and associative. Find the identity element for '$*$'.Also prove that every element of R – {1} is invertible.

8. If A = $Q \times Q$ and '$*$' be a binary operation defined by $(a, b) * (c, d) = (ac, b + ad)$,

for (a, b), (c, d)$\in$ A .Then with respect to '$*$' on A

(i) examine whether '$*$' is commutative & associative

(i) find the identity element in A,

(ii) find the invertible elements of A.

TOPIC 2
INVERSE TRIGONOMETRIC FUNCTIONS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XI Ed. 2007
Inverse Trigonometric Functions	(i).Principal value branch Table	**	Ex 2.1 QNo- 11, 14
	(ii). Properties of Inverse Trigonometric Functions	***	Ex 2.2 Q No- 7,13, 15 Misc Ex Q.No. 9,10,11,12

SOME IMPORTANT RESULTS/CONCEPTS

* Domain & Range of the Inverse Trigonometric Function :

	Functions	Domain	Range (Principal value Branch)
i.	$\sin^{-1}$:	$[-1,1]$	$[-\pi/2, \pi/2]$
ii.	$\cos^{-1}$:	$[-1,1]$	$[0,\pi]$
iii.	$\operatorname{cosec}^{-1}$:	$R-(-1,1)$	$[-\pi/2,\pi/2]-\{0\}$
iv.	$\sec^{-1}$:	$R-(-1,1)$	$[0,\pi]-\{\pi/2\}$
v.	$\tan^{-1}$:	R	$(-\pi/2,\pi/2)$
vi.	$\cot^{-1}$:	R	$(0,\pi)$

* Properties of Inverse Trigonometric Function for suitable value of domain

1. (i) $\sin^{-1}(\sin x)=x$ & $\sin(\sin^{-1} x)=x$ ii. $\cos^{-1}(\cos x)=x$ & $\cos(\cos^{-1} x)=x$

   iii. $\tan^{-1}(\tan x)=x$ & $\tan(\tan^{-1} x)=x$ iv. $\cot^{-1}(\cot x)=x$ & $\cot(\cot^{-1} x)=x$

   v. $\sec^{-1}(\sec x)=x$ & $\sec(\sec^{-1} x)=x$ vi. $\operatorname{cosec}^{-1}(\operatorname{cosec} x)=x$ & $\operatorname{cosec}(\operatorname{cosec}^{-1} x)=x$

2. i. $\operatorname{cosec}^{-1} x=\sin^{-1}\frac{1}{x}$ & $\sin^{-1} x=\operatorname{cosec}^{-1}\frac{1}{x}$ ii. $\sec^{-1} x=\cos^{-1}\frac{1}{x}$

   iii. $\cot^{-1} x=\tan^{-1}\frac{1}{x}$, $x>0$ & $\cot^{-1} x=\pi+\tan^{-1}\frac{1}{x}$, $x>0$

3. i $\sin^{-1}(-x)=-\sin^{-1} x$ iv $\cos^{-1}(-x)=\pi-\cos^{-1} x$

   ii. $\tan^{-1}(-x)=-\tan^{-1} x$ v $\sec^{-1}(-x)=\pi-\sec^{-1} x$

   iii. $\operatorname{cosec}^{-1}(-x)=-\operatorname{cosec}^{-1} x$ vi $\cot^{-1}(-x)=\pi-\cot^{-1} x$

4. i $\sin^{-1} x+\cos^{-1} x=\frac{\pi}{2}$ ii. $\tan^{-1} x+\cot^{-1} x=\frac{\pi}{2}$

   iii. $\operatorname{cosec}^{-1} x+\sec^{-1} x=\frac{\pi}{2}$

5. $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

6. $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy < 1$

$\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy > 1$

$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$ if $xy > -1$

ASSIGNMENTS

(i). Principal value branch Table

LEVEL I

Write the principal value of the following :

1. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\sin^{-1}\left(-\frac{1}{2}\right)$

3. $\tan^{-1}\left(-\sqrt{3}\right)$

4. $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

LEVEL II

Write the principal value of the following :

1. $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ **[CBSE 2011]**

2. $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$

3. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

(ii). Properties of Inverse Trigonometric Functions

LEVEL I

1. Evaluate $\cot[\tan^{-1}a + \cot^{-1}a]$

2. Prove $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right)$

3. Find x if $\sec^{-1}\left(\sqrt{2}\right) + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

LEVEL II

1. Write the following in simplest form : $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$

2. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

3. Prove that $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

4. Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$ **[CBSE 2011]**

5. Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$ **[CBSE 2012]**

LEVEL III

1. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2},\ x\in\left(0,\frac{\pi}{4}\right)$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$ **[CBSE 2011]**

3. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

4. Solve $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

5.Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

6. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ **[CBSE 2012]**

Questions for self evaluation

1. Prove that $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\tan^{-1}\frac{63}{16}$

2. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\quad x\in\left[-\frac{1}{\sqrt{2}},1\right]$

3. Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

4. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$

5. Prove that $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)=\frac{\pi}{4}$

6. Write in the simplest form $\cos\left[2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right]$

7. Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

8. Solve $\tan^{-1}2x+\tan^{-1}3x=\pi/4$

TOPIC 3
MATRICES & DETERMINANTS
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	References NCERT Text Book XI Ed. 2007
Matrices & Determinants	(i) Order, Addition, Multiplication and transpose of matrices	***	Ex 3.1 –Q.No 4,6 Ex 3.2 –Q.No 7,9,13,17,18 Ex 3.3 –Q.No 10
	(ii) Cofactors &Adjoint of a matrix	**	Ex 4.4 –Q.No 5 Ex 4.5 –Q.No 12,13,17,18
	(iii)Inverse of a matrix & applications	***	Ex 4.6 –Q.No 15,16 Example –29,30,32 ,33 MiscEx 4–Q.No 4,5,8,12,15
	(iv)To find difference between $\|A\|$, $\|adj A\|$, $\|kA\|$, $\|A.adjA\|$	*	Ex 4.1 –Q.No 3,4,7,8
	(v) Properties of Determinants	**	Ex 4.2–Q.No 11,12,13 Example –16,18

SOME IMPORTANT RESULTS/CONCEPTS

A matrix is a rectangular array of $m \times n$ numbers arranged in m rows and n columns.

$$A = \begin{bmatrix} a_{11} & a_{12} \ldots\ldots\ldots a_{1n} \\ a_{21} & a_{22} \ldots\ldots\ldots a_{2n} \\ \\ a_{m1} & a_{m2} \ldots\ldots\ldots a_{mn} \end{bmatrix}_{m\times n} \quad \text{OR } A = [a_{ij}]_{m\times n}, \text{ where } \mathbf{i} = 1, 2,\ldots, m ; \mathbf{j} = 1, 2,\ldots,n.$$

* **Row Matrix**: A matrix which has one row is called row matrix. $A = [a_{ij}]_{1\times n}$

* **Column Matrix** : A matrix which has one column is called column matrix. $A = [a_{ij}]_{m\times 1}$.

* **Square Matrix**: A matrix in which number of rows are equal to number of columns, is called a square matrix $A = [a_{ij}]_{m\times m}$

* **Diagonal Matrix** : A square matrix is called a Diagonal Matrix if all the elements, except the diagonal elements are zero. $A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$, $\mathbf{i} \neq j$.

$a_{ij} \neq 0$, $\mathbf{i} = j$.

* **Scalar Matrix**: A square matrix is called scalar matrix it all the elements, except diagonal elements are zero and diagonal elements are same non-zero quantity.

$A = [a_{ij}]_{n\times n}$, where $a_{ij} = 0$, $\mathbf{i} \neq j$.

$a_{ij} \neq \alpha$, $\mathbf{i} = j$.

* **Identity or Unit Matrix** : A square matrix in which all the non diagonal elements are zero and diagonal elements are unity is called identity or unit matrix.

* **Null Matrices** : A matrices in which all element are zero.
* **Equal Matrices** : Two matrices are said to be equal if they have same order and all their corresponding elements are equal.
* **Transpose of matrix** : If A is the given matrix, then the matrix obtained by interchanging the rows and columns is called the transpose of a matrix.\
* **Properties of Transpose** :

If A & B are matrices such that their sum & product are defined, then

(i). $(A^T)^T = A$ (ii). $(A+B)^T = A^T + B^T$ (iii). $(KA)^T = K.A^T$ where K is a scalar.

(iv). $(AB)^T = B^T A^T$ (v). $(ABC)^T = C^T B^T A^T$.

* **Symmetric Matrix** : A square matrix is said to be symmetric if $A = A^T$ i.e. If $A = [a_{ij}]_{m\times m}$, then $a_{ij} = a_{ji}$ for all i, j. Also elements of the symmetric matrix are symmetric about the main diagonal
* **Skew symmetric Matrix** : A square matrix is said to be skew symmetric if $A^T = -A$.

If $A = [a_{ij}]_{m\times m}$, then $a_{ij} = -a_{ji}$ for all i, j.

***Singular matrix**:A square matrix 'A' of order 'n' is said to be singular, if $|A| = 0$.

* **Non -Singular matrix** : A square matrix 'A' of order 'n' is said to be non-singular, if $|A| \neq 0$.

***Product of matrices:**

(i) If A & B are two matrices, then product AB is defined, if

Number of column of A = number of rows of B.

i.e. $A = [a_{ij}]_{m\times n}$, $B = [b_{jk}]_{n\times p}$ then AB = $AB = [C_{ik}]_{m\times p}$.

(ii) Product of matrices is not commutative. i.e. $AB \neq BA$.

(iii) Product of matrices is associative. i.e $A(BC) = (AB)C$

(iv) Product of matrices is distributive over addition.

***Adjoint of matrix** :

If $A = [a_{ij}]$ be a n-square matrix then transpose of a matrix $[A_{ij}]$,

where A_{ij} is the cofactor of A_{ij} element of matrix A, is called the adjoint of A.

Adjoint of A = Adj. A = $[A_{ij}]^T$.

A(Adj.A) = (Adj. A)A = | A| I.

***Inverse of a matrix** :Inverse of a square matrix A exists, if A is non-singular or square matrix

A is said to be invertible and $A^{-1} = \frac{1}{|A|}$ Adj.A

***System of Linear Equations** :

$a_1x + b_1y + c_1z = d_1$.

$a_2x + b_2y + c_2z = d_2$.

$a_3x + b_3y + c_3z = d_3$.

$$\begin{bmatrix} a_1 & b_2 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \Rightarrow A\,X = B \;\Rightarrow X = A^{-1}B \quad ; \{\,|A| \neq 0\}.$$

***Criteria of Consistency**.

(i) If $|A| \neq 0$, then the system of equations is said to be consistent & has a unique solution.

(ii) If $|A| = 0$ and (adj. A)B = 0, then the system of equations either consistent or inconsistent according as the system may be have either infinite many solutions or no solutions.

(iii) If $|A| = 0$ and (adj. A)B $\neq 0$, then the system of equations is inconsistent and has no solution.

*** Determinant** :

To every square matrix we can assign a number called determinant

If $A = [a_{11}]$, det. $A = |A| = a_{11}$.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $|A| = a_{11}a_{22} - a_{21}a_{12}$.

*** Properties** :

(i) The determinant of the square matrix A is unchanged when its rows and columns are interchanged.

(ii) The determinant of a square matrix obtained by interchanging two rows(or two columns) is negative of given determinant.

(iii) If two rows or two columns of a determinant are identical, value of the determinant is zero.

(iv) If all the elements of a row or column of a square matrix A are multiplied by a non-zero number k, then determinant of the new matrix is k times the determinant of A.

If elements of any one column(or row) are expressed as sum of two elements each, then determinant can be written as sum of two determinants.

Any two or more rows(or column) can be added or subtracted proportionally.

If A & B are square matrices of same order, then $|AB| = |A|\,|B|$

ASSIGNMENTS

(i). Order, Addition, Multiplication and transpose of matrices:

LEVEL I

1. If a matrix has 5 elements, what are the possible orders it can have? **[CBSE 2011]**

2. Construct a 3 × 2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then find A –2 B.

4. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, write the order of AB and BA.

LEVEL II

1. For the following matrices A and B, verify $(AB)^T = B^TA^T$,

   where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = [-1 \quad 2 \quad 1]$

2. Give example of matrices A & B such that AB = O, but BA ≠ O, where O is a zero matrix and

A, B are both non zero matrices.

3. If B is skew symmetric matrix, write whether the matrix (ABA^T) is Symmetric or skew symmetric.
4. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b so that $A^2 + aI = bA$

LEVEL III

1. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of $A^2 - 3A + 2I$

2. Express the matrix A as the sum of a symmetric and a skew symmetric matrix, where:

$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

3. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$, $n \in N$

(ii) Cofactors &Adjoint of a matrix

LEVEL I

1. Find the co-factor of a_{12} in $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
2. Find the adjoint of the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

LEVEL II

Verify $A(\text{adj}A) = (\text{adj}A)A = |A|I$ if

1. $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$
2. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

(iii)Inverse of a Matrix & Applications

LEVEL I

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A **CBSE 2011**
2. If A is square matrix satisfying $A^2 = I$, then what is the inverse of A ?
3. For what value of k , the matrix $A = \begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible ?

LEVEL II

1. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$. Hence find A^{-1}

2. If A, B, C are three non zero square matrices of same order, find the condition on A such that $AB = AC \Rightarrow B = C$.

3. Find the number of all possible matrices A of order 3 × 3 with each entry 0 or 1 and for which A $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has exactly two distinct solutions.

4. Show that the matrix A = $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A + 1 = 0$ where I is 2 × 2 identify and 0 is 2 × 2 zero matrix using this equation, find A^{-1}.

LEVEL III

1 If A = $\begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:
2x – 3y + 5z = 11, 3x + 2y – 4z = - 5, x + y – 2z = - 3

2. Using matrices, solve the following system of equations:

a. x + 2y - 3z = - 4
2x + 3y + 2z = 2
3x - 3y – 4z = 11 **[CBSE 2011]**

b. 4x + 3y + 2z = 60
x + 2y + 3z = 45
6x + 2y + 3z = 70 **[CBSE 2011]**

3. Find the product AB, where A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the equations x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

4. Using matrices, solve the following system of equations:

$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$

$\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$

5. Using elementary transformations, find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

(iv)To Find The Difference Between $|A|, |adjA|, |kA|$

LEVEL I

1. Evaluate $\begin{vmatrix} cos15^8 & sin15^o \\ sin75^o & cos75^o \end{vmatrix}$ **[CBSE 2011]**

2. What is the value of $|3I|$, where I is identity matrix of order 3?

3. If A is non singular matrix of order 3 and $|A| = 3$, then find $|2A|$

4. For what valve of a, $\begin{bmatrix} 2a & -1 \\ -8 & 3 \end{bmatrix}$ is a singular matrix?

LEVEL II

1. If A is a square matrix of order 3 such that $|adjA| = 64$, find $|A|$
2. If A is a non singular matrix of order 3 and $|A| = 7$, then find $|adjA|$

LEVEL III

1. If $A = \begin{bmatrix} a & 2 \\ 2 & a \end{bmatrix}$ and $|A|^3 = 125$, then find a.
2. A square matrix A, of order 3, has $|A| = 5$, find $|A.adjA|$

(v).Properties of Determinants

LEVEL I

1. Find positive valve of x if $\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$

2. Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

LEVEL II

1. Using properties of determinants, prove the following :

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

[CBSE 2012]

2. $$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

3. $$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x)$$

4. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$ **[CBSE 2012]**

LEVEL III

1. Using properties of determinants, solve the following for x :

a. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$ **[CBSE 2011]**

b. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ **[CBSE 2011]**

c. $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$ **[CBSE 2011]**

2. If a, b, c, are positive and unequal, show that the following determinant is negative:

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

3. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

4. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$ **[CBSE 2012]**

5. $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$

6. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

7. $\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

8. If p, q, r are not in G.P and $\begin{vmatrix} 1 & \frac{q}{p} & \alpha+\frac{q}{p} \\ 1 & \frac{r}{q} & \alpha+\frac{r}{q} \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$, show that $p\alpha^2+2p\alpha+r=0$.

9. If a, b, c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$

   Show that either a + b +c = 0 or a = b = c.

QUESTIONS FOR SELF EVALUTION

1. Using properties of determinants, prove that : $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

2. Using properties of determinants, prove that : $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$

3. Using properties of determinants, prove that : $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

4. .Express $A = \begin{bmatrix} 3 & 2 & 3 \\ 4 & 5 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

5. Let $A = \begin{bmatrix} -1 & -4 \\ 1 & 3 \end{bmatrix}$, prove by mathematical induction that : $A^n = \begin{bmatrix} 1-2n & -4n \\ n & 1+2n \end{bmatrix}$.

6. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y such that $A^2 + xI = yA$. Hence find A^{-1}.

7. Let $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Prove that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

8. Solve the following system of equations : $x + 2y + z = 7$, $x + 3z = 11$, $2x - 3y = 1$.

9. Find the product AB, where $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve

the equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

10. Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

TOPIC 4
CONTINUITY AND DIFFRENTIABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
Continuity& Differentiability	1.Limit of a function		
	2.Continuity	***	Ex 5.1 Q.No- 21, 26,30
	3.Differentiation	*	Ex 5.2 Q.No- 6 Ex 5.3 Q.No- 4,7,13
	4.Logrithmic Differentiation	***	Ex 5.5 QNo- 6,9,10,15
	5 Parametric Differentiation	***	Ex 5.6 QNo- 7,8,10,11
	6. Second order derivatives	***	Ex 5.7 QNo- 14,16,17
	7. Mean Value Theorem	**	Ex 5.8 QNo- 3,4

SOME IMPORTANT RESULTS/CONCEPTS

* A function f is said to be continuous at x = a if Left hand limit = Right hand limit = value of the function at x = a

i.e. $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$

i.e. $\lim_{h\to 0} f(a-h) = \lim_{h\to 0} f(a-h) = f(a)$.

* A function is said to be differentiable at x = a if $Lf'(a) = Rf'(a)$ i.e

$$\lim_{h\to 0}\frac{f(a-h)-f(a)}{-h} = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

(i) $\frac{d}{dx}(x^n) = n\,x^{n-1}$.

(ii) $\frac{d}{dx}(x) = 1$

(iii) $\frac{d}{dx}(c) = 0, \forall\, c \in R$

(iv) $\frac{d}{dx}(a^x) = a^x \log a,\ a > 0,\ a \neq 1$.

(v) $\frac{d}{dx}(e^x) = e^x$.

(vi) $\frac{d}{dx}(\log_a x) =, \frac{1}{x\log a}\ a > 0,\ a \neq 1,\ x$

(vii) $\frac{d}{dx}(\log x) = \frac{1}{x},\ x > 0$

(xiii) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(xiv) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(xv) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(xvi) $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$.

(xvii) $\frac{d}{dx}(\cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$.

(xviii) $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

(xix) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

(xx) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$

(xxi) $\frac{d}{dx}(\operatorname{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$

(xxii) $\frac{d}{dx}(|x|) = \frac{x}{|x|},\ x \neq 0$

(xxiii) $\frac{d}{dx}(ku) = k\frac{du}{dx}$

(xxiv) $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(viii) $\frac{d}{dx}(\log_a\|x\|) = \frac{1}{x\log a}$, $a > 0, a \neq 1, x \neq 0$ (ix) $\frac{d}{dx}(\log\|x\|) = \frac{1}{x}$, $x \neq 0$ (x) $\frac{d}{dx}(\sin x) = \cos x, \forall x \in R$. (xi) $\frac{d}{dx}(\cos x) = -\sin x, \forall x \in R$. (xii) $\frac{d}{dx}(\tan x) = \sec^2 x, \forall x \in R$.	(xxv) $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$ (xxvi) $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

2.Continuity

LEVEL-I

1.Examine the continuity of the function $f(x)=x^2 + 5$ at x=-1.

2. Examine the continuity of the function $f(x)=\frac{1}{x+3}$, $x \in R$.

3. Show that f(x)=4x is a continuous for all $x \in R$.

LEVEL-II

1. Give an example of a function which is continuous at x=1,but not differentiable at x=1.

2. For what value of k,the function $\begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at x=2.

3.Find the relationship between “a” and “b” so that the function ‘f’ defined by:

[CBSE 2011]

$f(x)=\begin{cases} ax+1 & \text{if } x \leq 3 \\ bx+3 & \text{if } x > 3 \end{cases}$ is continuous at x=3.

4. If $f(x)=\begin{cases} \frac{\sin 3x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$. Find whether f(x) is continuous at x=0.

LEVEL-III

1.For what value of k, the function $f(x)=\begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x=0?

2. If function $f(x)=\frac{2x+3\sin x}{3x+2\sin x}$, for $x \neq 0$ is continuous at x=0, then Find f(0).

3.Let $f(x)\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2} \\ a & \text{if } x=\dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & \text{if } x>\dfrac{\pi}{2} \end{cases}$ = If f(x) be a continuous function at x=$\dfrac{\pi}{2}$, find a and b.

4.For what value of k,is the function $f(x)=\begin{cases} \dfrac{\sin x + x\cos x}{x} & , \text{when } x\neq 0 \\ k & , \text{when } x=0 \end{cases}$ continuous at x= 0?

3.Differentiation

LEVEL-I

1. Discuss the differentiability of the function $f(x)=(x-1)^{2/3}$ at x=1.

2.Differentiate $y=\tan^{-1}\dfrac{2x}{1-x^2}$.

3. If $y=\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$, Find $\dfrac{dy}{dx}$.

LEVEL-II

1. Find $\dfrac{dy}{dx}$, $y=\cos(\log x)^2$.

2. Find $\dfrac{dy}{dx}$ of $y=\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-1}{x}\right]$

3.If $y=e^{ax}\sin bx$, then prove that $\dfrac{d^2y}{dx^2}-2a\dfrac{dy}{dx}+(a^2+b^2)y=0$.

4.Find $\dfrac{d^2y}{dx^2}$, if $y=\dfrac{3at}{1+t}$, $x=\dfrac{2at^2}{1+t}$.

LEVEL-III

1.Find $\dfrac{dy}{dx}$, if $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

2.Find $\dfrac{dy}{dx}$ $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, $0<x<\dfrac{\pi}{2}$.

3. If $y=\sin^{-1}\left(\dfrac{a+b\cos x}{b+a\cos x}\right)$, show that $\dfrac{dy}{dx}=\dfrac{-\sqrt{b^2-a^2}}{b+a\cos x}$.

4.Prove that $\frac{d}{dx}\left[\frac{1}{4\sqrt{2}}\log\left|\frac{x^2+\sqrt{2}\,x+1}{x^2-\sqrt{2}\,x+1}\right|+\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2}\,x}{1-x^2}\right)\right]=\frac{1}{1+x^4}$.

4.Logrithmic Differentiation

LEVEL-I

1.Differentiate $y=\log_7(\log x)$.

2. Differentiate , sin(log x),with respect to x.

3.Differentiate $y=\tan^{-1}(\log x)$

LEVEL-II

1. If $y.\sqrt{x^2+1}=\log[\sqrt{x^2+1}-x]$,show that $(x^2+1)\frac{dy}{dx}+xy+1=0$.
2. Find $\frac{dy}{dx}$, $y=\cos(\log x)^2$.
3. Find $\frac{dy}{dx}$ if $(\cos x)^y=(\cos y)^x$ **[CBSE 2012]**

LEVEL-III

1.If $x^p.y^q=(x+y)^{p+q}$, prove that $\frac{dy}{dx}=\frac{y}{x}$

2. $y=(\log x)^{\cos x}+\frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$

3. If $x^y=e^{x-y}$, Show that $\frac{dy}{dx}=\frac{\log x}{\{\log(xe)\}^2}$ **[CBSE 2011]**

4. Find $\frac{dy}{dx}$ when $y=x^{\cot x}+\frac{2x^2-3}{x^2+x+2}$ **[CBSE 2012]**

5 Parametric Differentiation

LEVEL-II

1.If y = tanx, prove that $\frac{d^2y}{dx^2}=2y\frac{dy}{dx}$

2. .If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$ find $\frac{d^2y}{dx^2}$at $\theta=\frac{\pi}{4}$.

3. If $x=\tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2}+(2x-a)=0$**[CBSE 2011]**

6. Second order derivatives

LEVEL-II

1. If y = a cos (log x) + b sin(log x), prove that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

2.If $y=(\sin^{-1} x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

3.If $(x-a)^2 + (x-b)^2 = c^2$ for some c>0.Prove that $\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant, independent

7. Mean Value Theorem

LEVEL-II

1.It is given that for the function $f(x)=x^3-6x^2+px+q$ on[1,3] , Rolle's theorem holds with $c=2+\frac{1}{\sqrt{3}}$. Find the values p and q.

2. Verify Rolle's theorem for the function f(x) = sinx, in $[0, \pi]$.Find c, if verified

3.VeifyLagrange's mean Value Theorem $f(x) = \sqrt{x^2-4}$ in the interval [2,4]

Questions for self evaluation

1.For what value of k is the following function continuous at x = 2 ?

$$f(x) = \begin{cases} 2x+1 ; x<2 \\ k ; x=2 \\ 3x-1 ; x>2 \end{cases}$$

2.If $f(x) = \begin{cases} 3ax+b, & \text{if } x>1 \\ 11 & \text{if } x=1 \\ 5ax-2b, & \text{if } x<1 \end{cases}$, continuous at x = 1, find the values of a and b.**[CBSE 2012 Comptt.]**

3. Discuss the continuity of $f(x) = |x-1| + |x-2|$ at x = 1 & x = 2.

4. If f(x), defined by the following is continuous at x = 0, find the values of a, b, c

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & , x<0 \\ c & , x=0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & , x>0 \end{cases}$$

5.If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$.

6. If $y = (\log x)^{\cos x} + \frac{x^2+1}{x^2-1}$, find $\frac{dy}{dx}$.

7. If $xy + y^2 = \tan x + y$, find $\frac{dy}{dx}$.

8. If $y = \sqrt{x^2+1} - \log\left(\frac{1}{x} + \sqrt{1+\frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.

9.If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

10. Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$

11.If y = a cos (log x) + b sin(log x), prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

12.If $x^p.y^q = (x+y)^{p+q}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

TOPIC 5
APPLICATIONS OF DERIVATIVES
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of importance	Refrences NCERT Text Book XII Ed. 2007
Application of Derivative.	1.Rate of change	*	Example 5 Ex 6.1 Q.No- 9,11
	2.Increasing & decreasing functions	***	Ex 6.2 Q.No- 6 Example 12,13
	3.Tangents & normals	**	Ex 6.3 Q.No- 5,8,13,15,23
	4.Approximations	*	Ex 6.4 QNo- 1,3
	5 Maxima & Minima	***	Ex 6.5Q.No- 8,22,23,25 Example 35,36,37,

SOME IMPORTANT RESULTS/CONCEPTS

** Whenever one quantity y varies with another quantity x, satisfying some rule y = f (x) , then $\frac{dy}{dx}$ (or $f'(x)$) represents the rate of change of y with respect to x and $\left[\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at $x = x_0$.

** Let I be an open interval contained in the domain of a real valued function f. Then f is said to be

(i) increasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \le f(x_2)$ for all $x_1, x_2 \in$ I.

(ii) strictly increasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) < f(x_2)$ for all $x_1, x_2 \in$ I.

(iii) decreasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in$ I.

(iv) strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow$ $f(x_1) > f(x_2)$ for all $x_1, x_2 \in$ I.

** (i) f is strictly increasing in (a, b) if $f'(x) > 0$ for each $x \in (a, b)$

(ii) f is strictly decreasing in (a, b) if $f'(x) < 0$ for each $x \in (a, b)$

(iii) A function will be increasing (decreasing) in **R** if it is so in every interval of **R**.

** Slope of the tangent to the curve y = f (x) at the point (x_0, y_0) is given by $\left[\frac{dy}{dx}\right]_{(x_0,y_0)}$ $\left(= f'(x_0)\right)$.

** The equation of the tangent at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = f'(x_0)(x - x_0)$.

** Slope of the normal to the curve y = f (x) at (x_0, y_0) is $-\frac{1}{f'(x_0)}$.

** The equation of the normal at (x_0, y_0) to the curve y = f (x) is given by $y - y_0 = -\frac{1}{f'(x_0)}(x - x_0)$.

** If slope of the tangent line is zero, then $\tan\theta = 0$ and so $\theta = 0$ which means the tangent line is parallel to the x-axis. In this case, the equation of the tangent at the point (x0, y0) is given by $y = y_0$.

** If $\theta \to \frac{\pi}{2}$, then $\tan \theta \to \infty$, which means the tangent line is perpendicular to the x-axis, i.e., parallel to the y-axis. In this case, the equation of the tangent at (x_0, y_0) is given by $x = x_0$.

** Increment Δy in the function $y = f(x)$ corresponding to increment Δx in x is given by $\Delta y = \frac{dy}{dx}\Delta x$.

** Relative error in y = $\frac{\Delta y}{y}$.

** Percentage error in y = $\frac{\Delta y}{y} \times 100$.

** Let f be a function defined on an interval I. Then

(a) f is said to have a maximum value in I, if there exists a point c in I such that $f(c) \geq f(x)$, for all $x \in I$. The number f (c) is called the maximum value of f in I and the point c is called a point of maximum value of f in I.

(b) f is said to have a minimum value in I, if there exists a point c in I such that $f(c) \leq f(x)$, for all $x \in I$. The number f (c), in this case, is called the minimum value of f in I and the point c, in this case, is called a point of minimum value of f in I.

(c) f is said to have an extreme value in I if there exists a point c in I such that f (c) is either a maximum value or a minimum value of f in I.
The number f (c), in this case, is called an extreme value of f in I and the point c is called an extreme point.

* * Absolute maxima and minima

Let f be a function defined on the interval I and $c \in I$. Then

(a) f(c) is absolute minimum if $f(x) \geq f(c)$ for all $x \in I$.

(b) f(c) is absolute maximum if $f(x) \leq f(c)$ for all $x \in I$.

(c) $c \in I$ is called the critical point of f if $f'(c) = 0$

(d) Absolute maximum or minimum value of a continuous function f on [a, b] occurs at a or b or at critical points off (i.e. at the points where f' is zero)

If $c_1, c_2, \ldots, c_n$ are the critical points lying in [a , b], then

$$\text{absolute maximum value of } f = \max\{f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\}$$

$$\text{and absolute minimum value of } f = \min\{f(a), f(c_1), f(c_2), \ldots, f(c_n), f(b)\}.$$

** Local maxima and minima

(a)A function f is said to have a local maxima or simply a maximum value at x = a if $f(a \pm h) \leq f(a)$ for sufficiently small h

(b)A function f is said to have a local minima or simply a minimum value at x = a if $f(a \pm h) \geq f(a)$.

** First derivative test : A function f has a maximum at a point x = a if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from + ve to –ve in the neighbourhood of 'a' (points taken from left to right).

However, f has a minimum at x = a, if

(i) $f'(a) = 0$, and

(ii) $f'(x)$ changes sign from –ve to +ve in the neighbourhood of 'a'.

If $f'(a) = 0$ and $f'(x)$ does not change sign, then f(x) has neither maximum nor minimum and the point 'a' is called point of inflexion

The points where $f'(x) = 0$ are called stationary or critical points. The stationary points at which the function attains either maximum or minimum values are called extreme points.

** Second derivative test

(i) a function has a maxima at x a if f ′(x) 0 and f ″ (a) <0

(ii) a function has a minima at x = a if f ′(x) = 0 and f ″(a) > 0.

35 (iii) the test fails if f (a) = 0 and f (a) = 0. In this case we apply 1st derivative test. I II

OR

f (a) & f III IV(a) if f (aIII) 1 0 then f has max or min value at x = a (is called point) if f (a) = 0

m m III

and f IV(a) < 0 then f is maxm at x = 4 and f III(a) = 0, & f IV(a) > 0 then f is minm at x = a

ASSIGNMENTS

1.Rate of change

LEVEL -I

1. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

2 .The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long ?

3. The radius of a circle is increasing at the rate of 0.7 cm/sec. what is the rate of increase of its circumference ?

LEVEL –II

1. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate?

2. A man 2 metre high walks at a uniform speed of 6km /h away from a lamp post 6 metre high. Find the rate at which the length of his shadow increases. Also find the rate at which the tip of the shadow is moving away from the lamp post.

3. The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3cm/sec. find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm

LEVEL III

1. A particle moves along the curve $6y = x^3 + 2$., Find the points on the curve at which y-coordinate is changing 8 times as fast as the x-coordinate.

2. Water is leaking from a conical funnel at the rate of 5 cm^3/sec. If the radius of the base of the funnel is 10 cm and altitude is 20 cm, Find the rate at which water level is dropping when it is 5 cm from top.

3. From a cylinder drum containing petrol and kept vertical, the petrol is leaking at the rate of 10 ml/sec. If the radius of the drum is 10cm and height 50cm, find the rate at which the level of the petrol is changing when petrol level is 20 cm from bottom.

2.Increasing & decreasing functions

LEVEL I

1. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all $x \in R$.

2. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on (0,1)

3. Find the intervals in which the function $f(x) = \sin x - \cos x$, $0 < x < 2\pi$ is increasing or decreasing

decreasing.

LEVEL II

1. Indicate the interval in which the function $f(x) = \cos x,\ 0 \le x \le 2\pi$ is decreasing.

2. Show that the function $f(x) = \frac{\sin x}{x}$ is strictly decreasing on $(0, \pi/2)$

3. Find the intervals in which the function $f(x) = \frac{\log x}{x}$ increasing or decreasing.

LEVEL III

1. Find the interval of monotonocity of the function $f(x) = 2x^2 - \log x,\ x \neq 0$

2. Prove that the function $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $[0, \pi/2]$

[CBSE 2011]

3.Tangents &Normals

LEVEL-I

1.Find the equations of the normals to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$.

2. Find the point on the curve $y = x^2$ where the slope of the tangent is equal to the x-coordinate of the point.

3. At what points on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x axis ?

LEVEL-II

1. Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3)

2. For the curve $y = 2x^2 + 3x + 18$, find all the points at which the tangent passes through the origin.

3. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$

4. Show that the equation of tangent at (x_1, y_1) to the parabola $y^2=4ax$ is $yy_1 = 2a(x + x_1)$. **[CBSE 2012Comptt.]**

LEVEL- III

1 .Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 2$ which is parallel to the line $4x - 2y + 3 = 0$

2. Show that the curve $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 2y = 0$ cut orthogonally at the point (0,0)

3. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$ to intersect orthogonally.

4.Approximations

LEVEL-I

Q.1 Evaluate $\sqrt{25.3}$

Q.2 Use differentials to approximate the cube root of 66

Q.3 Evaluate $\sqrt{0.082}$

Q.4 Evaluate $\sqrt{49.5}$ **[CBSE 2012]**

LEVEL-II

1. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its surface area

5 Maxima & Minima

LEVEL I

1. Find the maximum and minimum value of the function $f(x) = 3 - 2 \sin x$

2. Show that the function $f(x) = x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value

3. Find two positive numbers whose sum is 24 and whose product is maximum

LEVEL II

1. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.

2.A piece of wire 28(units) long is cut into two pieces. One piece is bent into the shape of a circle and other into the shape of a square. How should the wire be cut so that the combined area of the two figures is as small as possible.

3. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

LEVEL III

1 .Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one extremity of major axis.

2.An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.**[CBSE 2012 Comptt.]**

3.A window is in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

[CBSE 2011]

Questions for self evaluation

1.Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

2. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

3. Find the intervals in which the following function is strictly increasing or decreasing:

$f(x) = -2x^3 - 9x^2 - 12x + 1$

4. Find the intervals in which the following function is strictly increasing or decreasing:

$f(x) = \sin x + \cos x$, $0 \le x \le 2\pi$

5. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$ (b) perpendicular to the line $5y - 15x = 13$.

7. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 = 1$.

8. Using differentials, find the approximate value of each of the following up to 3places of decimal :

(i) $(26)^{\frac{1}{3}}$ (ii) $(32.15)^{\frac{1}{5}}$

9. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

10. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

TOPIC 6
INDEFINITE & DEFINITE INTEGRALS
SCHEMATIC DIAGRAM

Topics	Concept	Degree of Importance	References Text book of NCERT, Vol. II 2007 Edition
Indefinite Integrals	(i) Integration by substitution	*	Exp 5&6 Page301,303
	(ii)) Application of trigonometric function in integrals	**	Ex 7 Page 306, Exercise 7.4Q13&Q24
	(iii) Integration of some particular function $\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}},$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx, \int \frac{dx}{ax^2 + bx + c},$ $\int \frac{dx}{\sqrt{ax^2 + bx + c}},$ $\int \frac{(px+q)dx}{ax^2 + bx + c},$ $\int \frac{(px+q)dx}{\sqrt{ax^2 + bx + c}}$	***	Exp 8, 9, 10 Page 311,312 Exercise 7.4 Q 3,4,8,9,13&23
	(iv) Integration using Partial Fraction	***	Exp 11&12 Page 318 Exp 13 319,Exp 14 & 15 Page320
	(v) Integration by Parts	**	Exp 18,19&20 Page 325
	(vi)Some Special Integrals $\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2 - a^2}\, dx$	***	Exp 23 &24 Page 329
	(vii) Miscellaneous Questions	***	Solved Ex.41
Definite Integrals	(i) Definite Integrals based upon types of indefinite integrals	*	Exercise 27 Page 336, Q 2,3,4,5,9,11,16 Exercise 7.9
	(ii) Definite integrals as a limit of sum	**	Exp 25 &26 Page 333, 334 Q3, Q5 & Q6 Exercise 7.8
	(iii) Properties of definite Integrals	***	Exp 31 Page 343*,Exp 32*,34&35 page 344 Exp 36***Exp 346 Exp 44 page351 Exercise 7.11 Q17 & 21
	(iv) Integration of modulus function	**	Exp 30 Page 343,Exp 43 Page 351 Q5& Q6 Exercise 7.11

SOME IMPORTANT RESULTS/CONCEPTS

* $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad ; n \neq -1$
* $\int 1.dx = x + c$
* $\int \frac{1}{\sqrt{x}} = 2\sqrt{x} + c$
* $\int \frac{1}{x} dx = \log x + c$
* $\int e^x dx = e^x + c$
* $\int a^x dx = \frac{a^x}{\log a} + c$
* $\int \sin x dx = -\cos x + c$
* $\int \cos x dx = \sin x + c$
* $\int \sec^2 x \, dx = \tan x + c$
* $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$
* $\int \sec x . \tan x \, dx = \sec x + c$
* $\int \operatorname{cosec} x . \cot x \, dx = -\operatorname{cosec} x + c$
* $\int \tan x \, dx = -\log|\cos x| + c = \log|\sec x| + c$
* $\int \cot x \, dx = \log|\sin x| + C$
* $\int \sec x \, dx = \log|\sec x + \tan x| + C$

  $= \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$
* $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + C$

  $= -\log|\operatorname{cosec} x + \cot x| + C$

  $= \log\left|\tan\frac{x}{2}\right| + C$
* $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + C$
* $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$
* $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + C$
* $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log|x + \sqrt{x^2 + a^2}| + C$
* $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}| + C$
* $\int \sqrt{x^2 + a^2} dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$
* $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$
* $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
* $\int \{f_1(x) \pm f_2(x) \pm \ldots\ldots\ldots f_n(x)\} dx$

  $= \int f_1(x) dx \pm \int f_2(x) dx \pm \ldots\ldots\ldots \pm \int f_n(x) dx$
* $\int \lambda f(x) dx = \lambda \int f(x) dx + C$
* $\int u.v \, dx = u.\int v.dx - \int\left[\int v.dx\right]\frac{du}{dx}.dx$
* $\int_a^b f(x) \, dx = F(b) - F(a)$, where $F(x) = \int f(x) \, dx$

*** General Properties of Definite Integrals.**

* $\int_a^b f(x) \, dx = \int_a^b f(t) \, dt$
* $\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$
* $\int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$
* $\int_a^b f(x) \, dx = \int_a^b f(a + b - x) \, dx$
* $\int_0^a f(x) \, dx = \int_0^a f(a - x) \, dx$
* $\int_{-a}^a f(x) \, dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function of x.} \\ 0 & \text{if } f(x) \text{ is an odd function of x} \end{cases}$

* $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C, = -\frac{1}{a}\cot^{-1}\frac{x}{a} + C$`	* $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)dx, & \text{if } f(2a-x) = f(x). \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$.

Assignments

(i) Integration by substitution

LEVEL I

1. $\int \frac{\sec^2(\log x)}{x} dx$ 2. $\int \frac{e^{m\tan^{-1}x}}{1+x^2} dx$ 3. $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}} dx$

LEVEL II

1. $\int \frac{1}{\sqrt{x}+x} dx$ 2. $\int \frac{1}{x\sqrt{x^6-1}} dx$ 3. $\int \frac{1}{e^x-1} dx$

LEVEL III

1. $\int \frac{\sqrt{\tan x}}{\sin x.\cos x} dx$ 2. $\int \frac{\tan x}{\sec x + \cos x} dx$ 3. $\int \frac{1}{\sin x.\cos^3 x} dx$

(ii) Application of trigonometric function in integrals

LEVEL I

1. $\int \sin^3 x.dx$ 2. $\int \cos^2 3x.dx$ 3. $\int \cos x.\cos 2x.\cos 3x.dx$

LEVEL II

1. $\int \sec^4 x.\tan x.dx$ 2. $\int \frac{\sin 4x}{\sin x} dx$

LEVEL III

1. $\int \cos^5 x.dx$ 2. $\int \sin^2 x.\cos^3 x.dx$

(iii) Integration using standard results

LEVEL I

1. $\int \frac{dx}{\sqrt{4x^2-9}}$ 2. $\int \frac{1}{x^2+2x+10} dx$ 3. $\int \frac{dx}{9x^2+12x+13}$

LEVEL II

1. $\int \frac{x}{x^4+x^2+1} dx$ 2. $\int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$ 3. $\int \frac{dx}{\sqrt{7-6x-x^2}}$

LEVEL III

1. $\int \frac{2x}{\sqrt{1-x^2-x^4}}dx$ 2. $\int \frac{x^2+x+1}{x^2-x+1}dx$ 3. $\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$

4. $\int \sqrt{\frac{1-x}{1+x}}\,dx$ 5. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}$ **[CBSE 2011]**

(iv) Integration using Partial Fraction

LEVEL I

1. $\int \frac{2x+1}{(x+1)(x-1)}dx$ 2. $\int \frac{x^2}{(x-1)(x-2)(x-3)}dx$ 3. $\int \frac{3x-2}{(x+1)^2(x+3)}dx$

LEVEL II

1. $\int \frac{x^2+2x+8}{(x-1)(x-2)}dx$ 2. $\int \frac{x^2+x+1}{x^2(x+2)}dx$ 3. $\int \frac{x^2+1}{(x-1)^2(x+3)}dx$

LEVEL III

1. $\int \frac{8}{(x+2)(x^2+4)}dx$ 2. $\int \frac{dx}{\sin x+\sin 2x}$ 3. $\int \frac{1}{1+x^3}dx$

(v) Integration by Parts

LEVEL I

1. $\int x.\sec^2 x.dx$ 2. $\int \log x.dx$ 3. $\int e^x(\tan x+\log\sec x)dx$

LEVEL II

1. $\int \sin^{-1} x.dx$ 2. $\int x^2.\sin^{-1} x.dx$ 3. $\int \frac{x.\sin^{-1} x}{\sqrt{1-x^2}}dx$

4. $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right).dx$ 5. $\int \sec^3 x.dx$

LEVEL III

1. $\int \cos(\log x)dx$ 2. $\int \frac{e^x(1+x)}{(2+x)^2 dx}$ 3. $\int \frac{\log x}{(1+\log x)^2}dx$

4. $\int \frac{2+\sin x}{1+\cos 2x}e^x.dx$ 5. $\int e^{2x}.\cos 3x.dx$

(vi) Some Special Integrals

LEVEL I

1. $\int \sqrt{4+x^2}.dx$ 2. $\int \sqrt{1-4x^2}.dx$

LEVEL II

1. $\int \sqrt{x^2+4x+6}.dx$ 2. $\int \sqrt{1-4x-x^2}.dx$

LEVEL III

1. $\int (x+1)\sqrt{1-x-x^2}.dx$ 2. $\int (x-5)\sqrt{x^2+x}\ dx$

(vii) Miscellaneous Questions

LEVEL II

1. $\int \frac{1}{2-3\cos 2x}dx$ 2. $\int \frac{1}{3+\sin 2x}dx$ 3. $\int \frac{dx}{4\sin^2 x+5\cos^2 x}$

4. $\int \frac{dx}{1+3\sin^2 x+8\cos^2 x}$ 5. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx$ 6. $\int \frac{\sec x}{5\sec x+4\tan x}dx$

LEVEL III

1. $\int \frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx$ 2. $\int \frac{dx}{1-\tan x}$ 3. $\int \frac{x^4}{x^4-1}dx$

4. $\int \frac{x^2+1}{x^4+x^2+1}dx$ 5. $\int \frac{x^2-1}{x^4+1}dx$ 6. $\int \sqrt{\tan x}.dx$ 7. $\int \left(\sqrt{\tan x}+\sqrt{\cot x}\right) dx$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I

1. $\int_0^1 \frac{2x+3}{5x^2+1}$ 2. $\int_0^{\pi/2} \sqrt{\sin x}.\cos^5 x.dx$ 3. $\int_0^2 x\sqrt{x+2}\,dx$

LEVEL II

1. $\int_1^2 \frac{5x^2}{x^2+4x+3}dx$ 2. $\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}\ dx$

(ii) Definite integrals as a limit of sum

LEVEL I

1. Evaluate $\int_0^2 (x+2)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^4 (1+x)\,dx$ definite integral as the limit of a sum.

LEVEL II

1. Evaluate $\int_1^2 (3x^2 - 1)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^3 (x^2 + 1)\,dx$ as the limit of a sum.

LEVEL III

1. Evaluate $\int_1^2 (x^2 + x + 2)dx$ as the limit of a sum.

2. Evaluate $\int_2^4 \left(e^{2x} + x^2\right) dx$ as the limit of a sum.

(iii) Properties of definite Integrals

LEVEL I

1. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$ 2. $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}\,dx$ 3. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx$

LEVEL II

1. $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$ 2. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx$ 3. $\int_0^{\pi} \frac{x \tan x}{\sec x . \operatorname{cosec}}\,dx$

4. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **[CBSE 2011]**

LEVEL III

1. $\int_0^{\pi} \frac{x + \sin x}{1+\cos x}\,dx$ **[CBSE 2011]** 2. $\int_0^{\pi/2} \log \sin x\; dx$ 3. $\int_0^{\pi/4} \log(1 + \tan x)dx$

[CBSE 2011]

(iv) Integration of modulus function

LEVEL III

1. $\int_2^5 (|x-2| + |x-3| + |x-4|)dx$ 2. $\int_{-1}^{2} \left|x^3 - x\right| dx$ 3. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|]dx$

Questions for self evaluation

1. Evaluate $\int \frac{(2x-3)dx}{x^2 - 3x - 18}$ 2. Evaluate $\int \frac{(3x+1).dx}{\sqrt{5-2x-x^2}}$

3. Evaluate $\int \cos^4 x.dx$

4. Evaluate $\int \frac{dx}{3+2\sin x+\cos x}$

5. Evaluate $\int \frac{2\sin x+3\cos x}{3\sin x+4\cos x}dx$

6. Evaluate $\int \frac{x.\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

7. Evaluate $\int_0^{\pi/2} \sqrt{\sin x}.\cos^5 x.dx$

8. Evaluate $\int_{-1}^{3/2} |x\sin \pi x|\,dx$

9. Evaluate $\int_0^{\pi/2} \log\sin x\,dx$

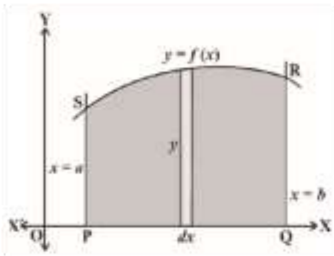
10. Evaluate $\int_1^4 (|x-1|+|x-2|+|x-3|)dx$

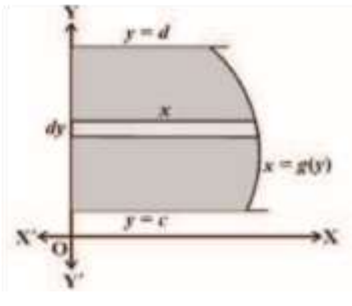
TOPIC 7
APPLICATIONS OF INTEGRATION
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	Reference NCERT Text BookEdition 2007
Applications of Integration	(i)Area under ***Simple Curves***	*	Ex.8.1 Q.1,2,5
	(ii) Area of the region enclosed between ***Parabola and line***	***	Ex. 8.1 Q 10,11 Misc.Ex.Q 7
	(iii) Area of the region enclosed between ***Ellipse and line***	***	Example 8, page 369 Misc.Ex. 8
	(iv) Area of the region enclosed between***Circle and line***	***	Ex. 8.1 Q 6
	(v) Area of the region enclosed between ***Circle and parabola***	***	Ex 8.2 Q1, Misc.Ex.Q 15
	(vi) Area of the region enclosed between ***Two Circles***	***	Example 10, page370 Ex 8.2 Q2
	(vii) Area of the region enclosed between ***Two parabolas***	***	Example 6, page368
	(viii) Area of triangle ***when vertices are given***	***	Example 9, page370 Ex 8.2 Q4
	(ix) Area of triangle ***when sides are given***	***	Ex 8.2 Q5 ,Misc.Ex. Q 14
	(x) ***Miscellaneous Questions***	***	Example 10, page374 Misc.Ex.Q 4, 12

SOME IMPORTANT RESULTS/CONCEPTS

** Area of the region PQRSP = $\int_a^b dA = \int_a^b y\, dx = \int_a^b f(x)\, dx$.





** The area A of the region bounded by the curve x = g (y), y-axis and the lines y = c, y = d is given by A= $\int_c^d x\, dy = \int_c^d g(y)\, dy$

ASSIGNMENTS

(i) Area under ***Simple Curves***

LEVEL I

1. Sketch the region of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and find its area, using integration,

2. Sketch the region $\{(x, y) : 4x^2 + 9y^2 = 36\}$ and find its area, using integration.

(ii) Area of the region enclosed between ***Parabola and line***

LEVEL II

1. Find the area of the region included between the parabola $y^2 = x$ and the line $x + y = 2$.
2. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

LEVEL III

1. Find the area of the region : $\{(x, y): y \le x^2 + 1,\ y \le x + 1, 0 \le x \le 2\}$

(iii) Area of the region enclosed between ***Ellipse and line***

LEVEL II

1. Find the area of smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ and the straight line $\frac{x}{4}+\frac{y}{5}=1$.

(iv) Area of the region enclosed between ***Circle and line***

LEVEL II

1. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

LEVEL III

1. Find the area of the region : $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$

(v) Area of the region enclosed between ***Circle and parabola***

LEVEL III

1. Draw the rough sketch of the region $\{(x, y): x^2 \le 6y,\ x^2 + y^2 \le 16\}$ an find the area enclosed by the region using the method of integration.
2. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.

(vi) Area of the region enclosed between ***Two Circles***

LEVEL III

1. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

(vii) Area of the region enclosed between ***Two parabolas***

LEVEL II

1. Draw the rough sketch and find the area of the region bounded by two parabolas

$4y^2 = 9x$ and $3x^2 = 16y$ by using method of integration.

(viii) Area of triangle ***when vertices are given***

LEVEL III

1. Using integration compute the area of the region bounded by the triangle whose vertices are (2 , 1), (3 , 4), and (5 , 2).
2. Using integration compute the area of the region bounded by the triangle whose vertices are (–1 , 1), (0 , 5), and (3 , 2).

(ix) Area of triangle ***when sides are given***

LEVEL III

1. Using integration find the area of the region bounded by the triangle whose sides are $y = 2x + 1$, $y = 3x + 1$, $x = 4$.

2. Using integration compute the area of the region bounded by the linesx $+ 2y = 2$, $y - x = 1$, and $2x + y = 7$.

(x) Miscellaneous Questions

LEVEL III

1. Find the area of the region bounded by the curves $y = |x - 1|$ and $y = -|x - 1| + 1$.
2. Find the area bounded by the curve $y = x$ and $y = x^3$.
3. Draw a rough sketch of the curve y = sinx and y = cosx as x varies from $x = 0$ to $x = \frac{\pi}{2}$ and find the area of the region enclosed by them and x-axis
4. Sketch the graph of $y = |x + 1|$.Evaluate $\int_{-3}^{1} |x + 1| dx$ .What does this value represent on the graph.
5. Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.
6. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = -6$ to $x = 0$. **[CBSE 2011]**

Questions for self evaluation

1. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.
2. Find the area bounded by the parabola $y = x^2$ and $y = |x|$.
3. Find the area of the region : $\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$
4. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.
5. Find the area of the region : $\{(x, y): x^2 + y^2 \le 1, \le x + y\}$
6. Find the area lying above the x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$.
7. Find the area bounded by the curves $x^2 + y^2 = 4$ and $(x + 2)^2 + y^2 = 4$ using integration.

8. Using integration compute the area of the region bounded by the triangle whose vertices are (2 , 1), (3 , 4), and (5 , 2).

9. Using integration compute the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$, and $x - 3y + 5 = 0$.

10. Sketch the graph of : $f(x) = \begin{cases} |x-2| + 2, & x \le 2 \\ x^2 - 2, & x > 2 \end{cases}$.

Evaluate $\int_0^4 f(x)dx$. What does the value of this integral represent on the graph ?

TOPIC 8
DIFFERENTIAL EQUATIONS
SCHEMATIC DIAGRAM

		(ii).General and particular solutions of a differential equation	**	Ex. 2,3 pg384
		(iii).Formation of differential equation whose general solution is given	*	Q. 7,8,10 pg 391
		(iv).Solution of differential equation by the method of separation of variables	*	Q.4,6,10 pg 396
		(vi).Homogeneous differential equation of first order and first degree	**	Q. 3,6,12 pg 406
		(vii)Solution of differential equation of the type dy/dx +py=q where p and q are functions of x And solution of differential equation of the type dx/dy+px=q where p and q are functions of y	***	Q.4,5,10,14 pg 413,414

SOME IMPORTANT RESULTS/CONCEPTS

** Order of Differential Equation : Order of the heighest order derivative of the given differential equation is called the order of the differential equation.

** Degree of the Differential Equation : Heighest power of the heighest order derivative when powers of all the derivatives involved in the given differential equation is called the degree of the differential equation

** Homogeneous Differential Equation : $\frac{dy}{dx} = \frac{f_1(x,y)}{f_2(x,y)}$, where $f_1(x,y)$ & $f_2(x,y)$ be the homogeneous function of same degree.

** Linear Differential Equation :

i. $\frac{dy}{dx} + py = q$, where p & q be the function of x or constant.

Solution of the equation is : $y.e^{\int p\,dx} = \int e^{\int p\,dx}.q\,dx$, where $e^{\int p\,dx}$ is Integrating Factor (I.F.)

ii. $\frac{dx}{dy} + px = q$, where p & q be the function of y or constant.

Solution of the equation is : $x.e^{\int p\,dy} = \int e^{\int p\,dy}.q\,dy$, where $e^{\int p\,dy}$ is Integrating Factor (I.F.)

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ASSIGNMENTS

1. *Order and degree of a differential equation*

LEVEL I

1. Write the order and degree of the following differential equations

   (i) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$

2. *General and particular solutions of a differential equation*

LEVEL I

1. Show that $y = e^{-x} + ax + b$ is the solution of $e^x \frac{d^2y}{dx^2} = 1$

3. *Formation of differential equation*

LEVEL II

1. Obtain the differential equation by eliminating a and b from the equation $y = e^x(a\cos x + b\sin x)$

LEVEL III

1. Find the differential equation of the family of circles $(x - a)^2 - (y - b)^2 = r^2$

2. Obtain the differential equation representing the family of parabola having vertex at the origin and axis along the positive direction of x-axis

4. *Solution of differential equation by the method of separation of variables*

LEVEL II

1. Solve $\frac{dy}{dx} = 1 + x + y + xy$ 2. Solve $\frac{dy}{dx} = e^{-y}\cos x$ given that y(0)=0.

3. Solve $\left(1 + x^2\right)\frac{dy}{dx} - x = \tan^{-1} x$

5. *Homogeneous differential equation of first order and first degree*

LEVEL II

1. Solve $(x^2 + xy)dy = (x^2 + y^2)dx$

LEVEL III

Show that the given differential equation is homogenous and solve it.

1. $(x - y)\frac{dy}{dx} = x + 2y$ 2. $ydx + x\log(\frac{y}{x})dy - 2xdy = 0$

3.Solve $xdy - ydx = \sqrt{x^2 - y^2}dx$ 4.Solve $x^2ydx - (x^3 + y^3)dy = 0$

5.Solve $xdy - ydx = \sqrt{(x^2 + y^2)}dx$ **CBSE2011** 6.Solve $\left(y + 3x^2\right)\frac{dx}{dy} = x$

7. Solve $x\ dy + (y - x^3)dx = 0$ **CBSE2011** 8.Solve $x\ dy + (y + 2x^2)dx = 0$

6. *Linear Differential Equations*

LEVEL I

1.Find the integrating factor of the differential $x\frac{dy}{dx} - y = 2x^2$

LEVEL II

1.Solve $\frac{dy}{dx} + 2y\tan x = \sin x$ 2. Solve $(1+x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

3. Solve $x\frac{dy}{dx} + y = x\log x$

LEVEL III

1. Solve $\frac{dy}{dx} = \cos(x+y)$ 2.Solve $ye^y dx = (y^3 + 2xe^y)dy$

3. Solve $x^2\frac{dy}{dx} = y(x+y)$ 4. Solve $\frac{dy}{dx} + \frac{4x}{x^2+1}y = -\frac{1}{(x^2+1)^3}$

5. Solve the differential equation $\left(x + 2y^2\right)\frac{dy}{dx} = y$;given that when x=2,y=1

Questions for self evaluation

1. Write the order and degree of the differential equation $\left(\frac{d^3y}{dy^3}\right)^2 + \frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$

2. Form the differential equation representing the family of ellipses having foci on x –axis and centre at origin .

3. Solve the differential equation : $(\tan^{-1} y - x)dy = (1 + y^2)dx$, given that y = 0 when x = 0.

4. Solve the differential equation :$xdy - y\ dx = \sqrt{x^2 + y^2}dx$

5. Solve the differential equation : $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$.

6. Solve the differential equation : $x^2 dy + (y^2 + xy)\ dx = 0$, y(1) = 1

7. Show that the differential equation $2y.e^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$ is homogeneous andfind its particular solution given that y(0) = 1 .

8. Find the particular solution of differential equation

$$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x, \text{ given that } y\left(\frac{\pi}{2}\right) = 0 .$$

TOPIC 9
VECTOR ALGEBRA
SCHEMATIC DIAGRAM

Topic	**Concept**	**Degree of importance**	**Refrence** NCERT Text Book Edition 2007
Vector algebra	(i)Vector and scalars	*	Q2 pg428
	(ii)Direction ratio and direction cosines	*	Q 12,13 pg 440
	(iii)Unit vector	* *	Ex 6,8 Pg 436
	(iv)Position vector of a point and collinear vectors	* *	Q 15 Pg 440 , Q 11Pg440 , Q 16 Pg448
	(v)Dot product of two vectors	**	Q6 ,13 Pg445
	(vi)Projection of a vector	* * *	Ex 16 Pg 445
	(vii)Cross product of two vectors	* *	Q 12 Pg458
	(viii)Area of a triangle	*	Q 9 Pg 454
	(ix)Area of a parallelogram	*	Q 10 Pg 455

SOME IMPORTANT RESULTS/CONCEPTS

* Position vector of point $A(x, y, z) = \vec{OA} = x\hat{i} + y\hat{j} + z\hat{k}$

* If $A(x_1, y_1, z_1)$ and point $B(x_2, y_2, z_2)$ then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

* If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$; $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

* Unit vector parallel to $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

* Scalar Product (dot product) between two vectors: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$; θ is angle between the vectors

* $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

* If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ then $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$

* If $\vec{a}$ is perpendicular to $\vec{b}$ then $\vec{a}.\vec{b} = 0$

* $\vec{a}.\vec{a} = |\vec{a}|^2$

* Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

* Vector product between two vectors:

  $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$; $\hat{n}$ is the normal unit vector which is perpendicular to both $\vec{a}$ & $\vec{b}$

* $\hat{n} = \dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

* If $\vec{a}$ is parallel to $\vec{b}$ then $\vec{a}\times\vec{b} = 0$

* Area of triangle (whose sides are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}|\vec{a}\times\vec{b}|$

* Area of parallelogram (whose adjacent sides are given by $\vec{a}$ and $\vec{b}$) $= |\vec{a}\times\vec{b}|$

* Area of parallelogram (whose diagonals are given by $\vec{a}$ and $\vec{b}$) $= \dfrac{1}{2}|\vec{a}\times\vec{b}|$

  or (whose diagonals are given by $\vec{p}$ & $\vec{q}$) $= \dfrac{1}{2}|\vec{p}\times\vec{q}|$

ASSIGNMENTS

(i) Vector and scalars, Direction ratio and direction cosines&Unit vector

LEVEL I

1. If $\vec{a} = \hat{i} + \hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} - 4\hat{j} + 3\hat{k}$ find a unit vector parallel to $\vec{a} + \vec{b}$
2. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$
3. If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + \hat{k}$; $\vec{c} = -\hat{i} + \hat{j} + \hat{k}$ find a unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$
4. Find a unit vector in the direction of the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ **[CBSE 2011]**
5. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$, whose magnitude is 7

LEVEL II

1. Find a vector of magnitude 5 units, perpendicular to each of the vectors $(\vec{a} + \vec{b})$,$(\vec{a} - \vec{b})$ where

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

2. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

LEVEL – III

1. If a line make α,β, γ with the X - axis , Y– axis and Z – axis respectively, then find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$
2. For what value of p, is $(\hat{i} + \hat{j} + \hat{k})\,p$ a unit vector?
3. What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with Y-axis
4. Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.

(ii)Position vector of a point and collinear vectors

LEVEL – I

1. Find the position vector of the midpoint of the line segment joining the points A($5\hat{i} + 3\hat{j}$) and B($3\hat{i} - \hat{j}$).

2. In a triangle ABC, the sides AB and BC are represents by vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 3\hat{j} + 5\hat{k}$ respectively. Find the vector representing CA.

3. Show that the points (1,0), (6,0) ,(0,0) are collinear.

LEVEL – II

1.Write the position vector of a point R which divides the line joining the points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 2 : 1 externally.

2.Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ respectively, externally in the ratio 1:2. Also, show that P is the mid-point of the line segment RQ

(iii) Dot product of two vectors

LEVEL – I

1.Find $\vec{a} \cdot \vec{b}$ if $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 3\hat{k}$.

2.If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$. Then find the angle between $\vec{a}$ and $\vec{b}$.

3.Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a} \cdot \vec{b} = \sqrt{6}$ **[CBSE 2011]**

LEVEL – II

1. The dot products of a vector with the vectors $\hat{i} - 3\hat{j}$, $\hat{i} - 2\hat{j}$ and $\hat{i} + \hat{j} + 4\hat{k}$ are 0 , 5 and 8 respectively. Find the vectors.

2. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$.

3. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, find the value of λ.

LEVEL – III

1. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$.

2. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

3. For what values of λ, vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{a} = \lambda\hat{i} - 4\hat{j} + 8\hat{k}$ are
   (i) Orthogonal (ii) Parallel `

4..Find$|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a})=15$.

5. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \mu\hat{k}$, find μ, such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

6. Show that the vector $2\hat{i} - \hat{j} + \hat{k}$, $-3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form sides of a right angled triangle.

7.Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 18$.

8. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

9. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three vectors such that $\left|\vec{a}\right| = 3$, $\left|\vec{b}\right| = 4$, $\left|\vec{c}\right| = 5$ and each of them being perpendicular

to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$.

(iv) Projection of a vector

LEVEL – I

1. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}\,.\,\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$.

2. Write the projection of the vector $\hat{i} - \hat{j}$ on the vector $\hat{i} + \hat{j}$ **[CBSE 2011]**

3.Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$

4. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$

LEVEL – II

1.Three vertices of a triangle are A(0, -1, -2), B(3,1,4) and C(5,7,1). Show that it is a right angled triangle. Also find the other two angles.

2.Show that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

3.If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non - zero and non – coplanar vectors, prove that $\vec{a} - 2\vec{b} + 3\vec{c}$, $-3\vec{b} + 5\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are also coplanar

LEVEL – III

1.If a unit vector $\vec{a}$ makes angles $\pi/4$, with $\hat{i}$, $\pi/3$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the component of $\vec{a}$ and angle θ.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$.

3.If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$,and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

4.Show that the points A, B, C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ respectively form the vertices of a right angled triangle.

5. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ, prove that

(i) $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ (ii) $\tan\frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}$

(vii)Cross product of two vectors

LEVEL – I

1. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a} \cdot \vec{b} = 9$. Find $|\vec{a} \times \vec{b}|$

2. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

3. Find $|\vec{x}|$, if $\vec{p}$ is a unit vector and , $(\vec{x} - \vec{p}).(\vec{x} + \vec{p}) = 80$.

4. Find p, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

LEVEL – II

1. Find λ, if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

2. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

3. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} \times \vec{b}| = 6$.

4. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, prove that $\vec{a} = \pm 2(\vec{a} \times \vec{b})$.

LEVEL – III

1. Find the value of the following: $\hat{i}.(\hat{j} \times \hat{k}) + \hat{i}.(\hat{i} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$

2. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$, and $\vec{a} \times \vec{b}$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$

3. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.

4. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show that $(\vec{a} - \vec{d})$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

5. Express $2\hat{i}-\hat{j}+3\hat{k}$ as the sum of a vector parellal and perpendicular to $2\hat{i}+4\hat{j}-2\hat{k}$.

(viii)Area of a triangle & Area of a parallelogram

LEVEL – I

1.Find the area of Parallelogram whose adjacent sides are represented by the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$.

2.If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a Parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.

3. Find the area of triangle having the points A(1,1,1) , B(1,2,3) and C(2,3,1) as its vertices.

LEVEL – II

1.Show that the area of the Parallelogram having diagonals ($3\hat{i} + \hat{j} - 2\hat{k}$) and ($\hat{i} - 3\hat{j} + 4\hat{k}$) is $5\sqrt{3}$ Sq units.

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors of the vertices of a ΔABC, show that the area of the ΔABC is

$$\frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|.$$

3.Using Vectors, find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5) **[CBSE 2011]**

Questions for self evaluation

1.The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$.

3. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

4. Dot product of a vector with $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$, and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5, 8 respectively. Find the vector.

5. Find the components of a vector whic is perpendicular to the vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} - \hat{j} + 2\hat{k}$.

TOPIC 10
THREE DIMENSIONAL GEOMETRY
SCHEMATIC DIAGRAM

Topic	Concept	Degree of importance	Refrence NCERT Text Book Edition 2007
Three Dimensional Geometry	(i) Direction Ratios and Direction Cosines	*	Ex No 2 Pg -466 Ex No 5 Pg – 467 Ex No 14 Pg - 480
	(ii)Cartesian and Vector equation of a line in space & conversion of one into another form	**	Ex No 8 Pg -470 Q N. 6, 7, - Pg 477 QN 9 – Pg 478
	(iii)Co-planer and skew lines	*	Ex No 29 Pg -496
	(iv) Shortest distance between two lines	***	Ex No 12 Pg -476 Q N. 16, 17 - Pg 478
	(v) Cartesian and Vector equation of a plane in space & conversion of one into another form	**	Ex No 17 Pg -482 Ex No 18 Pg – 484 Ex No 19 Pg – 485 Ex No 27 Pg – 495 Q N. 19, 20 - Pg 499
	(vi) Angle Between (iv) Two lines (v) Two planes (vi) Line & plane	 * * **	Ex No 9 Pg -472 Q N. 11 - Pg 478 Ex No 26 Pg – 494 Q N. 12 - Pg 494 Ex No 25 Pg - 492
	(vii) Distance of a point from a plane	**	Q No 18 Pg -499 Q No 14 Pg – 494
	(viii)Distance measures parallel to plane and parallel to line	**	
	(ix)Equation of a plane through the intersection of two planes	***	Q No 10 Pg -493
	(x) Foot of perpendicular and image with respect to a line and plane	**	Ex. N 16 Pg 481

SOME IMPORTANT RESULTS/CONCEPTS

** Direction cosines and direction ratios :

If a line makes angles α, β and γ with x, y and z axes respectively the $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are the direction cosines denoted by l, m, n respectively and $l^2 + m^2 + n^2 = 1$

Anythreenumbers proportional to direction cosines are direction ratios denoted by a,b,c

$\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$ $\quad l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},\quad m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},\quad n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}},$

* Direction ratios of a line segment joining $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ may be taken as $x_2-x_1,\ y_2-y_1,\ z_2-z_1$
* Angle between twolines whosedirection cosines are l_1,m_1,n_1 and l_2,m_2,n_2 is given by

$$\cos\theta=l_1l_2+m_1m_2+n_1n_2=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{\left(a_1^{\,2}+b_1^{\,2}+c_1^{\,2}\right)\left(a_2^{\,2}+b_2^{\,2}+c_2^{\,2}\right)}}$$

* For parallel lines $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ and

for perpendicular lines $a_1a_2+b_1b_2+c_1c_2=0$ or $l_1l_2+m_1m_2+n_1n_2=0$

** STRAIGHTLINE:

* Equation of line passing through a point (x_1,y_1,z_1) with direction cosines a, b, c: $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$
* Equation of line passing through a point (x_1,y_1,z_1) and parallel to the line: $\frac{x-\alpha}{a}=\frac{y-\beta}{b}=\frac{z-\gamma}{c}$ is

$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

* Equation of line passing through two point (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$
* Equation of line (Vector form)

Equation of line passing through a point $\vec{a}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$

* Equation of line passing through two points $\vec{a}$ & $\vec{b}$ and in the direction of $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\left(\vec{b}-\vec{a}\right)$
* Shortest distance between two skew lines : if lines are $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ $\vec{r}=\vec{a_2}+\lambda\vec{b_2}$

then Shortest distance $=\frac{\left(\vec{a_2}-\vec{a_1}\right)\left(\vec{b_1}\times\vec{b_2}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|}$ $\quad ;\vec{b_1}\times\vec{b_2}\neq 0$

$\frac{\left|\left(\vec{a_2}-\vec{a_1}\right)\times\vec{b_1}\right|}{\left|\vec{b_1}\right|}$ $\quad ;\vec{b_1}\times\vec{b_2}=0$

** PLANE:

* Equation of plane is $ax+by+cz+d=0$ where a, b & c are direction ratios of normal to the plane
* Equation of plane passing through a point (x_1,y_1,z_1) is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$
* Equation of plane in intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, where a, b, c are int ercepts on the axes
* Equation of plane in normal form $lx+my+nz=p$ where l, m, n are direction cos ines of normal to the plane p is length of perpendicular form origin tot eplane

* Equation of plane passing through three points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and $(x_3, y_3, z\ )$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

* Equation of plane passing through two points $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and perpendicular to the plane

$a_1x + b_1y + c_1z + d_1 = 0$ or parralal to the line $\frac{x-\alpha_1}{a_1} = \frac{y-\beta_1}{b_1} = \frac{z-\gamma_1}{c_1}$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$

* Equation of plane passing through the point (x_1, y_1, z_1) and perpendicular to the

planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z\ + d_2 = 0$ or parralal to the lines $\frac{x-\alpha_1}{a_1} = \frac{y-\beta_1}{b_1} = \frac{z-\gamma_1}{c_1}$

and $\frac{x-\alpha_2}{a_2} = \frac{y-\beta_2}{b_2} = \frac{z-\gamma_2}{c_2}$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

* Equation of plane contaning the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and passing through the point (x_2, y_2, z_2) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$$

* Condition for coplaner lines : $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplaner if

$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$ and equation of common plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

* Equation of plane passing through the intersection of two planes $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$ is $(a_1x + b_1y + c_1z) + \lambda(a_2x + b_2y + c_2z) = 0$

* Perpendicular distance from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$ is $\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$

* Distance between two parallel planes $ax + by + cz + d_1 = 0, ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

ASSIGNMENTS

(i)Direction Ratios and Direction Cosines

LEVEL-I

1. Write the direction-cosines of the line joining the points (1,0,0) and (0,1,1) **[CBSE 2011]**

2.Find the direction cosines of the line passing through the following points (-2,4,-5), (1,2,3).

3.Write the direction cosines of a line equally inclined to the three coordinate axes

LEVEL-II

1.Write the direction cosines of a line parallel to the line $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$.

2.Write the direction ratios of a line parallel to the line $\frac{5-x}{3} = \frac{y+7}{-2} = \frac{z+2}{6}$.

3. If the equation of a line AB $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+6}{3}$ Find the direction cosine.

4. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axis.

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

LEVEL-I

1.Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$. **[CBSE 2011]**

2. Write the equation of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point(1,2,3).

3.Express the equation of the plane $\vec{r} = (\hat{\imath} - 2\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ in the Cartesian form.

4.Express the equation of the plane $\vec{r} \cdot (2\hat{\imath} - 3\hat{\jmath} + \hat{k}) + 4 = 0$ in the Cartesian form.

(iii) Co-planer and skew lines

LEVEL-II

1.Find whether the lines $\vec{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} + \hat{\jmath})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath}) + \mu(\hat{\imath} + \hat{\jmath} - \hat{k})$ intersect or not. If intersecting , find their point of intersection.

2.Show that the four points (0,-1,-1) , (4,5,1) , (3,9,4) and (-4,4,4,) are coplanar. Also, find the equation of the plane containing them.

3.Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.

LEVEL-III

1. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

2. The points A(4,5,10) , B(2,3,4) a d C(1,2,-1) are three vertices of a parallelogram ABCD. Find

the vector equation of the sides AB and BC and also find the coordinates

3.Find the equations of the line which intersects the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point (1,1,1).

4. Show that The four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .

(iv) Shortest distance between two lines

LEVEL-II

1. Find the shortest distance between the lines l_1 and l_2 given by the following:

(a) $l_1 : \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{1}$ $\quad$ $l_2 : \frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$

(b) $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$

$\vec{r} = (4\hat{\imath} + 2\mu)\hat{\imath} + (5 + 3\mu)\hat{\jmath} + (6 + \mu)\hat{k}$.

2. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.

3.. Find the shortest distance between the lines

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$

4.Find the shortest distance between the lines

$\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{\jmath} + (3 - t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} + (2s + 1)\hat{k}$ **[CBSE 2011]**

5. Find the distance between the parallel planes x + y – z = -4 and 2x + 2y – 2z + 10 = 0.

6. Find the vector equation of the line parallel to the line $\frac{x-1}{5} = \frac{3-y}{2} = \frac{z+1}{4}$ and passing through (3,0,-4). Also, find the distance between these two lines.

(v) Cartesian and Vector equation of a plane in space & conversion of one into another form

LEVEL I

1.Find the equation of a plane passing through the origin and perpendicular to x-axis

2.Find the equation of plane with intercepts 2, 3, 4 on the x ,y, z –axis respectively.

3.Find the direction cosines of the unit vector perpendicular to the plane

$\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) + 1 = 0$ passing through the origin.

4.Find the Cartesian equation of the following planes:

(a) $r \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ $\quad$ (b) $r \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$

6

LEVEL II

1. Find the vector and cartesian equations of the plane which passes through the point (5, 2, – 4) and perpendicular to the line with direction ratios 2, 3, – 1.
2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector 3 iˆ + 5 ˆj - 6 kˆ.
3.Find the vector and cartesian equations of the planes that passes through the point (1, 0, – 2)and the normal to the plane is iˆ + ˆj - kˆ.

(vi) Angle Between(i)Two lines (ii)Two planes (iii)Line & plane

LEVEL-I

1. Find the angle between the lines whose direction ratios are (1, 1, 2) and ($\sqrt{3}$–1 , –$\sqrt{3}$ –1 , 4).

2.Find the angle between line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and the plane 3x + 4y + z + 5 = 0.

3.Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane
3x – y – 2z = 7.

4.Find the angle between the planes whose vector equations are
r· (2 iˆ + 2 ˆj - 3 kˆ) = 5 and r · (3 iˆ - 3 ˆj + 5 kˆ) = 3

5.Find the angle between the line$\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$and the plane 10 x + 2y – 11 z = 3.

LEVEL-II

1.Find the value of p, such that the lines $\frac{x}{1} = \frac{y}{3} = \frac{z}{2p}$ and $\frac{x}{-3} = \frac{y}{5} = \frac{z}{2}$ are perpendicular to each other.
2. A line makes angles α, β, γ, δ with the four diagonals of a cube, Prove that
$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

(vii) Distance of a point from a plane

LEVELI

1.Write the distance of plane 2x– y + 2z + 1 = 0 from the origin.
2.Find the point through which the line 2x = 3y = 4z passes.
3. Find the distance of a point (2, 5, – 3)from the plane r · (6 î − 3 ĵ + 2 k̂) = 4
4. Find the distance of the following plane from origin: 2x – y + 2z + 1 = 0

5.Find the distance of the point (a,b,c) from x-axis

LEVELII

1..Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

2.Find the distance of the point (3,4,5) from the plane x + y + z =2 measured parallel to the line 2x = y = z.

3. Find the distance between the point P(6, 5, 9) and the plane determinedby the points A (3, – 1, 2), B (5, 2, 4) and C(– 1, – 1, 6).

4.Find the distance of the point $(-1,-5,-10)$from the point of intersection of theline

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$and the plane $\vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$

[CBSE2011]

LEVEL III

1.Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane 2x – y + z + 3 = 0. Find also, the image of the point in the plane.

2.Find the distance of the point P(6,5,9) from the plane determined by the points A(3,-1,2) , B(5,2,4) and C(-1,-1,6).

3.Find the equation of the plane containing the lines $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ and $\vec{r} = \hat{\imath} + \hat{\jmath} + \mu(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$.Find the distance of this plane from origin and also from the point (1,1,1).

(viii) Equation of a plane through the intersection of two planes

LEVELII

1.Find the equation of plane passing through the point (1,2,1) and perpendicular to the line joining the points (1,4,2) and (2,3,5). Also find the perpendicular distance of the plane from the origin.

2.Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z +8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0.

3.Find the equation of the plane that contains the point (1,-1,2) and is perpendicular to each of the planes 2x + 3y -2 z = 5 and x + 2y – 3z = 8.

LEVEL-III

1.Find the equation of the plane passing through the point (1,1,1) and containing the line $\vec{r} = (-3\hat{\imath} + \hat{\jmath} + 5\hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} - 5\hat{k})$. Also, show that the plane contains the line $\vec{r} = (-\hat{\imath} + 2\hat{\jmath} + 5\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$.

2.Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.

3.Find the Cartesian equation of the plane passing through the points A(0,0,0) and B(3,-1,2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

4. Find the equation of the perpendicular drawn from the point P(2,4,-1) to the line

$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

(ix)Foot of perpendicular and image with respect to a line and plane

LEVEL II

1. Find the coordinates of the point where the line through (3,-4,-5) and (2,-3,1) crosses the plane determined by points A(1,2,3) , B(2,2,1) and C(-1,3,6).
2. Find the foot of the perpendicular from P(1,2,3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, obtain the equation of the plane containing the line and the point (1,2,3).
3. Prove that the image of the point (3,-2,1) in the plane 3x – y + 4z =2 lies on the plane, x + y + z + 4 = 0.

LEVEL-III

1. Find the foot of perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).
2. Find the image of the point (1, –2, 1) in the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2}$.
3. The foot of the perpendicular from the origin to the plane is (12, – 4, 3). Find the equation of the plane
4. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(3,2,1) from the plane 2x – y+z+1=0. Find also, the image of the point in the plane.

Questions for self evaluation

1. Find the equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.
2. Find the vector equation of a line joining the points with position vectors $\hat{i} - 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 4\hat{k}$, and $2\hat{i} + \hat{j} + 2\hat{k}$. Also find the cartesian equivalent of this equation.
3. Find the foot of perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).
4. Find the shortest distance between the lines
$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$
5. Find the image of the point (1, –2, 1) in the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z+3}{2}$.
6. Show that the four points (0, –1, –1), (4, 5, 1), (3, 9, 4) and (– 4, 4, 4) are coplanar and find the equation of the common plane .
7. The foot of the perpendicular from the origin to the plane is (12, – 4, 3). Find the equation of the plane.
8. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.
9. A line makes angles α, β, γ, δ with the four diagonals of a cube, Prove that
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

TOPIC 11
LINEAR PROGRAMMING
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Linear Programming	(i) LPP and its Mathematical Formulation	**	**Articles 12.2 and 12.2.1**
	(ii) Graphical method of solving LPP (bounded and unbounded solutions)	**	**Article 12.2.2** Solved Ex. 1 to 5 Q. Nos 5 to 8 Ex.12.1
	(iii) Diet Problem	***	Q. Nos 1, 2 and 9 Ex. 12.2 Solved Ex. 9 Q. Nos 2 and 3 Misc. Ex. Solved Ex. 8 Q. Nos 3,4,5,6,7 of Ex.
	(iv) Manufacturing Problem	***	12.2 Solved Ex.10 Q. Nos 4 & 10 Misc. Ex.
	(v) Allocation Problem	**	Solved Example 7Q. No 10 Ex.12.2, Q. No 5 & 8 Misc. Ex.
	(vi) Transportation Problem	*	Solved Ex.11 Q. Nos 6 & 7 Misc. Ex.
	(vii) Miscellaneous Problems	**	Q. No 8 Ex. 12.2

SOME IMPORTANT RESULTS/CONCEPTS

** Solving linear programming problem using **Corner Point Method**. The method comprises of the following steps:

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function Z = $ax + by$ at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is **bounded**, M and m are the maximum and minimum values of Z.
   (ii) In case, the feasible region is **unbounded**, we have:
4. (a) M is the maximum value of Z, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.
   (b) Similarly, m is the minimum value of Z, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

ASSIGNMENTS

(i) LPP and its Mathematical Formulation

LEVEL I

1. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food 'I' contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food 'II' contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem.

(ii) Graphical method of solving LPP (bounded and unbounded solutions)

LEVEL I

Solve the following Linear Programming Problems graphically:

1.Minimise $Z = -3x + 4y$ subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.

2.Maximise $Z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

3.Minimise $Z = 3x + 5y$ suchthat $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

(iii) Diet Problem

LEVEL II

1.A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1,400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs. 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of mineral and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost? Also find the least cost.

2. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Rs. 10 per kg and rice Rs. 20 per kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 gm and 200 gm respectively. In what quantities, should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of protein and carbohydrates at minimum cost ?

(iv) Manufacturing Problem

LEVEL II

1.A company manufactures two articles A and B. There are two departments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs. 6 for each unit of A and Rs. 8 for each unit of B, find the number of units of A and B to be produced per week in order to have maximum profit.

2. A company sells two different produces A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that for B is 125. Profit on each unit of A is Rs. 20 and that on B is Rs. 15. How many units of A and B should be produced to maximize the profit? Solve it graphically

LEVEL III

1.A manufacture makes two types of cups, A and B. Three machines are required to manufacture the cups and the time in minutes required by each is as given below:

Type of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise, and on B it is 50 paise, show that the 15 cups of type A and 30 cups of type B should be manufactured per day to get the maximum profit.

(v) Allocation Problem

LEVEL II

1. Ramesh wants to invest at most Rs. 70,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 10,000 in Bond A and at least Rs. 30,000 in Bond B. If the rate of interest on bond A is 8 % per annum and the rate of interest on bond B is 10 % per annum , how much money should he invest to earn maximum yearly income ? Find also his maximum yearly income.

2. An oil company requires 12,000, 20,000 and 15,000 barrels of high grade, medium grade and low grade oil respectively. Refinery A produces 100, 300 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200, 400 and 100 barrels per day respectively. If A costs Rs. 400 per day and B costs Rs. 300 per day to operate, how many days should each be run to minimize the cost of requirement?

LEVEL III

1. An aeroplane can carry a maximum of 250 passengers. A profit of Rs 500 is made on each executive class ticket and a profit of Rs 350 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit?

(vi) Transportation Problem

LEVEL III

1. A medicine company has factories at two places A and B . From these places, supply is to be made to each of its three agencies P, Q and R. The monthly requirement of these agencies are respectively 40, 40 and 50 packets of the medicines, While the production capacity of the factories at A and B are 60 and 70 packets are respectively. The transportation cost per packet from these factories to the agencies are given:

Transportation cost per packet (in Rs.)		
From / **To**	**A**	**B**
P	**5**	**4**
Q	**4**	**2**
R	**3**	**5**

How many packets from each factory be transported to each agency so that the cost of transportation is minimum ? Also find the minimum cost.

Questions for self evaluation

1. Solve the following linear programming problem graphically : Maximize $z = x - 7y + 190$ subject to the constraints $x + y \leq 8$, $x \leq 5$, $y \leq 5$, $x + y \geq 4$, $x \geq 0$, $y \geq 0$.

2. Solve the following linear programming problem graphically : Maximize $z = 3x + 5y$ subject to the constraints $x + y \geq 2$, $x + 3y \geq 3$, $x \geq 0$, $y \geq 0$.

3. Kellogg is a new cereal formed of a mixture of bran and rice that contains at least 88 grams of protein and at least 36 milligrams of iron. Knowing that bran contains, 80 grams of protein and 40 milligrams of iron per kilogram, and that rice contains 100 grams protein and 30 milligrams of iron per kilogram, find the minimum cost of producing this new cereal if bran costs Rs. 5 per kilogram and rice costs Rs. 4 per kilogram.

4. A shopkeeper deals only in two items — tables and chairs. He has Rs. 6,000 to invest and a space to store at most 20 pieces. A table costs him Rs. 400 and a chair Rs. 250. He can sell a table at a profit of Rs. 25 and a chair at a profit of Rs. 40. Assume that he can sell all items that he buys. Using linear programming formulate the problem for maximum profit and solve it graphically.

5. A small firm manufactures items A and B. The total number of items A and B it can manufacture a day is at most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs. 300 and one unit of item B be Rs. 160, how many of each type of item be produced to maximize the profit ? Solve the problem graphically.

6. A chemist requires 10, 12 and 12 units of chemicals A, B and C respectively for his analysis. A liquid product contains 5, 2, and 1 units of A, B and C respectively and it costs Rs. 3 per jar. A dry product contains 1, 2, and 4 units of A, B and C per carton and costs Rs. 2 per carton. How many of each should he purchase in order to minimize the cost and meet the requirement ?

7. A person wants to invest at most Rs. 18,000 in Bonds A and B. According to the rules, he has to invest at least Rs. 4,000 in Bond A and at least Rs. 5,000 in Bond B. If the rate of interest on bond A is 9 % per annum and the rate of interest on bond B is 11 % per annum , how much money should he invest to earn maximum yearly income ?

8. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to to stitch at least 60 shirts and 32 pants at a minimum labourcost.

TOPIC 12
PROBABILITY
SCHEMATIC DIAGRAM

Topic	Concepts	Degree of Importance	References NCERT Book Vol. II
Probability	(i) Conditional Probability	***	**Article 13.2 and 13.2.1** Solved Examples 1 to 6 Q. Nos 1 and 5 to 15 Ex. 13.1
	(ii)Multiplication theorem on probability	**	**Article 13.3** SolvedExamples 8 & 9 Q. Nos 2, 3, 13 14 & 16 Ex.13.2
	(iii) Independent Events	***	**Article 13.4** Solved Examples 10 to 14 Q. Nos 1, 6, 7, 8 and 11 Ex.13.2
	(iv) Baye's theorem, partition of sample space and Theorem of total probability	***	**Articles 13.5, 13.5.1, 13.5.2** Solved Examples 15 to 21, 33 & 37 ,Q. Nos 1 to 12 Ex.13.3 Q. Nos 13 & 16 Misc. Ex.
	(v) Random variables & probability distribution Mean & variance of random variables	***	**Articles 13.6, 13.6.1, 13.6.2 & 13.6.2** Solved Examples 24 to 29 Q. Nos 1 & 4 to 15 Ex. 13.4
	(vi) Bernoulli,s trials and Binomial Distribution	***	**Articles 13.7, 13.7.1 & 13.7.2** Solved Examples 31 & 32 Q. Nos 1 to 13 Ex.13.5

SOME IMPORTANT RESULTS/CONCEPTS

** **Sample Space and Events** :

The set of all possible outcomes of an experiment is called the sample space of that experiment. It is usually denoted by S. The elements of S are called events and a subset of S is called an event.

ϕ ($\subset$ S) is called an impossible event and

S($\subset$ S) is called a sure event.

** **Probability of an Event.**

(i) If E be the event associated with an experiment, then probability of E, denoted by P(E) is

defined as $P(E) \; \frac{\text{number of outcomes in E}}{\text{number of total outcomes in sample space S}}$

it being assumed that the outcomes of the experiment in reference are equally likely.

(ii) P(sure event or sample space) = P(S) = 1 and P(impossible event) = P(ϕ) = 0.

(iii) If $E_1, E_2, E_3, \ldots, E_k$ are mutually exclusive and exhaustive events associated with an experiment

(i.e. if $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_k) = S$ and $E_i \cap E_j = \phi$ for $i, j \in \{1, 2, 3, \ldots, k\}$ $i \neq j$), then

$$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_k) = 1.$$

(iv) $P(E) + P(E^C) = 1$

** If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P (E|F) is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)} \text{ provided } P(F) \neq 0$$

** Multiplication rule of probability : $P(E \cap F) = P(E)\, P(F|E)$

$= P(F)\, P(E|F)$ provided $P(E) \neq 0$ and $P(F) \neq 0$.

** Independent Events : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.

Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E \cap F) = P(E) \,.\, P(F)$.

** Bayes' Theorem : If $E_1, E_2, \ldots, E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots, E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots \cup E_n = S$ andA is any event of nonzero probability, then

$$P(Ei|A) = \frac{P(E_i).P(A|E_i)}{\sum_{j=1}^{n} P(E_j).P(A|E_j)} \quad \text{for any } i = 1, 2, 3, \ldots, n$$

** The probability distribution of a random variable X is the system of numbers

X :	x_1	x_2	...	x_n
P(X) :	p_1	p_2	...	p_n

where, $p_i > 0$, $\sum_{i=1}^{n} p_i = 1$, i = 1, 1, 2,...,

** **Binomial distribution:** The probability of x successes P (X = x) is also denoted by P (x) and is given by $P(x) = {}^nC_x\, q^{n-x} p^x$, $x = 0, 1, \ldots, n.$ $(q = 1 - p)$

ASSIGNMENTS

(i) Conditional Probability

LEVEL I

1. If P(A) = 0.3, P(B) = 0.2, find P(B/A) if A and B are mutually exclusive events.
2. Find the probability of drawing two white balls in succession from a bag containing 3 red and 5 white balls respectively, the ball first drawn is not replaced.

LEVEL II

1.A dice is thrown twice and sum of numbers appearing is observed to be 6. what is the conditional probability that the number 4 has appeared at least once.

LEVEL III

1.If$P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{2}$, find $P(\overline{A}/\overline{B})$ and $P(\overline{B}/\overline{A})$

(ii)Multiplication theorem on probability

LEVEL II

1.A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement, find what is the probability that none is red.

2. The probability of A hitting a target is $\frac{3}{7}$ and that of B hitting is $\frac{1}{3}$. They both fire at the target. Find the probability that (i) at least one of them will hit the target, (ii) Only one of them will hit the target.

LEVEL III

1.A class consists of 80 students; 25 of them are girls and 55 are boys, 10 of them are rich and the remaining poor; 20 of them are fair complexioned. what is the probability of selecting a fair complexioned rich girl.

2.Two integers are selected from integers 1 through 11. If the sum is even, find the probability that both the numbers are odd.

(iii) Independent Events

LEVEL I

1. A coin is tossed thrice and all 8 outcomes are equally likely.
E : "The first throw results in head" F : "The last throw results in tail"
Are the events independent ?

2. Given $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{3}$ and $P(A \cup B) = \frac{3}{4}$. Are the events independent ?

3. If A and B are independent events, Find P(B) if $P(A \cup B) = 0.60$ and $P(A) = 0.35$.

(iv) Baye's theorem, partition of sample space and Theorem of total probability

LEVEL I

1. A bag contains 6 red and 5 blue balls and another bag contains 5 red and 8 blue balls. A ball is drawn from the first bag and without noticing its colour is put in the second bag. A ball is drawn from the second bag . Find the probability that the ball drawn is blue in colour.

2. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts . Find the probability of the lost card being a heart.

3. An insurance company insured 2000 scooter and 3000 motorcycles . The probability of an accident involving scooter is 0.01 and that of motorcycle is 0.02 . An insured vehicle met with an accident. Find the probability that the accidental vehicle was a motorcycle.

4. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled at random from one of the two purses, what is the probability that it is a silver coin.

5. Two thirds of the students in a class are boys and the rest are girls. It is known that the probability of a girl getting first class is 0.25 and that of a boy is getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

LEVEL II

1. Find the probability of drawing a one-rupee coin from a purse with two compartments one of which contains 3 fifty-paise coins and 2 one-rupee coins and other contains 2 fifty-paise coins and 3 one-rupee coins.
2. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. An orator is chosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women.
3. A company has two plants to manufacture bicycles. The first plant manufactures 60 % of the bicycles and the second plant 40 % . Out of that 80 % of the bicycles are rated of standard quality at the first plant and 90 % of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality. Find the probability that it comes from the second plant.

LEVEL III

1. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible. What is the probability that the letter has come from (i) LONDON (ii) CLIFTON ?
2. A test detection of a particular disease is not fool proof. The test will correctly detect the disease 90 % of the time, but will incorrectly detect the disease 1 % of the time. For a large population of which an estimated 0.2 % have the disease, a person is selected at random, given the test, and told that he has the disease. What are the chances that the person actually have the disease.
3. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III , there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold ? **[CBSE 2011]**

(v) Random variables & probability distribution Mean & variance of random variables

LEVEL I

1. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of spades

2. 4 defective apples are accidentally mixed with 16 good ones. Three apples are drawn at random from the mixed lot. Find the probability distribution of the number of defective apples.

3. A random variable X is specified by the following distribution

X	2	3	4
P(X)	0.3	0.4	0.3

Find the variance of the distribution.

LEVEL III

1. A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice.Find the probability distribution of the number of tails.

2.The sum of mean and variance of a binomial distribution for 5 trials be 1.8. Find the probability distribution.

3. The mean and variance of a binomial distribution are $\frac{4}{3}$ and $\frac{8}{9}$ respectively. Find $P(X \geq 1)$.

(vi) Bernoulli,s trials and Binomial Distribution

LEVEL II

1. If a die is thrown 5 times, what is the chance that an even number will come up exactly 3 times.
2. An experiment succeeds twice as often it fails. Find the probability that in the next six trials, there will be at least 4 success.

3. A pair of dice is thrown 200 times. If getting a sum 9 is considered a success, find the mean and variance of the number of success.

Questions for self evaluation

1. A four digit number is formed using the digits 1, 2, 3, 5 with no repetitions. Find the probability that the number is divisible by 5.

2. The probability that an event happens in one trial of an experiment is 0.4. Three independent trials of an experiment are performed. Find the probability that the event happens at least once.

3. A football match is either won, draw or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches.

4. A candidate has to reach the examination center in time. Probability of him going by bus ore scooter or by other means of transport are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probability that he will be late is $\frac{1}{4}$ and $\frac{1}{3}$ respectively. But he reaches in time if he uses other mode of transport. He reached late at the centre. Find the probability that he traveled by bus.

5. Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission in x number of colleges. It is given that

$$P(X = x) = \begin{cases} kx, & \text{if } x = 0, \text{ or } 1 \\ 2kx, & \text{if } x = 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \end{cases}, \text{ k is a + ve constant.}$$

Find the mean and variance of the probability distribution. 1

6. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

7. On a multiple choice examination with three possible answers(out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

8. Two cards are drawn simultaneously (or successively) from a well shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

ANSWERS

TOPIC 1 RELATIONS& FUNCTIONS

(i) Domain , Co domain & Range of a relation

LEVEL I

1. R = { (3,5),(4,4),(5,3) },Domain = {3, 4, 5}, Range = {3, 4, 5}

2. Domain = {1, 2, 3,}, Range = {8, 9, 10}

(iii).One-one , onto & inverse of a function

LEVEL I

1. - f(x) 6. $\frac{1+x}{1-x}$

LEVEL II

2. $f^{-1}(x) = \frac{(4x+7)}{2}$ **3.**6 **5.**$f^{-1}(x) = \frac{(2x-5)}{3}$

(iv).Composition of function

LEVEL II

5.$f \circ f(x) = x$ 6 . $4x^2 - 12x + 9$

(v)Binary Operations

LEVEL I

5. 15 2. 4 3. e = 5 4.50

Questions for self evaluation

2. {1, 5, 9} 3. T_1 is related to T_3 6. $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$

7. e = 0, $a^{-1} = \frac{a}{a-1}$ 8. Identity element (1 , 0), Inverse of (a , b) is $\left(\frac{1}{a}, \frac{-b}{a}\right)$

TOPIC 2 INVERSE TRIGONOMETRIC FUNCTION

1. Principal value branch Table

LEVEL I

1.$\frac{\pi}{6}$ 2.$-\frac{\pi}{6}$ 3.$-\frac{\pi}{3}$ 4.$\frac{3\pi}{4}$

LEVEL II

1. π 2. $\frac{\pi}{5}$ 3. $\frac{5\pi}{6}$

2. Properties of Inverse Trigonometric Functions

LEVEL I

1. 0 3. $\sqrt{2}$

LEVEL II

1. $\frac{1}{2}\tan^{-1} x$

LEVEL III

3. $\frac{1}{6}$ 4. $\frac{1}{4}$ 5. $\pm\frac{1}{\sqrt{2}}$

Questions for self evaluation

6. x   7. $\pm\frac{1}{\sqrt{2}}$ 8. $\frac{1}{6}$

TOPIC 3 MATRICES & DETERMINANTS

1. Order, Addition, Multiplication and transpose of matrices:

LEVEL I

1. $1\times 5,\ 5\times 1$ **2.** $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix}$ **3.** $\begin{bmatrix} -3 & -4 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ **4.** $2\times 2,\ 3\times 3$

LEVEL II

3.skew symmetric   **4.** $a = 8, b = 8$

LEVEL III.

1. $\begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & 4 \\ -3 & 2 & 0 \end{bmatrix}$ **2.** $\begin{bmatrix} 3 & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$

(ii). Cofactors &Adjoint of a matrix

LEVEL I

1. 46   2. $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

(iii)Inverse of a Matrix & Applications

LEVEL I

1. $A^{-1} = -A$ **2**. $A^{-1} = A$ **3**. $k = 17$

LEVEL II

1. $\begin{bmatrix} -2/14 & -5/14 \\ -4/14 & -3/14 \end{bmatrix}$ **3. 0**   4. $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

LEVEL III

1.$x = 1,\ y = 2,\ z = 3.$ **2**. $x = 3,\ ,y = -2, z = 1.$ **3.** $AB = 6I,\ x = 2,\ y = -1,\ z = 4$

4.$x = ½,\ y = -1,\ z = 1.$ **5.** $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

(iv). To Find The Difference Between $|A|, |adjA|, |kA|$

LEVEL I

1. $\frac{1}{2}$ **2.** 27 **3.** 24 **4.** $\frac{4}{3}$

LEVEL II

1. 8 **2.** 49

LEVEL III

1. a = 3 **2.** 125

(v). Properties of Determinants

LEVEL I

1. x = 4 **2.** $a^2 + b^2 + c^2 + d^2$

LEVEL II

2. [Hint: Apply $C_1 \to -bC_3$ and $C_2 \to aC_3$]

LEVEL III

1a. 4 **1b.** $0, 0, 3a$ **1c.** $-\frac{a}{3}$

2. HINT $\Delta = \frac{-1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (a - b)^2]$

3.[Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively]

4.[Hint : Apply $R_1 \to R_1 + R_3$ and take common a + b + c]

5.Hint : Apply $R_1 \to aR_1$, $R_2 \to bR_2$, and $R_3 \to cR_3$]

6.[Hint : Multiply R_1, R_2 and R_3 by a, b and c respectively and then take a, b, and c common from C_1, C_2 and C_3 respectively and then apply $R_1 \to R_1 + R_2 + R_3$]

Questions for self evaluation

4. $\begin{bmatrix} 3 & 3 & 5/2 \\ 3 & 5 & 7/2 \\ 5/2 & 7/2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1/2 \\ 1 & 0 & 1/2 \\ -1/2 & 1/2 & 0 \end{bmatrix}$

6. x = 8, y = 8 and $A^{-1} = \frac{1}{8}\begin{bmatrix} 5 & -1 \\ -7 & 3 \end{bmatrix}$

8. $A^{-1} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$, x = 2, y = 1, z = 3

9. $AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$, x = 3, y = – 2, z = –1

10. $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

TOPIC 4 CONTINUITY AND DIFFRENTIABILITY

2.Continuity

LEVEL-I

1.Continuous   2.Not Continuous

LEVEL-II

2.3/4   3.$3a - 3b = 2$ 4. Not Continuous

LEVEL-III

1. 1 [Hint: Use 1-cos $2\theta = 2sin^2\theta$]2. 1 [Hint: Use $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]
3.a = 1/2, b = 4   4. K = 2

3.Differentiation

LEVEL- I

1.Not Differentiable   2. $\frac{2}{1+x^2}$

3. $\frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left(\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{(6x+4)}{3x^2+4x+5}\right)$

LEVEL-II

1. $2\log x\sin(\log x)^2/x$ 2.$\frac{1}{2(1+x^2)}$ 4.$-\frac{3}{2a}\left(\frac{1+t}{1-t}\right)^3$

LEVEL-III

1 .$\frac{x}{\sqrt{1-x^4}}$ [hint: Put $x^2 = \cos 2\theta$]   2. $\frac{1}{2}$[Hint: use $1 \pm sinx = (\cos\frac{x}{2} \pm \sin\frac{x}{2})^2$]

4.Logrithmic Differentiation

LEVEL-I

1.y′ = 1/(xlogxlog7)   2. $\frac{\cos(\log x)}{x}$

3. $\frac{dy}{dx} = \frac{1}{x(1+(\log x)^2)}$ [Hint: Use log(ex) = loge+logx=1+logx]

LEVEL-II

2. $2\log x\sin(\log x)^2/x$ 3. $\frac{y\tan x+\log\cos y}{x\tan y+\log\cos x}$

LEVEL-III

2 $\frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x\log(\log x)\right] - \frac{4x}{(x^2-1)^2}$ 4. $x^{\cot x}(\cot x + x\log\sin x) + \frac{2x^2+14x+3}{(x^2+x+2)^2}$

(v) Parametric differentiation

2. $\frac{2\sqrt{2}}{a}$

Questions for self evaluation

1. k = 5 2.a = 3, b = 2, 4. $a = -\frac{3}{2}, c = \frac{1}{2}, b \in R$

5. $\left[\frac{dy}{dx}\right]_{\theta=\pi/4} = 1$ 6. $(\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x.\log(\log x)\right] - \frac{4x}{(x^2-1)^2}$

7. $\frac{y-\sec^2 x}{1-x-2y}$ 9. [Hint: Put $x = \sin\theta$; $y = \sin\varphi$] 10. $\frac{y\tan x + \log\cos y}{x\tan y + \log\operatorname{cox}}$

TOPIC 5 APPLICATIONS OF DERIVATIVES

1. Rate of change

LEVEL I 1. $\frac{27\pi}{8}(2x+1)^2$ 2.64 cm^2/min 3. 4.4 cm/sec

LEVEL II 1.(2 , 4) 2. 9 km/h 3.8 cm^2/sec

LEVEL III 1. (4 , 11) and $\left(-4, \frac{-31}{3}\right)$ 2. $\frac{4}{45}$ πcm/sec 3. $\frac{1}{10}$ πcm/sec

2. Increasing & decreasing functions

LEVEL I 3.(0, 3π/4) U (7π/4, 2π) and (3π/4, 7π/4)

LEVEL II 1. (0 ,π) 3. (0 , e) and (e, ∞)

LEVEL III 1. (- ½ , 0) U (½ , ∞) &(-∞ , -1/2) U (0,1/2)

3. Tangents &normals

LEVEL I 1. x + 3y - 8 = 0 & x + 3y + 8 =0 2. (0 , 0)

3. (1 , 0) & (1 , 4)

LEVEL II 1. $2x + 3my - 3am^4 - 2am^2 = 0$ 2. (3 , 45) & (–3 , 27)

3. x + 14y – 254 = 0 & x + 14y + 86 = 0

LEVEL III 1.80x – 40y – 103 = 0 3. $a^2 = b^2$[Hint: Use $m_1m_2 = -1$]

4. Approximations

LEVEL I 1. 5.03 2. 4.042 3.0.2867 4. 7.036

LEVEL II 1. 2.16 π cm

5 Maxima & Minima

LEVEL I 1.1 & 5 3. 12, 12

LEVEL II 2. $\frac{112}{\pi+4}$ cm , $\frac{28\pi}{\pi+4}$ cm. 3. Length = $\frac{20}{\pi+4}$ m , bredth $\frac{10}{\pi+4}$ m .

LEVEL III 1. $\frac{3\sqrt{3}}{4}$ ab 3. $\frac{4(6+\sqrt{3})}{11}$ m , $\frac{30-6\sqrt{3}}{11}$

Questions for self evaluation

1. $\frac{1}{48\pi}$ cm/s 2. $b\sqrt{3}\text{cm}^2/\text{s}$ 3. $\uparrow$ in $(-2,-1)$ and $\downarrow$ in $(-\infty,-2)\cup(-1,\infty)$

4. $\uparrow$ in $\left[0,\frac{\pi}{4}\right)\cup\left(\frac{5\pi}{4},2\pi\right]$ and $\downarrow$ in $\left(\frac{\pi}{4},\frac{5\pi}{4}\right)$ 5. (0 , 0) , (1 , 2), (– 1, – 2)

6. (a) y – 2x – 3 = 0 , (b) 36y + 12x – 227 = 0 8.(i) 2.962 (ii) 2.962

10. $\frac{200}{7}$ m

TOPIC 6 INDEFINITE& DEFINITE INTEGRALS

(i) Integration by substitution

LEVEL I 1. $\tan(\log_e x) + C$ 2. $\frac{1}{m}e^{m\tan^{-1}x} + C$ 3. $e^{\sin^{-1}x} + C$

LEVEL II 1. $2\log_e\left|1+\sqrt{x}\right| + C$ 2. $\frac{1}{3}\sec^{-1}x^3 + C$ 3. $\log_e\left|1-e^x\right| + C$

LEVEL III 1. $2\sqrt{\tan x} + C$ 2. $-\tan^{-1}(\cos x) + C$ 3. $\frac{\tan^2 x}{2} + \log_e\left|\tan x\right| + C$

(ii)) Application of trigonometric function in integrals

LEVEL I 1. $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x + C$ 2. $\frac{1}{2}\left[x + \frac{\sin 6x}{6}\right] + C$

3. $\frac{x}{4} + \frac{1}{4}\sin 6x + \frac{1}{16}\sin 4x + \frac{1}{8}\sin 2x + C$

LEVEL II 1. $\frac{1}{4}\sec^4 x + C$ OR $\frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + C$ 2. $\frac{2}{3}\sin 3x + 2\sin x + C$

LEVEL III 1. $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C$ 2. $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

(iii) Integration using Standard results

LEVEL I 1. $\frac{1}{2}\log_e\left|x + \frac{1}{2}\sqrt{4x^2-9}\right| + C$ 2. $\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right) + C$ 3. $\frac{1}{9}\tan^{-1}\left(\frac{3x+2}{3}\right) + C$

LEVEL II 1. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + C$ 2. $\tan^{-1}(\sin x + 2) + C$ 3. $\sin^{-1}\left(\frac{2x-1}{5}\right) + C$

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LEVEL III $1.\ \sin^{-1}\left(\frac{2x^2-1}{5}\right)+C$ $2.\ x+\log\left|x^2-x+1\right|+\frac{2}{\sqrt{3}}\log\left|\frac{2x-1}{\sqrt{3}}\right|+C$

$3.\ \sqrt{x^2+5x+6}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+C$

$4.\ \sin^{-1}x+\sqrt{1-x^2}+C$ [Hint: Put x=cos2θ]

$5.\ 6\sqrt{x^2-9x+20}+34\log\left|\left(\frac{2x-9}{2}\right)+\sqrt{x^2-9x+20}\right|+C$

(iv) Integration using Partial Fraction

LEVEL I $1.\ \frac{1}{3}\log(x+1)+\frac{5}{3}\log(x-2)+C$ $2.\ \frac{1}{2}\log(x-1)-2\log(x-2)+\frac{3}{2}\log(x-3)+C$

$3.\ \frac{11}{4}\log\left(\frac{x+1}{x+3}\right)+\frac{5}{2(x+1)}+C$

LEVEL II 1.x – 11log(x – 1) + 16log(x – 2) + C $2.\ \frac{1}{4}\log x-\frac{1}{2x}+\frac{3}{4}\log(x+2)+C$

$3.\ \frac{3}{8}\log(x-1)-\frac{1}{2(x-1)}+\frac{5}{8}\log(x+3)+C$

LEVEL III $1.\log(x+2)-\frac{1}{2}\log\left(x^2+4\right)+\tan^{-1}\frac{x}{2}$ $2.\ \frac{\log(1-\cos x)}{6}+\frac{\log(1+\cos x)}{2}-\frac{2\log(1+2\cos x)}{3}+C$

$3.\ \frac{1}{3}\log(1+x)-\frac{1}{6}\log\left(1-x+x^2\right)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+C$ [Hint: Partial fractions]

(v) Integration by Parts

LEVEL I 1.x.tanx + logcosx + C 2.xlogx – x + C 3.e^x.logsecx + C

LEVEL II $1.\ x\sin^{-1}x+\sqrt{1-x^2}+C$ $2.\ \frac{x^3}{3}\sin^{-1}x+\frac{\left(x^2+2\right)\sqrt{1-x^2}}{9}+C$

$3.\ -\sqrt{1-x^2}\sin^{-1}x+x+C$ $4.\ 2x\tan^{-1}x-\log\left(1+x^2\right)+C$

$5.\ \frac{1}{2}(\sec x.\tan x+\log(\sec x+\tan x))+C$

LEVEL III $1.\ \frac{x}{2}[\cos(\log x)+\sin(\log x)]+C$ $2.\ \frac{e^x}{2+x}+C$ [Hint: $\int[e^x f(x)+f'(x)]dx= e^x f(x)+c$]

$3.\ \frac{x}{1+\log x}+C$

4. e^x.tanx + C $5.\ \frac{e^{2x}}{13}(3\sin 3x+2\cos 3x)+C$

(vi) Some Special Integrals

LE EL I $1.\ \frac{x\sqrt{4+x^2}}{2}+2\log\left|x+\sqrt{4+x^2}\right|+C$ $2.\ \frac{x\sqrt{1-4x^2}}{2}+\frac{1}{4}\sin^{-1}2x+C$

LEVEL II 1. $\frac{(x+2)\sqrt{x^2+4x+6}}{2}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$

2. $\frac{(x+2)\sqrt{1-4x-x^2}}{2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$

LEVEL III 1. $-\frac{1}{3}\left(1-x-x^2\right)^{3/2}+\frac{1}{8}(2x-1)\sqrt{1-x-x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)+C$

2. $\frac{1}{3}\left(x^2+x\right)^{3/2}-\frac{11}{8}(2x+1)\sqrt{x^2+x}+\frac{11}{16}\log\left[(2x+1)+2\sqrt{x^2+x}\right]+C$

(vii) Miscellaneous Questions

LEVEL II 1. $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x-1}{\sqrt{5}\tan x+1}\right|+C$ 2. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{3\tan x+1}{2\sqrt{2}}\right)+C$

3. $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+C$ 4. $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

5. $\tan^{-1}\left(\tan^2 x\right)+C$ [Hint: divide Nr. and Dr. by x^2] 6. $\frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}+4}{3}\right)+C$

LEVEL III 1. $-\frac{12}{13}x-\frac{5}{13}\log|3\cos x+2\sin x|+C$ 2. $\frac{x}{2}-\frac{1}{2}\log|\cos x-\sin x|+C$

3. $x+\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$ 4. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+C$

5. $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right|+C$

6. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+\frac{1}{2\sqrt{2}}\left|\frac{\tan x-\sqrt{2\tan x}+1}{\tan x+\sqrt{2\tan x}+1}\right|+C$

Definite Integrals

(i) Definite Integrals based upon types of indefinite integrals

LEVEL I 1. $\frac{1}{5}\log\ 6+\frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$ 2. $\frac{64}{231}$ 3. $\left[\log\frac{3}{2}-9\log\frac{5}{4}\right]$

LEVEL II 1. $5+\frac{5}{2}\left[\log\frac{3}{2}-9\log\frac{5}{4}\right]$ 2. $\frac{e^2}{4}\left(e^2-2\right)$

(ii) Definite integrals as a limit of sum

LEVEL I 1. 6 2. 12

(iii) Properties of definite Integrals

LE EL I 1. $\frac{\pi}{4}$ 2. 1 3. $\frac{\pi}{4}$

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LEVEL II 1. $\frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$ 2. $\frac{\pi^2}{4}$ 3. $\frac{\pi^2}{4}$

4. $\frac{\pi}{12}$

LEVEL III 1. $\frac{\pi}{2}$ 2. $-\frac{\pi}{2}\log 2$ 3. $\frac{\pi}{8}\log 2$

(iv) Integration of modulus function

LEVEL III 1. $\frac{19}{2}$ 2. $\frac{11}{4}$ 3. 4

Questions for self evaluation

1. $\log\left|x^2+3x-18\right|-\frac{2}{3}\log\left|\frac{x-3}{x+6}\right|+c$

2. $-3\sqrt{5-2x-x^2}-2\sin^{-1}\left(\frac{x+1}{\sqrt{6}}\right)+c$

3. $\left[\frac{1}{8}\left(3x+2\sin 2x+\frac{\sin 4x}{4}\right)+c\right]$

4. $\tan^{-1}\left(1+\tan\frac{x}{2}\right)+c$

5. $\frac{18}{15}x+\frac{1}{25}\log\left|3\sin x+4\cos x\right|+c$

6. $x-\sqrt{1-x^2}\sin^{-1}x+c$

7. $\frac{64}{231}$

8. $\frac{3}{\pi}+\frac{1}{\pi^2}$

9. $-\frac{\pi}{2}\log 2$

10. 19/2

TOPIC 7 APPLICATIONS OF INTEGRATION

(i)Area under ***Simple Curves***

LEVEL I 1. 20π Sq. units 2. 6π Sq. units

(ii) Area of the region enclosed between ***Parabola and line***

LEVEL II 1. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ Sq. units 2. $\frac{32-8\sqrt{2}}{3}$ Sq. units

LEVEL III 1. $\frac{23}{6}$ Sq. units

(iii) Area of the region enclosed between ***Ellipse and line***

LEVEL II 1. $5(\pi-2)$ Sq. units

(iv) Area of the region enclosed between***Circle and line***

LEVEL II 1. 4π Sq. units

LEVEL III 1. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ Sq. units

( ) Area of the region enclosed between ***Circle and parabola***

LEVEL III 1. $2\left(4\pi - \frac{4\sqrt{3}}{3}\right)$ Sq. units 2. $\frac{4}{3}(8+3\pi)$ Sq. units

(vi) Area of the region enclosed between ***Two Circles***

LEVEL III 1. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ Sq. units

(vii) Area of the region enclosed between ***Two parabolas***

LEVEL II 1. 4 Sq. units

(viii) Area of triangle ***when vertices are given***

LEVEL III 1.4 Sq. units 2. $\frac{15}{2}$ Sq. units

(ix) Area of triangle ***when sides are given***

LEVEL III 1.8 Sq. units 2. 6 Sq. units

(x) ***Miscellaneous Questions***

LEVEL III 1. $\frac{1}{2}$ Sq. units 2. $\frac{1}{2}$ Sq. units

3. $\left(2-\sqrt{2}\right)$ Sq. units 4. 2 Sq. units

5. $\frac{64}{3}$ Sq. units 6. 9 Sq. units

Questions for self evaluation

1. $\frac{9}{8}$ sq. units 2. $\frac{1}{3}$ sq. units 3. $\frac{23}{6}$ sq. units

4. $\frac{3}{4}(\pi-2)$ sq. units 5. $\left(\frac{\pi}{4}-\frac{1}{2}\right)$ sq. units 6. $\frac{4}{3}(8+3\pi)$ sq. units

7. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units 8. 4 sq. units

TOPIC 8 DIFFERENTIAL EQUATIONS

1. *Order and degree of a differential equation*

LEVEL I 1.order 2 degree 2

3. *Formation of differential equation*

LEVEL II 1. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

LEVEL III 1. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$ 2. $y^2 - 2xy\frac{dy}{dx} = 0$ [Hint: $y^2=4ax$]

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4. *Solution of differential equation by the method of separation of variables*

LEVEL II 1. $\log|1+y| = x + \frac{1}{2}x^2 + c$ 2. $e^y = \sin x + 1$

3. $y = \frac{1}{2}\log\left|1+x^2\right| + (\tan^{-1} x)^2 + c$

5. *Homogeneous differential equation of first order and first degree*

LEVEL II 1. $\log|x| - \log|x-y| - \frac{y}{x} + c = 0$

LEVEL III 1. $cy = \log\frac{y}{x} - 1$ 2. $\sin^{-1}\left(\frac{y}{x}\right) = \log|x| + c$ 3. $y = ce^{\frac{x^3}{3y^3}}$

4. $y + \sqrt{x^2 + y^2} = cx^2$ 5. $y = 3x^2 + cx$ 6. $y = \frac{x^3}{4} + \frac{c}{x}$

7. $y = -\frac{2}{3}x^2 + \frac{c}{x}$

6. *Linear Differential Equations*

LEVEL I 1. 1/x

LEVEL II 1. y=cosx+c cos2x 2. $\frac{y}{x+1} = \frac{1}{3}e^{3x} + c$ 3. xy = $\frac{x^2}{4}$ (2logx -1)+c

LEVEL III 1 $\tan\left(\frac{x+y}{2}\right) = x + c$ 2. $x = -y^2e^{-y} + cy^2$ 3. $-\frac{x}{y} = \log|x| + c$

4. $(x^2+1)^2 = -\tan^{-1}x + c$ [Hint: Use $\frac{dy}{dx} + Py = Q$] 5. $x = 2y^2$

Questions for self evaluation

1. Order 2, Degree not defined 2. $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

3. $x = \left(\tan^{-1} y - 1\right) + Ce^{-\tan^{-1} y}$ 4. $y + \sqrt{x^2 + y^2} = Cx^2$

5. ylogx = $\frac{-2}{x}\left(1+\log|x|\right)+C$ 6. y + 2x = $3x^2y$ [Hint: use $v = \frac{x}{y}$]

7. $2e^{\frac{x}{y}} + \log|y| = 2$

8. $y = x^2 - \frac{\pi^2}{4\sin x}$

TOPIC 9
VECTOR ALGEBRA

(i)Vector and scalars, Direction ratio and direction cosines &Unit vector

LEVEL I

1. $\frac{2}{\sqrt{17}}\hat{i} - \frac{3}{\sqrt{17}}\hat{j} - \frac{2}{\sqrt{17}}\hat{k}$ 2. $5\hat{i} - 10\hat{j} + 10\hat{k}$ 3. $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

4. $\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$ 5. $7(\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j})$

LEVEL II

1. $5(\frac{-1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k})$ 2. $\sqrt{3}$ 3. $2\hat{i} - \hat{j} + 4\hat{k}$

LEVEL III

1. 2 2. $P = \pm\frac{1}{\sqrt{3}}$ 3. Cosine of the angle with y –axis is $\frac{1}{2}$ 4. $P = \frac{2}{3}$

(ii)Position vector of a point and collinear vectors

LEVEL I

1. $4\hat{i} + \hat{j}$ 2 $\overrightarrow{CA} = -(3\hat{i} + 2\hat{j} + 7\hat{k})$

LEVEL II

1. $-3\hat{i} + 3\hat{k}$

(iii). Dot product of two vectors

LEVEL I

1. 1. $\vec{a}\cdot\vec{b} = 9$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{4}$

LEVEL II

1. $\vec{a} = 15\hat{i} - 27\hat{j} + 5\hat{k}$ 2. $\theta = \frac{\pi}{4}$ 3. $\lambda = 8$

LEVEL III

2. $\frac{\pi}{2}$ 3. (i) $\lambda = \frac{-40}{3}$ (ii) $\lambda = 6$ 4. $|\vec{x}| = 4$ 5.[Hint: Use $(\vec{a} - \vec{b}).(\vec{a} + \vec{b}) = 0$]

7. $\vec{d} = 64\hat{i} - 2\hat{j} - 28\hat{k}$ 9. $5\sqrt{2}$

(iv)Projection of a vector

LEVEL I

1. $\frac{8}{7}$ [Hint: Use projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$] 2. 0 3. $\cos^{-1}\frac{5}{7}$

4. $\frac{60}{\sqrt{114}}$

LEVEL III

1. [$\frac{1}{\sqrt{2}}\hat{i}, \frac{1}{2}\hat{j}, \frac{1}{2}\hat{k}$, $\Theta = \pi/3$] 3. $\vec{\beta}_1 = \frac{1}{2}(3\hat{i} - \hat{j})$, $\vec{\beta} = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

(vii)Cross product of two vectors

LEVEL I

1. 12 2. $19\sqrt{2}$ 3. $|\vec{x}| = 9$ 4. $p = \frac{27}{2}$

LEVEL II

1. $\lambda = -3$ 3. $\theta = \frac{\pi}{6}$

LEVEL III

1. 1 [Hint: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$] **2.** $\theta = \frac{\pi}{3}$ 3. $\vec{c} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$

5. $\left(-\frac{\hat{i}}{2} - \hat{j} + \frac{\hat{k}}{2}\right) + \frac{5}{2}(\hat{i} + \hat{j})$

(viii)Area of a triangle & Area of a parallelogram

LEVEL I

92

1. $10\sqrt{3}$ Sq. units 2. $|\vec{a} \times \vec{b}|$ 3. $\frac{\sqrt{21}}{2}$ Squnits [Hint : Use area$\Delta = \frac{1}{2}|\overrightarrow{AB}\ x\ \overrightarrow{AC}|$]

LEVEL II

3. $\frac{\sqrt{61}}{2}$.

Questions for self evaluation

1. $\lambda=1$ 2. $5\sqrt{2}$ 3. $\frac{\pi}{2}$

4. $\hat{i} + 2\hat{j} + \hat{k}$ 5. $\pm\frac{3}{\sqrt{83}}\vec{i}, \mp\frac{5}{\sqrt{83}}\vec{j}, \mp\frac{7}{\sqrt{83}}\vec{k}$

TOPIC 10THREE DIMENSIONAL GEOMETRY

(i)Direction Ratios and Direction Cosines

LEVEL I

1. $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ **2.** $\left[\text{Ans.} \frac{3}{\sqrt{77}}, -\frac{2}{\sqrt{77}}, \frac{8}{\sqrt{77}}\right]$

3. $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

LEVEL II

1. $-\frac{3}{7}, -\frac{2}{7}, \frac{6}{7}$ 2. $<-3,-2,6>$ 3. $\frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ 4. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

LEVEL I

1. $\vec{r} = (5\hat{\imath} - 4\hat{\jmath} + 6\hat{k}) + \lambda(3\hat{\imath} + 7\hat{\jmath} - 2\hat{k})$ 2. $\left[\frac{x-1}{-3} = \frac{y-2}{2} = \frac{z-3}{6}\right]$

3. $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-1}{2} = \lambda$ 4. $2x - 3y + z + 4 = 0$

(iii)plane and skew lines

LEVEL II

1. Lines are intersecting & point of intersection is (3,0,-1).

[Hint: For Coplanarity use $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 a_2 & & a_3 \\ b_1 b_2 & & b_3 \end{vmatrix}$

LEVEL III

2. Equation of AB is $\vec{r} = (4\hat{\imath} + 5\hat{\jmath} + 10\hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$.

3. Equation of BC is $\vec{r} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + 5\hat{k})$. Coordinates of D are (3,4,5).

(iv) Shortest distance between two lines

LEVEL II

1(a) $\frac{3\sqrt{2}}{2}$ *units*, 4. $\frac{8}{\sqrt{29}}$

1(b) $\frac{3}{\sqrt{19}}$ units   5. $\frac{1}{\sqrt{3}}$

3. 0   6. Vector equation $\vec{r} = (3\hat{\imath} - 4\hat{k}) + \lambda(5\hat{\imath} - 2\hat{\jmath} + 4\hat{k})$ and distance = 7.75 units

(v)Cartesian and Vector equation of a plane in space & conversion of one into another form

LEVEL I

1. x = 0   2. 12x + 4y + 3z = 12   3. $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$ 4. (a) $x + y - z = 2$ (b) $2x + 3y - 4z = 1$

LEVEL II

1. $2x + 3y - z = 20$ 2. $\vec{r} \cdot \left(\frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}\right) = 7$

3. $[r - (\hat{\imath} - 2\hat{k})] \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 0;\ x + y - z = 3$

(vi) Angle Between (i)Two lines(ii)Two planes(iii)Line & plane

LEVEL-I

1. 60^0 2. $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$ 3. $\lambda = -3$ 4. $\cos^{-1}\frac{15}{\sqrt{731}}$ 5. $\sin^{-1}\frac{8}{21}$

LEVEL-II

1. p = –3

(vii) *Distance of a point from a plane*

LEVEL-I

1. 1/3 2. (0, 0, 0) 3. $\frac{13}{7}$

4. $\frac{1}{3}$ 5. [$\sqrt{b^2+c^2}$]

LEVEL-II

1. (4 , 3, 7) 2. 6 units 3. $\frac{3\sqrt{34}}{17}$ 4. 13

LEVEL-III

1. Foot of perpendicular (-1,4,3), Image (-3,5,2), Distance $= \sqrt{6}$ units

2. 3x – 4y + 3z -19 = 0

3. x + y – z – 2 = 0 , $\frac{2}{\sqrt{3}}$ *units* , $\frac{1}{\sqrt{3}}$ units.

(viii).Equation of a plane through the intersection of two planes

LEVEL-II

1. x – y + 3z – 2 = 0 , $\frac{2\sqrt{11}}{11}$ 2. Ans. 51x + 15y – 50z + 173 = 0

3. 5x – 4y –z = 7

LEVEL-III

1. x – 2y + z = 0 3. x– 19y – 11z = 0 4. $\frac{x-2}{-6} = \frac{y-4}{-3} = \frac{z+1}{-2}$

(ix) Foot of perpendicular and image with respect to a line and plane

LEVEL-II

1. (1, -2, 7) 2. (3, 5, 9) 3. Image of the point = (0,-1,-3)

LEVEL-III

1. $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$ 2. $\left(\frac{39}{7},\frac{-6}{7},\frac{-37}{7}\right)$ 3. 12x - 4y + 3z = 169 4. (-1,4,-1)

Questions for self evaluation

1. 17x + 2y – 7z = 12

2. $\vec{r} = (\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$,

3. $\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

4.ZERO

5. $\left(\frac{39}{7}, \frac{-6}{7}, \frac{-37}{7}\right)$

8. [Hint: second line can also be written as $\frac{(x-4)}{-5} = \frac{(y-1)}{2} = \frac{(z-0)}{1}$]

TOPIC 11LINEAR PROGRAMMING

(i) LPP and its Mathematical Formulation

LEVEL I

1. $Z = 50x + 70y,\ 2x + y \geq 8,\ x + 2y \geq 10,\ x, y \geq 0$

(ii) Graphical method of solving LPP (bounded and unbounded solutions)

1. Minimum Z = – 12 at (4, 0),

2. Maximum $Z = \frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$

3. Minimum Z = 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$

(iii) Diet Problem

LEVEL II

1. Least cost = Rs.110 at x = 5 and y = 30

2. Minimum cost = Rs.6 at x = 400 and y = 200

(iv) Manufacturing Problem

LEVEL II

1.Maximum profit is Rs. 120 when 12 units of A and 6 units of B are produced

2. For maximum profit, 25 units of product A and 125 units of product B are produced

and sold.

(v) Allocation Problem

LEVEL II

1.Maximum annual income = Rs. 6,200 on investment of Rs. 40,000 on Bond A and Rs. 30,000 on Bond B.

2. A should run for 60 days and B for 30 days.

LEVEL III

1.For maximum profit, 62 executive class tickets and 188 economy class ticket should be sold.

(vi) Transportation Problem

LEVEL III

1. Minimum transportation cost is Rs. 400 when 10, 0 and 50 packets are transported from factory at A and 30, 40 and 0 packets are transported from factory at B to the agencies at P, Q and R respectively.

Questions for self evaluation

1. Minimum 155 at (0 , 5). 2. Minimum value is 5 at $\left(\frac{3}{2}, \frac{1}{2}\right)$

3. Maximum is Rs 4.60 at (0.6 , 0.4) 4.Maximum is Rs.800 at (0 , 20)

5.8 items of type A and 16 items of type B 6.1 jar of liquid and 5 cartons of dry product.

7.Rs.4,000 in Bond A and Rs.14,000 in Bond B8. Minimum cost Rs.1350 at (5 , 3)

TOPIC 12PROBABILITY

(i) Conditional Probability

LEVEL I 1. 0 2. $\frac{5}{14}$

LEVEL II 1. $\frac{2}{5}$

LEVEL III 1. $\frac{3}{4}$ and $\frac{3}{5}$

(ii)Multiplication theorem on probability

LEVEL II 1. $\frac{8}{65}$ 2.**(i)** $\frac{13}{21}$ (ii) $\frac{10}{21}$ [Hint : p(x>=1) = 1 – P(x<0)

LEVEL III 1. $\frac{5}{512}$ 2. $\frac{3}{5}$

(iii) Independent Events

LEVEL I 1.Yes 2.Yes [check: P(A∩B) = P(A).P(B)] 3. $\frac{5}{13}$

(iv) Baye's theorem, partition of sample space and Theorem of total probability

LEVEL I 1. $\frac{93}{154}$ 2. $\frac{11}{50}$ 3. $\frac{3}{4}$ 4. $\frac{19}{42}$ 5. 0.27

LEVEL II 1. $\frac{1}{2}$ 2. $\frac{2}{3}$ 3. $\frac{3}{7}$

LEVEL III 1.(i) $\frac{12}{17}$ (ii) $\frac{5}{17}$ 2.0.15 3. $\frac{2}{3}$

(v) Random variables & probability distribution , Mean & variance of random variables

LEVEL I 1.

X	0	1	2
P(X)	9/16	6/16	1/16

X	0	1	2	3
P(X)	28/57	24/57	24/285	1/285

3. 0.6

.

X	0	1	2
P(X)	9/16	6/16	1/16

2.. $\left(\frac{4}{5}+\frac{1}{5}\right)^5$ 3. $\frac{65}{81}$

(vi) Bernoulli's trials and Binomial Distribution

LEVEL II 1. $\frac{5}{16}$ 2. $\frac{496}{729}$ 3. $\frac{200}{9}$, $\frac{1600}{81}$ [Hint: mean =np, variance =npq]

Questions for self evaluation

1. $\frac{1}{4}$ 2.0.784 3. $\frac{1}{9}$

4. $\frac{9}{13}$ 5. $\frac{19}{8}$, $\frac{47}{64}$ 6. $\frac{625}{23328}$

7. $\frac{11}{243}$ 8. 1 and 1.47

SAMPLE PAPER-I

Mathematics
Class: 12

Time: 3hrs **Max.marks:100**

General Instructions:

1. All questions are compulsory
2. The question paper consists of 29 questions divided into three sections A,B and C. Section A comprises of 10 questions of 1 mark each, Section B comprises of 12 questions of 4 marks each and Section C comprises of 7 questions of 6 marks each.
3. Use of calculators is not permitted.

SECTION A

1. If A is square matrix of order 3 such that $|\text{adj } A| = 64$, find $|A|$
2. If A= $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, find ; $0<\theta<\frac{\pi}{2}$. when $A + A^t = I$.
3. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ .Find x
4. Let * be a binary operation on R given by a * b = $\frac{a^2-b}{3}$. Write the value of 3 * 4.
5. Evaluate : Sin $\{\pi/3 - \sin^{-1}(-1/2)\}$.
6. Evaluate $\int \frac{dx}{\sqrt{x}+x}$.
7. If $\int (e^{ax} + bx)\, dx = \frac{e^{4x}}{4} + \frac{3x^2}{2}$. find the values of a and b.
8. If $\vec{a} = i + j$, $\vec{b} = j + k$, find the unit vector in the direction of $\vec{a} + \vec{b}$.
9. If $|\vec{a}| = \sqrt{3}$,$|\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60. Find $(\vec{a}.\vec{b})$
10. What is the cosine of the angle which the vector $\sqrt{2}\, i + j + k$ makes with y-axis.

SECTION B

11. Prove that $\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$.

OR

Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

12 Consider

$f : R_+ \to [-$

f *is invertib*

SECTION C

23. Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

13. Show that

$$\begin{vmatrix} 1+a & 1 & \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

14. If x=a sin pt and y=b cos pt, find the value of $\frac{d^2y}{dx^2} at\, t = 0$.

15. If $y = \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ Prove that $\frac{dy}{dx} = -\frac{x}{\sqrt{1-x^2}}$

16 Find the intervals in which the function f(x) = sinx – cosx ; $0 \le x \le 2\pi$ is (i) increasing (ii) decreasing

17. Evaluate $\int \frac{x^2+4}{x^4+x^2+16} dx$.

OR

Evaluate $\int_0^{\pi/2} \frac{x+\sin x}{1+\cos x} dx$.

18. Form the differential equation representing the family of ellipse having foci on x-axis and centre at the origin.

OR

Form the differential equation of the family of circles having radii 3.

19. Find the equation of the plane determined by the points A(3,-1,2), B(5,2,4) and C(-1,-1,6) Also find the distance of the point P(6,5,9) from the plane

20. Solve the differential equation $\frac{dy}{dx} + y\sec^2 x = \tan x.\sec^2 x$; $y(0)=1$.

21. If a and b are unit vectors and θ is the angle between them, then prove that
cos θ/2 = ½ |a + b|

22. Find the probability distribution of the number of heads in a single throw of three coins .

OR

Three balls are drawn one by one without replacement from a bag containing 5 white and 4 green balls. Find the probability distribution of number of green balls drawn.

*****************00******************

24. Evaluate $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$

25. Show that the semi vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\frac{1}{3}$.

26. Using method of integration find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

OR

Using integration, find the area of the region

$$\left\{(x, y) : |x-1| \le y \le \sqrt{5-x^2}\right\}$$

27. Find the distance of the point (3,4,5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

OR

Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1,3,4) from the plane $2x - y + z + 3 = 0$.
And also find the image of the point in the plane.

28. A dealer in rural area wishes to purchase a number of sewing machines. He has only rupees 5760.00 to invest and has space for at most 20 items. Electronic sewing machines cost him rupees 360 and manually operated sewing machine rs.240 . He can sell an electronic sewing machine at a profit of rupees 22 and a manually operated sewing machine at a profit of Rs.18. Assuming that he can sell all the items he can buy, how should he invest his money in order to maximize his profit. Make it as a Linear Programming Problem and solve it graphically. Justify the values promoted for the selection of the manually operated machines.

29. A student is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
To get the probability as 1 , Which value to be promoted among students.

MARKING SCHEME

1. $|AdjA| = |A|^{n-1} \Rightarrow |A| = \pm 8$ ---------(1)
2. $\pi/3$
3. x = +6 or -6
4. 5/3
5. 1
6. $2\log\left|1+\sqrt{x}\right| + c$.
7. a = 4 and b = 3
8. i+ 2j + k / √6
9. √3
10. ½

(1 mark each for correct answer for Qs. 1 to 10)

11. Let,

$\alpha = \sin^{-1}\frac{5}{13},\ \beta = \sin^{-1}\frac{7}{25},$ (1)

$then\ \cos\alpha = \frac{12}{13}, \cos\beta = \frac{24}{25}$ (1)

$\therefore \cos(\alpha + \beta) = \frac{253}{325}$ (1)

$\therefore \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{7}{25} = \cos^{-1}\frac{253}{325}$ (1)

OR

$\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$ ½

$\Rightarrow (1-x) = \sin(\frac{\pi}{2} + 2\sin^{-1}x) = \cos(2\sin^{-1}x)$ ½

$\Rightarrow (1-x) = \cos(2\alpha)$ where $\sin^{-1}x = \alpha$ or $x = \sin\alpha$ ½

$\Rightarrow (1-x) = 1 - 2\sin^2\alpha = 1 - 2x^2,\ \therefore 2x^2 - x = 0$ 1

$\Rightarrow x(2x-1) = 0$ ½

$\therefore x = 0, \frac{1}{2}$

Since $x = \frac{1}{2}$ does not satisfy the given equation $\therefore x = 0$ 1

12. let $x_1, x_2 \in R+$ s.t $f(x_1) = f(x_2)$ → prove that $x_1 = x_2$ (1)
f is 1-1

$9x^2 + 6x - 5 = y$

$\Rightarrow (3x+1)^2 - 6 = y$

$3x + 1 = \sqrt{y+6}$

$x = \dfrac{\sqrt{(y+6)} - 1}{3} = f^{-1}(y)$

Therefore for every y $\in [-5, \infty)$, $\exists$ ($\sqrt{y+6}$ -1) / 3 $\in$ R+ s.t
f($\sqrt{y+6}$ -1) / 3) = y. f is onto. $(1\frac{1}{2})$
since f is 1-1 and onto , f is invertible $(\frac{1}{2})$

Therfore f^{-1}(x): from $[-5, \infty)$ to R+ defined as f^{-1}(x) = ($\sqrt{x+6}$ -1)/3 (1)

13.

$$\Delta = abc\begin{vmatrix} \frac{1}{a}+1 & \frac{1}{b} & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b}+1 & \frac{1}{c} \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} \quad (1)$$

$C_1 + C_2 + C_3 \Rightarrow$

$$\Delta = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)\begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 \end{vmatrix} \quad (1)$$

$$\Rightarrow \Delta = abc(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c})1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad (1)$$

$$= abc(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \quad (1)$$

14.

$\frac{dx}{dt} = ap\cos pt \qquad \frac{dy}{dt} = -bp\sin pt$ (1)

$\frac{dy}{dx} = -\frac{b}{a}\tan pt$ (1)

$\frac{d^2y}{dx^2} = -\frac{b}{a^2}\sec^3 pt$ (1)

$\left.\frac{d^2y}{dx^2}\right|_{at\ t=0} = -\frac{b}{a^2}$ (1)

15.

.Let x = cos t

$Y=\sin\left[2\tan^{-1}\sqrt{\frac{1-\cos t}{1+\cos t}}\right]=\sin\left[2\tan^{-1}\left(\tan\frac{t}{2}\right)\right]$ $(1\frac{1}{2})$

$Y=\sin t=\sqrt{1-x^2}$ (1)

$\frac{dy}{dx}=\frac{-x}{\sqrt{1-x^2}}$ $(1\frac{1}{2})$

16. f '(x)=cos(x) + sin(x)
f ' (x)=0 → tan(x) = -1 (1)
x=3π/4 and 7π/4
Intervals are [0,3π/4),(3π/4,7π/4) , (7π/4,2π] (1)
[0,3π/4)- f ' is positive, so f(x) is increasing
(3π/4,7π/4)- f ' is negative, so f(x) is decreasing
(7π/4,2π]-f ' is positive, so f(x) is increasing (2)

17. Consider

$$\int\frac{x^2+4}{x^4+x^2+16}dx=\int\frac{1+\frac{4}{x^2}}{x^2+1+\frac{16}{x^2}}dx$$ ------------(1)

$$=\int\frac{dt}{t^2+3^2}\ ,where\ t=x-\frac{4}{x}$$ ------------(1)

$$=\frac{1}{3}\tan^{-1}\frac{t}{3}+c$$ ------------(1)

$$=\frac{1}{3}\tan^{-1}\frac{x^2-4}{3x}+c$$ ------------(1)

OR

consider

$$\int_0^{\frac{\pi}{2}} \frac{x+\sin x}{1+\cos x}dx = \int_0^{\frac{\pi}{2}} \frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx \qquad \text{--------(1)}$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} x\sec^2\frac{x}{2}dx + \int_0^{\frac{\pi}{2}} \tan\frac{x}{2}dx \qquad \text{--------(1)}$$

By solving further we get given =

$$\frac{\pi}{2}\tan\frac{\pi}{4} = \frac{\pi}{2} \qquad \text{----------------(2)}$$

18. The equation of the ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (1)
$2x/a^2 + 2y/b^2 dy/dx = 0$
$(y/x)dy/dx = -b^2/a^2$ (1)
Differentiating again and getting the differential equation as
$(xy)d^2y/dx^2 + x(dy/dx)^2 - y dy/dx = 0$ (2)

OR

The equation of the family of circles is $(x-a)^2+(y-b)^2=9$ ---------------(i) ½

$$\Rightarrow 2(x-a)+2(y-b)\frac{dy}{dx}=0 \text{ or } (x-a)=-(y-b)\frac{dy}{dx} \qquad \text{----------------(ii)} \qquad 1$$

$$\Rightarrow 1+(y-b)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0 \Rightarrow (y-b)=-\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}} \qquad \text{----------------(iii)} \qquad 1$$

from (ii), $(x-a)=+\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}\cdot\frac{dy}{dx}$ ½

putting in (i) to get $\left[\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2\left(\frac{dy}{dx}\right)^2+\left[\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2=9$ ½

or $\left[\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2\left[\left(\frac{dy}{dx}\right)^2+1\right]=9 \Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3=9\left(\frac{d^2y}{dx}\right)^2$ ½

19. Equation of the plane

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0$$

$i.e., 3x-4y+3z-19=0$

(2)

Now the perp. Distance from (6,5,9)to this plane is

$$= \frac{|3.6-4.5+3.9-19|}{\sqrt{9+16+9}}$$

$$= \frac{6}{\sqrt{34}} units$$ ------------------ (2)

20. Which is in linear differential equation

For finding I.F.= $e^{\int \sec^2 x\, dx} = e^{\tan x}$ ------------------(1)

Solution y.I.F. = $\int e^{\tan x}.\tan x.(\sec(x))^2 dx + c$

(1)

=tan x. $e^{\tan x} - e^{\tan x} + c$ ----------------(1)

When x =0, y =1 $\Rightarrow c = 2$ *and writing the completed solution* -----------(1)

21. Consider $\left|\hat{a}+\hat{b}\right|^2 = 1+1+2\cos\theta$ ------------(1)

$=2(1+\cos\theta)$ -----------------------------(1)

$=4\cos^2\frac{\theta}{2}$

$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}\left|\hat{a}+\hat{b}\right|$ --------------(2)

22. Let X be the number of heads ,X=0,1,2,3

P(having head)=p=1/2 q=1/2 , n=3 .- ------------------------(1)

Now P(X=0)= $3_{C_0} p^0 q^3 = q^3 = \frac{1}{8}$ (1)

Probability distribution

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

-----------------------------------(2)

OR

Let X denotes the random variable, 'number of green balls,

X :	0	1	2	3	1
P(X) :	$\frac{5c_3}{9c_3}$	$\frac{5c_2 \cdot 4c_1}{9c_3}$	$\frac{5c_1 \cdot 4c_2}{9c_3}$	$\frac{4c_3}{9c_3}$	1
	$= \frac{5}{42}$	$\frac{10}{21}$	$\frac{5}{14}$	$\frac{1}{21}$	2

SECTION C

23.

$$Let\ B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} and\ C = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$$

$$|B| = 1 \neq 0 \quad and \quad |c| = -1 \neq 0$$

$$adj B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \ adjC = \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} \ ---------(3)$$

$$B^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \ and\ C^{-1} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

$$\therefore A = B^{-1}C^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}. \ ---------(3)$$

24.

$$I = \int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x + b^2 \sin^2 x}$$

$$I = \int_0^{\pi} \frac{(\pi - x) dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad (1)$$

$$2I = \pi \int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x} \qquad (1)$$

$$I = \pi \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$$

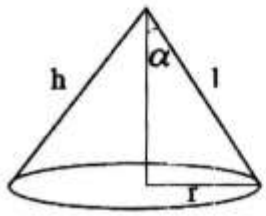
$$I = \pi \int_0^{\pi/2} \frac{\sec^2 x\, dx}{a^2 + b^2 \tan^2 x} \qquad (1)$$

For getting the answer as

$$\frac{\pi^2}{2ab}$$

(3)

25. Let r, h, l ,S and V be the radius, height, slant height. surface area and the volume of the cone.



S=

$\pi rl + \pi r^2$

$l = \dfrac{S - \pi r^2}{\pi r}$ -- (1)

$and\ V = \dfrac{1}{3}\pi r^2 h \Rightarrow V^2 = \dfrac{1}{9}\pi^2 r^4 h^2$

-- (1)

$\dfrac{dv^2}{dr} = 0$

$\dfrac{1}{9}(2rS^2 - 8S\pi r^3) = 0$

For max or min $\Rightarrow \dfrac{r}{l} = \dfrac{1}{3}$ ---------------------------(2)

$now\ \dfrac{d^2V^2}{dr^2}(at\ S = 4\pi r^2) < o$

$\therefore V^2$ is max imum.

$\therefore V$ is max imum.

---------------------------------- (1)

Now $\sin\alpha = \dfrac{r}{l} = \dfrac{1}{3}$

$\alpha = \sin^{-1}(\dfrac{1}{3}).$

-- (1)

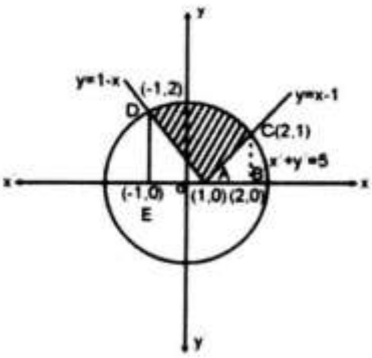
26. Let AB→ 2x + y =4
BC→ 3x-2y=6
and AC→ x-3y+5=0
Solving 1 and 2
B(2,0), C(4,3) and A(1,2) $(1\frac{1}{2})$

For the correct figure (1)

Area of triangle = $\int_1^4 \frac{x+5}{3}dx - \int_1^2 4 - 2x\,dx - \int_2^4 \frac{3x-6}{2}dx$ $(1\frac{1}{2})$

For integrating and getting area=7/2 sq.units (2)

OR



Equations of curves are

$x^2+y^2=5$ and $y=\begin{Bmatrix} 1-x, & x<1 \\ x-1, & x>1 \end{Bmatrix}$ ½

correct figure 1

Points of intersection are C(2, 1) ½

D(-1, 2)

Required Area = Area of region (EABCDE) - Area of (ADEA)- Area of (ABCA) ½

$$= \int_{-1}^{2}\sqrt{5-x^2}\,dx - \int_{-1}^{1}(1-x)dx - \int_{1}^{2}(x-1)dx -$$ 1

$$= \left[\frac{x}{2}\sqrt{5-x^2}+\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x-\frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2}-x\right]_{1}^{2}$$ 1

$$= \left[\left\{1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right\}-\left\{-\frac{1}{2}\times 2+\frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right\}\right]-\left[\left(1-\frac{1}{2}\right)-\left(-1-\frac{1}{2}\right)\right]$$
$$-\left[(2-2)-\left(\frac{1}{2}-1\right)\right]$$ 1

$= 1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}+1-\frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)-2-\frac{1}{2}$

$= -\frac{1}{2}+\frac{5}{2}\left[\text{Sin}^{-1}\frac{2}{\sqrt{5}}-\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right]$ ½

27. Line is x/1=y/2=z/2 -------------------- (1)

Line PQ through P(3,4,5)and II to the given line is -------------------- (1)
(x-3)/1=(y-4)/2 =(z-5)/2= λ
General point on the line is $Q(\lambda+3, 2\lambda+4, 2\lambda+5)$
If this point lies on the plane x+y+z=2,

$\therefore \lambda+3+2\lambda+4+2\lambda+5=2$ -------------------- (2)

$\Rightarrow \lambda=-2$

$\therefore Q(1,0,1)$ -------------------- (2)

$\therefore PQ=\sqrt{(3-1)^2+(4-0)^2+(5-1)^2}$

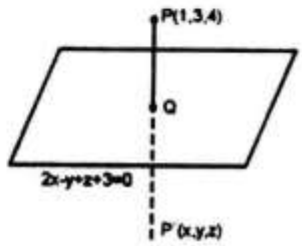
$=6.$

OR

Let Q be the foot of perpendicular from P to the plane and P' (x, y, z) be the image of P in the plane.

$\therefore$ The equations of line through P and Q is

$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}$ 1



The coordinates of Q (for some value of λ) are

$(2\lambda+1, -\lambda+3, \lambda+4)$ ½

Since Q lies on the plane, $\therefore 2(2\lambda+1)-1(-\lambda+3)+(\lambda+4)+3=0$ 1

Solving to get $\lambda=-1$ ½

$\therefore$ coordinates of foot of perpendicular (Q) are (-1, 4, 3) ½

Perpendicular distance (PQ) = $\sqrt{(2)^2+(-1)^2+(1)^2} = \sqrt{6}$ units ½

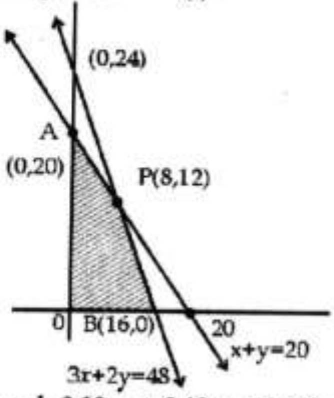
Since Q is mid point of PP'

$\therefore \frac{x+1}{2}=-1, \frac{y+3}{2}=4, \frac{z+4}{2}=3 \Rightarrow x=-3, y=5, z=2$ 1

$\therefore$ Image of P is (-3, 5, 2) 1

28. Suppose number of electronic operated machine = x and number of manually operated sewing machines = y 2

$x+y < 20$ - - - (i)



and, $360x + 240y < 5760$ or $3x+2y < 48$ - - - (ii)

$x>0$, $y>0$

To maximise $Z=22x + 18y$ 2

Corners of feasible region are A (0,20), P(8,12), B(16,0)

$Z_A = 18 \times 20 = 360$, $Z_P = 22 \times 8 + 18 \times 12 = 392$, $Z_B = 352\frac{1}{2}$

Z is maximum at x=8 and y=12

The dealer should invest in 8 electric and 12 manually operated machines (½)

Keeping the 'save environment' factor in mind the manually operated machine should be promoted so that energy could be saved. (1)

29.

Let E be the event that the student reports that 6 occurs in the throwing of die and let S_1 be the event that 6 occurs and S_2 be the event that 6 does not occur.

$P(S_1)=1/6$, $P(S_2)=5/6$, (1)

$P(E/S_1)=3/4$, $P(E/S_2)=1/4$ (2)

$$P(S_1/E)=\frac{P(S1).P(\frac{E}{S1})}{P(S1).P(\frac{E}{S1})+P(s2).P(\frac{E}{s2})}=3/8$$ (2)

To get the probability as one, the student should always speak truth.

The value to be promoted among students is truth value. (1)

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खण्ड अ

SECTION A

प्रश्न संख्या 1 से 10 तक प्रत्येक प्रश्न 1 अंक का है ।
Question numbers 1 to 10 carry 1 mark each.

1. द्विआधारी संक्रिया $* : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$, $a * b = 2a + b$ द्वारा परिभाषित है । $(2 * 3) * 4$ ज्ञात कीजिए ।

The binary operation $* : \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ is defined as $a * b = 2a + b$. Find $(2 * 3) * 4$.

2. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ का मुख्य मान ज्ञात कीजिए ।

Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

3. निम्नलिखित समीकरण से $x + y$ का मान ज्ञात कीजिए :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

Find the value of $x + y$ from the following equation :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

4. यदि $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ तथा $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ है, तो $A^T - B^T$ ज्ञात कीजिए ।

If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$.

5. माना A, 3×3 कोटि का एक वर्ग आव्यूह है । $|2A|$ का मान लिखिए, जहाँ $|A| = 4$ है ।

Let A be a square matrix of order 3×3. Write the value of $|2A|$, where $|A| = 4$.

6. मान ज्ञात कीजिए :

$$\int_0^2 \sqrt{4 - x^2}\, dx$$

Evaluate :

$$\int_0^2 \sqrt{4 - x^2}\, dx$$

7. दिया गया है कि $\int e^x (\tan x + 1) \sec x\, dx = e^x f(x) + c$ है ।

तो वह f(x) लिखिए जो उपर्युक्त को संतुष्ट करता है ।

Given $\int e^x (\tan x + 1) \sec x\, dx = e^x f(x) + c$.

Write f(x) satisfying the above.

8. $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$ का मान लिखिए ।

Write the value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$

9. सदिश $\overrightarrow{AB}$, जिसका प्रारंभिक बिन्दु A(2, 1) तथा अन्तिम बिन्दु B(– 5, 7) है, के अदिश घटक ज्ञात कीजिए ।

Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A(2, 1) and terminal point B(– 5, 7).

10. मूल बिन्दु से समतल $3x - 4y + 12z = 3$ की दूरी ज्ञात कीजिए ।

Find the distance of the plane $3x - 4y + 12z = 3$ from the origin.

खण्ड ब
SECTION B

प्रश्न संख्या 11 से 22 तक प्रत्येक प्रश्न 4 अंक का है ।
Question numbers 11 to 22 carry 4 marks each.

11. निम्न को सिद्ध कीजिए :

$$\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$$

Prove the following :

$$\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$$

12. सारणिकों के गुणधर्मों का प्रयोग कर दर्शाइए कि

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4\ abc$$

Using properties of determinants, show that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4\ abc$$

13. दर्शाइए कि $f : N \to N$, जो निम्न द्वारा प्रदत्त है,

$$f(x) = \begin{cases} x+1, & \text{यदि } x \text{ विषम है} \\ x-1, & \text{यदि } x \text{ सम है} \end{cases}$$

एकैकी तथा आच्छादक दोनों है ।

अथवा

द्विआधारी संक्रियाओं $* : R \times R \to R$ तथा $o : R \times R \to R$, जो $a * b = |a - b|$ तथा $a o b = a$, सभी $a, b \in R$ के लिए, द्वारा परिभाषित हैं, पर विचार कीजिए । दर्शाइए कि '$*$' क्रमविनिमेय है परन्तु साहचर्य नहीं है, 'o' साहचर्य है परन्तु क्रमविनिमेय नहीं है ।

Show that $f : N \to N$, given by

$$f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$$

is both one-one and onto.

OR

Consider the binary operations $* : R \times R \to R$ and $o : R \times R \to R$ defined as $a * b = |a - b|$ and $a \, o \, b = a$ for all $a, b \in R$. Show that '$*$' is commutative but not associative, 'o' is associative but not commutative.

14. यदि $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$ है, तो दर्शाइए कि $\frac{dy}{dx} = -\frac{y}{x}$.

अथवा

$\tan^{-1}\left[\frac{\sqrt{1+x^2} - 1}{x}\right]$ का x के सापेक्ष अवकलन कीजिए ।

If $x = \sqrt{a^{\sin^{-1} t}}$, $y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$.

OR

Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2} - 1}{x}\right]$ with respect to x.

15. यदि $x = a(\cos t + t \sin t)$ तथा $y = a(\sin t - t \cos t)$, $0 < t < \frac{\pi}{2}$ है, तो $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ तथा $\frac{d^2y}{dx^2}$ ज्ञात कीजिए ।

If $x = a(\cos t + t \sin t)$ and $y = a(\sin t - t \cos t)$, $0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}$, $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

16. एक 5 मी. लंबी सीढ़ी दीवार के सहारे झुकी है । सीढ़ी का नीचे का सिरा, पृथ्वी के अनुदिश, दीवार से दूर 2 सेमी/से. की दर से खींचा जाता है । दीवार पर इसकी ऊँचाई किस दर से घट रही है जबकि सीढ़ी के नीचे का सिरा दीवार से 4 मी. दूर है ?

A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall ?

17. मान ज्ञात कीजिए :

$$\int_{-1}^{2} \left| x^3 - x \right| dx$$

अथवा

मान ज्ञात कीजिए :

$$\int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

Evaluate :

$$\int_{-1}^{2} \left| x^3 - x \right| dx$$

OR

Evaluate :

$$\int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

18. द्वितीय चतुर्थांश में ऐसे वृत्तों के कुल का अवकल समीकरण ज्ञात कीजिए जो निर्देशांक अक्षों को स्पर्श करते हैं ।

अथवा

अवकल समीकरण $x(x^2 - 1) \frac{dy}{dx} = 1$ का विशिष्ट हल ज्ञात कीजिए जब $x = 2$ है, तो $y = 0$ है ।

Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

OR

Find the particular solution of the differential equation

$x(x^2 - 1)\dfrac{dy}{dx} = 1; \; y = 0$ when $x = 2$.

19. निम्नलिखित अवकल समीकरण को हल कीजिए :

$$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx; \; x \neq 0$$

Solve the following differential equation :

$$(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx; \; x \neq 0$$

20. माना $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ तथा $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ है। एक सदिश $\vec{p}$ ज्ञात कीजिए जो $\vec{a}$ तथा $\vec{b}$ दोनों पर लम्ब है तथा $\vec{p} \cdot \vec{c} = 18$ है।

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$.

21. उस बिंदु के निर्देशांक ज्ञात कीजिए जहाँ बिन्दुओं A(3, 4, 1) तथा B(5, 1, 6) को मिलाने वाली रेखा XY-तल को काटती है।

Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane.

22. ताश के 52 पत्तों की एक भली-भाँति फेंटी गई गड्डी में से दो पत्ते उत्तरोत्तर (बिना प्रतिस्थापना के) निकाले जाते हैं। लाल रंग के पत्तों की संख्या का माध्य तथा प्रसरण ज्ञात कीजिए।

Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

खण्ड स
SECTION C

प्रश्न संख्या 23 से 29 तक प्रत्येक प्रश्न 6 अंक का है ।
Question numbers 23 to 29 carry 6 marks each.

23. आव्यूहों का प्रयोग करके, निम्नलिखित समीकरण निकाय को हल कीजिए :

$$2x + 3y + 3z = 5, \quad x - 2y + z = -4, \quad 3x - y - 2z = 3$$

Using matrices, solve the following system of equations :

$$2x + 3y + 3z = 5, \quad x - 2y + z = -4, \quad 3x - y - 2z = 3$$

24. सिद्ध कीजिए कि एक दिए गए शंकु के अन्तर्गत महत्तम वक्र पृष्ठ क्षेत्रफल वाले लंब-वृत्तीय बेलन की त्रिज्या, शंकु की त्रिज्या की आधी होती है ।

अथवा

दिए गए c^2 वर्ग इकाई क्षेत्रफल वाले एक गत्ते में से ऊपर से खुला एक बक्सा जिसका आधार वर्गाकार है, बनाया जाना है । दर्शाइए कि बक्से का अधिकतम आयतन $\frac{c^3}{6\sqrt{3}}$ घन इकाई है ।

Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

OR

An open box with a square base is to be made out of a given quantity of cardboard of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

25. मान ज्ञात कीजिए :

$$\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} \, dx$$

अथवा

मान ज्ञात कीजिए :

$$\int \frac{x^2 + 1}{(x - 1)^2 (x + 3)} \, dx$$

Evaluate :

$$\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}}\, dx$$

OR

Evaluate :

$$\int \frac{x^2 + 1}{(x - 1)^2 (x + 3)}\, dx$$

26. $\{(x, y) : x^2 + y^2 \leq 4,\ x + y \geq 2\}$ द्वारा प्रदत्त क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Find the area of the region $\{(x, y) : x^2 + y^2 \leq 4,\ x + y \geq 2\}$.

27. यदि रेखाएँ $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ तथा $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ लम्बवत् हैं, तो k का मान ज्ञात कीजिए । अतः उस समतल का समीकरण ज्ञात कीजिए जो इन रेखाओं को अन्तर्विष्ट करता है ।

If the lines $\frac{x-1}{-3} = \frac{y-2}{-2k} = \frac{z-3}{2}$ and $\frac{x-1}{k} = \frac{y-2}{1} = \frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines.

28. माना एक लड़की एक पासा उछालती है । यदि उसे 5 या 6 की संख्या प्राप्त होती है, तो वह एक सिक्के को तीन बार उछालती है और चितों की संख्या नोट करती है । यदि उसे 1, 2, 3 या 4 की संख्या प्राप्त होती है, तो वह एक सिक्के को एक बार उछालती है और यह नोट करती है कि उसे चित या पट प्राप्त हुआ । यदि उसे ठीक एक चित प्राप्त होता है, तो उसके द्वारा उछाले गए पासे पर 1, 2, 3 या 4 प्राप्त होने की प्रायिकता क्या है ?

Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die ?

29. एक आहार-विज्ञानी दो प्रकार के भोज्यों को इस प्रकार मिलाना चाहता है कि मिश्रण में विटामिन A का घटक कम-से-कम 8 मात्रक और विटामिन C का घटक कम-से-कम 10 मात्रक हो । भोज्य I में 2 मात्रक विटामिन A प्रति किग्रा और 1 मात्रक विटामिन C प्रति किग्रा है जबकि भोज्य II में 1 मात्रक विटामिन A प्रति किग्रा और 2 मात्रक विटामिन C प्रति किग्रा है । प्रति किग्रा भोज्य I को खरीदने में ₹ 5 तथा प्रति किग्रा भोज्य II को खरीदने में ₹ 7 लगते हैं । उपर्युक्त को एक रैखिक प्रोग्रामन समस्या बनाकर ग्राफ़ द्वारा हल करके प्रति किग्रा मिश्रण का न्यूनतम मूल्य ज्ञात कीजिए ।

A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 5 per kg to purchase Food I and ₹ 7 per kg to purchase Food II. Determine the minimum cost of such a mixture. Formulate the above as a LPP and solve it graphically.

खण्ड – अ
SECTION – A

प्रश्न संख्या 1 से 10 तक प्रत्येक प्रश्न 1 अंक का है ।
Question numbers 1 to 10 carry 1 mark each.

1. $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान लिखिए ।

   Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

2. $\tan\left(2\tan^{-1}\frac{1}{5}\right)$ का मान लिखिए ।

   Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.

3. यदि $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$ है, तो a का मान ज्ञात कीजिए ।

   Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

4. यदि $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$ है, तो x का मान लिखिए ।

   If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

5. यदि $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$ है, तो आव्यूह A ज्ञात कीजिए ।

If $\begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix} = A + \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix}$, then find the matrix A.

6. अवकल समीकरण $x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$ की घात लिखिए ।

Write the degree of the differential equation $x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$.

7. यदि $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ तथा $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ दो समान सदिश हैं, तो $x + y + z$ का मान लिखिए ।

If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

8. यदि एक मात्रक सदिश $\vec{a}$, $\hat{i}$ के साथ $\frac{\pi}{3}$, $\hat{j}$ के साथ $\frac{\pi}{4}$ और $\hat{k}$ के साथ एक न्यूनकोण θ बनाता है, तो θ का मान ज्ञात कीजिए ।

If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ.

9. उस रेखा का कार्तीय समीकरण ज्ञात कीजिए जो बिंदु (–2, 4, –5) से होकर जाती है और रेखा $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$ के समांतर है ।

Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and is parallel to the line $\frac{x+3}{3} = \frac{4-y}{5} = \frac{z+8}{6}$.

10. किसी शहर की वायु में x-डीजल के वाहनों द्वारा जनित प्रदूषण की मात्रा में बढ़ोतरी $P(x) = 0.005x^3 + 0.02x^2 + 30x$ द्वारा प्रदत्त है । 3 डीजल के वाहनों के बढ़ने पर प्रदूषण की बढोतरी का सीमांत मान ज्ञात कीजिए तथा लिखिए कि उपरोक्त प्रश्न कौन सा मूल्य दर्शाता है ।

The amount of pollution content added in air in a city due to x-diesel vehicles is given by $P(x) = 0.005x^3 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.

खण्ड – ब
SECTION – B

प्रश्न संख्या **11** से **22** तक प्रत्येक प्रश्न के **4** अंक हैं ।
Question numbers **11** to **22** carry **4** marks each.

11. दर्शाइए कि $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$ में $f(x) = \frac{4x+3}{6x-4}$ द्वारा परिभाषित फलन f, एकैकी तथा आच्छादक है । अत: f^{-1} ज्ञात कीजिए ।

    Show that the function f in $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1}.

12. निम्न का मान ज्ञात कीजिए :

    $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $|x| < 1$, $y > 0$ तथा $xy < 1$.

    अथवा

    सिद्ध कीजिए : $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

    Find the value of the following :

    $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $|x| < 1$, $y > 0$ and $xy < 1$.

    OR

    Prove that : $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

13. सारणिकों के गुणधर्मों का प्रयोग करके, निम्न को सिद्ध कीजिए :

    $$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

    Using properties of determinants, prove the following :

    $$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2.$$

14. निम्न फलन का x के सापेक्ष अवकलन कीजिए :

    $(\log x)^x + x^{\log x}$

    Differentiate the following function with respect to x :

    $(\log x)^x + x^{\log x}$

15. यदि $y = \log\left[x + \sqrt{x^2 + a^2}\right]$ है, तो दर्शाइए कि $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

If $y = \log\left[x + \sqrt{x^2 + a^2}\right]$, show that $(x^2 + a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$.

16. दर्शाइए कि फलन $f(x) = |x - 3|, x \in \mathbb{R}, x = 3$ पर संतत है पर अवकलित नहीं है ।

अथवा

यदि $x = a \sin t$ तथा $y = a(\cos t + \log \tan {}^t/_2)$ है, तो $\frac{d^2y}{dx^2}$ ज्ञात कीजिए ।

Show that the function $f(x) = |x - 3|, x \in \mathbb{R}$, is continuous but not differentiable at $x = 3$.

OR

If $x = a \sin t$ and $y = a(\cos t + \log \tan {}^t/_2)$, find $\frac{d^2y}{dx^2}$.

17. मान ज्ञात कीजिए : $\int \frac{\sin(x - a)}{\sin(x + a)} dx$

अथवा

मान ज्ञात कीजिए : $\int \frac{5x - 2}{1 + 2x + 3x^2} dx$

Evaluate : $\int \frac{\sin(x - a)}{\sin(x + a)} dx$

OR

Evaluate : $\int \frac{5x - 2}{1 + 2x + 3x^2} dx$

18. मान ज्ञात कीजिए : $\int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$

Evaluate : $\int \frac{x^2}{(x^2 + 4)(x^2 + 9)} dx$

19. मान ज्ञात कीजिए : $\int_0^4 (|x| + |x - 2| + |x - 4|) dx$

Evaluate : $\int_0^4 (|x| + |x - 2| + |x - 4|) dx$

20. यदि दो सदिश $\vec{a}$ तथा $\vec{b}$ इस प्रकार के हैं कि $|\vec{a} + \vec{b}| = |\vec{a}|$ है, तो सिद्ध कीजिए कि सदिश $2\vec{a} + \vec{b}$ तथा सदिश $\vec{b}$ परस्पर लंबवत हैं ।

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

21. उस बिंदु के निर्देशांक ज्ञात कीजिए जहाँ रेखा $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$, समतल $x - y + z - 5 = 0$ को प्रतिच्छेद करती है । रेखा तथा समतल के बीच बनने वाला कोण भी ज्ञात कीजिए ।

अथवा

समतलों $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ और $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ की प्रतिच्छेदन रेखा को अंतर्विष्ट करने वाले तथा तल $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$ के लंबवत् तल का सदिश समीकरण ज्ञात कीजिए ।

Find the coordinates of the point, where the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ intersects the plane $x - y + z - 5 = 0$. Also find the angle between the line and the plane.

OR

Find the vector equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

22. A, 60% दशाओं में सत्य बोलता है जबकि B, 90% दशाओं में । एक ही तथ्य को कहने में वह कितनी प्रतिशत दशाओं में परस्पर खण्डन करेंगे ? क्या आप समझते हैं कि परस्पर खण्डन की स्थितियों में B के कथन को अधिक वरीयता दी जाएगी क्योंकि वह A से अधिक स्थितियों में सत्य बोलता है ?

A speaks truth in 60% of the cases, while B in 90% of the cases. In what percent of cases are they likely to contradict each other in stating the same fact ? In the cases of contradiction do you think, the statement of B will carry more weight as he speaks truth in more number of cases than A ?

खण्ड – स
SECTION – C

प्रश्न संख्या **23** से **29** तक प्रत्येक प्रश्न के **6** अंक हैं ।

Question numbers **23** to **29** carry **6** marks each.

23. एक विद्यालय अपने विद्यार्थियों को ईमानदारी, नियमितता तथा कठोर परिश्रम के मूल्यों के आधार पर ₹ 6,000 की कुल नकद राशि द्वारा पुरस्कृत करना चाहता है । मूल्य कठोर परिश्रम की पुरस्कार राशि का तीन गुना ईमानदारी के लिए पुरस्कार राशि में जोड़ने पर ₹ 11,000 बनते हैं । ईमानदारी तथा कठोर परिश्रम के मूल्यों की पुरस्कार राशियों का योग नियमितता के लिए दी गई पुरस्कार राशि का दुगुना है । उपरोक्त परिस्थिति का बीजगणितीय निरूपण कीजिए तथा आव्यूह विधि से प्रत्येक मूल्य के लिए दी गई पुरस्कार राशि ज्ञात कीजिए । उपरोक्त तीन मूल्यों, ईमानदारी, नियमितता तथा कठोर परिश्रम के अतिरिक्त एक अन्य मूल्य सुझाइए, जो आपके विचार में विद्यालय को पुरस्कार देने के लिए शामिल करना चाहिए ।

A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of ₹ 6,000. Three times the award money for Hardwork added to that given for honesty amounts to ₹ 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

24. सिद्ध कीजिए कि एक R त्रिज्या के गोले के अंतर्गत अधिकतम आयतन के बेलन की ऊँचाई $\frac{2R}{\sqrt{3}}$ है । अधिकतम आयतन भी ज्ञात कीजिए ।

अथवा

वक्र $x^2 = 4y$ के किसी बिंदु पर अभिलंब का समीकरण ज्ञात कीजिए जो बिंदु (1, 2) से हो कर जाता है । इस अभिलंब की संगत स्पर्श रेखा का समीकरण भी ज्ञात कीजिए ।

Show that the height of the cylinder of maximum volume, that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Find the equation of the normal at a point on the curve $x^2 = 4y$ which passes through the point (1, 2). Also find the equation of the corresponding tangent.

25. समाकलन के प्रयोग से, वक्र $x^2 = 4y$ एवं रेखा $x = 4y - 2$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

अथवा

समाकलन के प्रयोग से, दो वृत्तों $x^2 + y^2 = 4$ एवं $(x - 2)^2 + y^2 = 4$ के मध्यवर्ती क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

OR

Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

26. दर्शाइए कि अवकल समीकरण $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$ समघातीय है । यदि $x = 0$ जब $y = 1$ दिया हुआ है, तो इस अवकल समीकरण का विशिष्ट हल ज्ञात कीजिए ।

Show that the differential equation $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$ is homogeneous. Find the particular solution of this differential equation, given that $x = 0$ when $y = 1$.

27. उस समतल का सदिश समीकरण ज्ञात कीजिए जो उन तीन बिंदुओं से हो कर जाता है जिनके स्थिति सदिश $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ तथा $\hat{i} + 2\hat{j} + \hat{k}$ हैं । यह समतल, रेखा $\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ को जिस बिंदु पर प्रतिच्छेद करता है, उसके निर्देशांक भी ज्ञात कीजिए ।

Find the vector equation of the plane passing through three points with position vectors $\hat{i} + \hat{j} - 2\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$. Also find the coordinates of the point of intersection of this plane and the line $\vec{r} = 3\hat{i} - \hat{j} - \hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$.

28. किसानों की एक सहकारी समिति के पास दो प्रकार की फसलों A तथा B उगाने के लिए 50 हेक्टेयर भूमि है । फसलों A तथा B से क्रमश: प्रति हेक्टेयर अनुमानित लाभ ₹ 10,500 तथा ₹ 9,000 है । खरपतवार के नियंत्रण के लिए समिति एक तरल शाकनाशी दवा प्रयोग करती है जिसकी मात्रा फसलों A तथा B के लिए क्रमश: 20 लीटर तथा 10 लीटर प्रति हेक्टेयर है । उस तालाब, जो भूमि से जल निकासी करता है, में रहने वाली मछलियों तथा अन्य वन्य जीवन को बचाने के लिए, शाकनाशी का प्रयोग 800 लीटर से अधिक नहीं किया जाना चाहिए । यह मानते हुए कि लाभ कमाने की अपेक्षा मछलियों तथा अन्य वन्य जीवन की रक्षा अधिक आवश्यक है, प्रत्येक फसल के लिए कितनी-कितनी भूमि नियत की जाए कि लाभ अधिकतम हो ? उपरोक्त को एक रैखिक प्रोग्रामन समस्या बनाकर, आलेख द्वारा हल कीजिए । क्या आप इस संदेश से सहमत हैं कि पर्यावरण संतुलन के लिए वन्य जीवन का संरक्षण अति आवश्यक है ?

A cooperative society of farmers has 50 hectares of land to grow two crops A and B. The profits from crops A and B per hectare are estimated as ₹ 10,500 and ₹ 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops A and B at the rate of 20 litres and 10 litres per hectare, respectively. Further not more than 800 litres of herbicide should be used in order to protect fish and wildlife using a pond which collects drainage from this land. Keeping in mind that the *protection of fish and* other wildlife is more important than earning profit, how much land should be allocated to each crop so as to maximize the total profit ? Form an LPP from the above and solve it graphically. Do you agree with the message that the protection of wildlife is utmost necessary to preserve the balance in environment ?

29. मान लीजिए किसी रोगी को दिल का दौरा पड़ने का संयोग 40% है । यह मानते हुए कि ध्यान और योग विधि दिल का दौरा पड़ने के खतरे को 30% कम कर देती है और दवा द्वारा खतरे को 25% कम किया जा सकता है । किसी भी समय रोगी इन दोनों में से एक विकल्प का चयन समान प्रायिकता से कर सकता है । यह दिया गया है कि उपरोक्त विकल्पों से किसी एक का चुनाव करने वाले रोगियों में से यादृच्छया चुना गया रोगी दिल के दौरे से ग्रसित हो जाता है । रोगी द्वारा ध्यान और योग विधि का उपयोग किए जाने की प्रायिकता ज्ञात कीजिए । अपने उत्तर की व्याख्या कीजिए तथा बताइए कि उपरोक्त विधियों में से रोगी के लिए कौन सी विधि अधिक हितकारी है ।

Assume that the chances of a patient having a heart attack is 40%. Assuming that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga. Interpret the result and state which of the above stated methods is more beneficial for the patient.

SET-1

Series SSO

कोड नं.
Code No. **65/1/P**

रोल नं.
Roll No. | | | | | | | |

परीक्षार्थी कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Code on the title page of the answer-book.

- कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **12** हैं ।
- प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।
- कृपया जाँच कर लें कि इस प्रश्न-पत्र में **26** प्रश्न हैं ।
- **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, प्रश्न का क्रमांक अवश्य लिखें ।**
- इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।
- Please check that this question paper contains **12** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **26** questions.
- **Please write down the Serial Number of the question before attempting it.**
- 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 100*

Time allowed : 3 hours | *Maximum Marks : 100*

सामान्य निर्देश :

(i) **सभी** प्रश्न अनिवार्य हैं ।

(ii) कृपया जाँच कर लें कि इस प्रश्न–पत्र में **26** प्रश्न हैं ।

(iii) खण्ड अ के प्रश्न **1 – 6** तक अति लघु-उत्तर वाले प्रश्न हैं और प्रत्येक प्रश्न के लिए **1** अंक निर्धारित है ।

(iv) खण्ड ब के प्रश्न **7 – 19** तक दीर्घ-उत्तर I प्रकार के प्रश्न हैं और प्रत्येक प्रश्न के लिए **4** अंक निर्धारित हैं ।

(v) खण्ड स के प्रश्न **20 – 26** तक दीर्घ-उत्तर II प्रकार के प्रश्न हैं और प्रत्येक प्रश्न के लिए **6** अंक निर्धारित हैं ।

(vi) उत्तर लिखना प्रारम्भ करने से पहले कृपया प्रश्न का क्रमांक अवश्य लिखिए ।

General Instructions :

(i) ***All*** *questions are compulsory.*

(ii) *Please check that this question paper contains* ***26*** *questions.*

(iii) *Questions* ***1 – 6*** *in Section A are very short-answer type questions carrying* ***1*** *mark each.*

(iv) *Questions* ***7 – 19*** *in Section B are long-answer I type questions carrying* ***4*** *marks each.*

(v) *Questions* ***20 – 26*** *in Section C are long-answer II type questions carrying* ***6*** *marks each.*

(vi) *Please write down the serial number of the question before attempting it.*

खण्ड अ

SECTION A

प्रश्न संख्या 1 से 6 तक प्रत्येक प्रश्न का 1 अंक है ।

Question numbers 1 to 6 carry 1 mark each.

1. यदि $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ हो, तो A^{-1} लिखिए ।

If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then write A^{-1}.

2. वह अवकल समीकरण ज्ञात कीजिए जो वक्र $y = cx + c^2$ को निरूपित करता है ।

Find the differential equation representing the curve $y = cx + c^2$.

3. निम्न अवकल समीकरण का समाकलन गुणक लिखिए :

$$(1 + y^2)\, dx - (\tan^{-1} y - x)\, dy = 0$$

Write the integrating factor of the following differential equation :

$$(1 + y^2)\, dx - (\tan^{-1} y - x)\, dy = 0$$

4. $\vec{a} \cdot (\vec{b} \times \vec{a})$ का मान लिखिए ।

Write the value of $\vec{a} \cdot (\vec{b} \times \vec{a})$.

5. यदि $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ तथा $\vec{c} = 5\hat{i} - 4\hat{j} + 3\hat{k}$ है, तो $(\vec{a} + \vec{b}) \cdot \vec{c}$ का मान ज्ञात कीजिए ।

If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 5\hat{i} - 4\hat{j} + 3\hat{k}$, then find the value of $(\vec{a} + \vec{b}) \cdot \vec{c}$.

6. निम्न रेखा के दिक्-अनुपातों को लिखिए :

$$x = -3, \ \frac{y-4}{3} = \frac{2-z}{1}$$

Write the direction ratios of the following line :

$$x = -3, \ \frac{y-4}{3} = \frac{2-z}{1}$$

खण्ड ब

SECTION B

प्रश्न संख्या 7 से 19 तक प्रत्येक प्रश्न के 4 अंक हैं ।
Question numbers 7 to 19 carry 4 marks each.

7. एक अनाथालय के लिए धन एकत्रित करने हेतु, तीन विद्यालयों A, B तथा C के विद्यार्थियों ने एक प्रदर्शनी अपने मोहल्ले में आयोजित की । उन्होंने इस प्रदर्शनी में पुन: चक्रित कागज़ से बने कागज़ के थैले, स्क्रैप-पुस्तकें एवं हल्का रंगीन पेस्टल कागज़ क्रमश: ₹ 20, ₹ 15 और ₹ 5 प्रति इकाई से बेचा है । विद्यालय A ने 25 कागज़ के थैले, 12 स्क्रैप-पुस्तकें एवं 34 हल्के रंगीन पेस्टल कागज़ बेचे, विद्यालय B ने 22 कागज़ के थैले, 15 स्क्रैप-पुस्तकें एवं 28 हल्के रंगीन पेस्टल कागज़ और विद्यालय C ने 26 कागज़ के थैले, 18 स्क्रैप-पुस्तकें एवं 36 हल्के रंगीन पेस्टल कागज़ बेचे । आव्यूहों का प्रयोग करके, यह ज्ञात कीजिए कि इन विद्यार्थियों ने प्रति विद्यालय कितना धन अर्जित किया ।

इस प्रकार की प्रदर्शनी के आयोजन से विद्यार्थियों में किन मूल्यों का जनन होता है ?

To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their locality, where they sold paper bags, scrap-books and pastel sheets made by them using recycled paper, at the rate of ₹ 20, ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap-books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap-books and 28 pastel sheets while School C sold 26 paper bags, 18 scrap-books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

By such exhibition, which values are generated in the students ?

8. सिद्ध कीजिए :

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$$

अथवा

निम्नलिखित को x के लिए हल कीजिए :

$$\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4},\ |x|<1.$$

Prove that :

$$2\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left(\frac{a\cos x+b}{a+b\cos x}\right)$$

OR

Solve the following for x :

$$\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4},\ |x|<1.$$

9. यदि $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ है, तो $A^2 - 5A + 16I$ ज्ञात कीजिए ।

If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$, find $A^2 - 5A + 16I$.

10. सारणिकों के गुणधर्मों के प्रयोग से निम्नलिखित को सिद्ध कीजिए :

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$$

Using the properties of determinants, prove the following :

$$\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(1-x) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix} = 6x^2(1-x^2)$$

11. यदि $x = \alpha \sin 2t\ (1 + \cos 2t)$ तथा $y = \beta \cos 2t\ (1 - \cos 2t)$ है, तो दर्शाइए कि $\frac{dy}{dx} = \frac{\beta}{\alpha} \tan t$.

If $x = \alpha \sin 2t\ (1 + \cos 2t)$ and $y = \beta \cos 2t\ (1 - \cos 2t)$, show that $\frac{dy}{dx} = \frac{\beta}{\alpha} \tan t$.

12. ज्ञात कीजिए :

$$\frac{d}{dx} \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$$

Find :

$$\frac{d}{dx} \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$$

13. $x = 1$ पर निम्नलिखित फलन $f(x)$ का x के सापेक्ष अवकलज ज्ञात कीजिए :

$$\cos^{-1}\left[\sin\sqrt{\frac{1+x}{2}}\right] + x^x$$

Find the derivative of the following function $f(x)$ w.r.t. x, at $x = 1$:

$$\cos^{-1}\left[\sin\sqrt{\frac{1+x}{2}}\right] + x^x$$

14. मान ज्ञात कीजिए :

$$\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$$

अथवा

मान ज्ञात कीजिए :

$$\int_0^{3/2} |x \cdot \cos(\pi x)| dx$$

Evaluate :

$$\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$$

OR

Evaluate :

$$\int_0^{3/2} |x \cdot \cos(\pi x)| dx$$

15. मान ज्ञात कीजिए :

$$\int (\sqrt{\cot x} + \sqrt{\tan x}) dx$$

Evaluate :

$$\int (\sqrt{\cot x} + \sqrt{\tan x}) dx$$

16. ज्ञात कीजिए :

$$\int \frac{x^3 - 1}{x^3 + x} \, dx$$

Find :

$$\int \frac{x^3 - 1}{x^3 + x} \, dx$$

17. दर्शाइए कि चार बिन्दु A, B, C तथा D जिनके स्थिति सदिश क्रमश: $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ तथा $4(-\hat{i} + \hat{j} + \hat{k})$ हैं, समतलीय हैं ।

Show that four points A, B, C and D whose position vectors are $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$ respectively are coplanar.

18. दिखाइए कि निम्नलिखित दो रेखाएँ समतलीय हैं :

$$\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta} \text{ और } \frac{x - b + c}{\beta - \gamma} = \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma}$$

अथवा

समतल $5x - 4y + 7z - 13 = 0$ और y-अक्ष के बीच न्यून कोण ज्ञात कीजिए ।

Show that the following two lines are coplanar :

$$\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta} \text{ and } \frac{x - b + c}{\beta - \gamma} = \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \gamma}$$

OR

Find the acute angle between the plane $5x - 4y + 7z - 13 = 0$ and the y-axis.

19. A और B बारी-बारी से एक पासे को उछालते हैं जब तक कि उनमें से कोई एक पासे पर चार से बड़ी संख्या प्राप्त कर खेल को जीत नहीं लेता । यदि A खेल को शुरू करे, तो B के जीतने की प्रायिकता क्या है ?

अथवा

एक पासे को तीन बार उछालने के परीक्षण में घटना A तथा B को निम्न प्रकार से परिभाषित किया गया है :

A : पहली उछाल पर संख्या 5 और दूसरी उछाल पर संख्या 6 प्रकट होना ।

B : तीसरी उछाल पर संख्या 3 या 4 प्रकट होना ।

यदि A का घटित होना दिया गया है, तो घटना B की प्रायिकता ज्ञात कीजिए ।

A and B throw a die alternatively till one of them gets a number greater than four and wins the game. If A starts the game, what is the probability of B winning ?

OR

A die is thrown three times. Events A and B are defined as below :

A : 5 on the first and 6 on the second throw.

B : 3 or 4 on the third throw.

Find the probability of B, given that A has already occurred.

खण्ड स

SECTION C

प्रश्न संख्या 20 से 26 तक प्रत्येक प्रश्न के 6 अंक हैं ।

Question numbers 20 to 26 carry 6 marks each.

20. यदि फलन $f : R \to R$ परिभाषित है $f(x) = 2x - 3$ द्वारा तथा फलन $g : R \to R$ परिभाषित है $g(x) = x^3 + 5$ द्वारा, तो $(fog)^{-1}(x)$ का मान ज्ञात कीजिए ।

अथवा

माना कि $A = Q \times Q$, जबकि Q सभी परिमेय संख्याओं का समुच्चय है तथा $*$ एक द्विआधारी संक्रिया है जो A पर सभी $(a, b), (c, d) \in A$ के लिए $(a, b) * (c, d) = (ac, b + ad)$ द्वारा परिभाषित है, तो

(i) A में तत्समक अवयव ज्ञात कीजिए ।

(ii) A में व्युत्क्रमणीय अवयव ज्ञात कीजिए ।

If the function f : R → R be defined by f(x) = 2x − 3 and g : R → R by $g(x) = x^3 + 5$, then find the value of $(fog)^{-1}(x)$.

OR

Let A = Q × Q, where Q is the set of all rational numbers, and * be a binary operation defined on A by

(a, b) * (c, d) = (ac, b + ad), for all (a, b) (c, d) ∈ A.

Find

(i) the identity element in A.

(ii) the invertible element of A.

21. यदि फलन $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, जहाँ m > 0, p तथा q पर क्रमशः उच्चतम मान और निम्नतम मान प्राप्त करता है, जहाँ $p^2 = q$ है, तो m का मान ज्ञात कीजिए ।

If the function $f(x) = 2x^3 - 9mx^2 + 12m^2x + 1$, where m > 0 attains its maximum and minimum at p and q respectively such that $p^2 = q$, then find the value of m.

22. समाकलन विधि से, रेखाओं y = 2 + x, y = 2 − x और x = 2 से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए ।

Using integration, find the area of the region bounded by the lines y = 2 + x, y = 2 − x and x = 2.

23. ऐसी सभी सीधी रेखाओं के कुल का अवकल समीकरण ज्ञात कीजिए जो मूल-बिन्दु से मात्रक दूरी पर हैं ।

अथवा

दर्शाइए कि अवकल समीकरण $2xy\frac{dy}{dx} = x^2 + 3y^2$ समघातीय है और इसका हल ज्ञात कीजिए ।

Find the differential equation for all the straight lines, which are at a unit distance from the origin.

OR

Show that the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is homogeneous and solve it.

24. उस समतल, जो बिन्दु (1, 0, 0) व (0, 1, 0) से गुज़रता है तथा समतल $x + y = 3$ से $\frac{\pi}{4}$ का कोण बनाता है, के लम्ब के दिक्-अनुपात ज्ञात कीजिए तथा समतल का समीकरण भी ज्ञात कीजिए ।

Find the direction ratios of the normal to the plane, which passes through the points (1, 0, 0) and (0, 1, 0) and makes angle $\frac{\pi}{4}$ with the plane $x + y = 3$. Also find the equation of the plane.

25. एक महाविद्यालय के 40% विद्यार्थी छात्रावास में रहते हैं और बाकी के बाहर रहते हैं । वर्ष के अन्त में छात्रावास में रहने वाले 50% छात्र A ग्रेड (श्रेणी) में उत्तीर्ण होते हैं तथा बाहर रहने वालों में से केवल 30% छात्र ही A ग्रेड (श्रेणी) प्राप्त करते हैं । वर्ष के अन्त में एक छात्र यादृच्छया चुना जाता है और पाया जाता है कि उसने A ग्रेड (श्रेणी) प्राप्त किया है । प्रायिकता ज्ञात कीजिए कि यह छात्र छात्रावासी है ।

40% students of a college reside in hostel and the remaining reside outside. At the end of the year, 50% of the hostelers got A grade while from outside students, only 30% got A grade in the examination. At the end of the year, a student of the college was chosen at random and was found to have gotten A grade. What is the probability that the selected student was a hosteler ?

26. दीपावली के उत्सव पर, स्थानीय डाकघर का डाकपाल कुछ अतिरिक्त व्यक्तियों की सेवाएँ लेना चाहता है, क्योंकि इस समय कहीं अधिक डाकपत्रों को संभालना तथा वितरण करना होता है । ऑफिस जगह की कमी एवं वित्तीय समस्याओं के कारण वह 10 से अधिक अतिरिक्त व्यक्तियों की सेवाएँ नहीं ले सकता है । पहले के अनुभव से यह ज्ञात है कि एक पुरुष दिनभर में 300 लैटरों व 80 पैकेटों को सम्भाल सकता है, तथा एक महिला दिनभर में 400 लैटरों व 50 पैकेटों को सम्भाल सकती है । डाकपाल का यह मानना है कि प्रतिदिन कम-से-कम 3400 लैटरों व 680 पैकेटों को सम्भालना होगा । डाकपाल को प्रतिदिन ₹ 225 एक पुरुष को और ₹ 200 एक महिला को देने होंगे । ज्ञात कीजिए डाकपाल कितने पुरुष व कितनी महिलाएँ काम पर रखे कि तनख्वाह के रूप में कम-से-कम राशि देनी पड़े । इस प्रश्न को रैखिक प्रोग्रामन समस्या बनाकर ग्राफ़ द्वारा हल कीजिए ।

The postmaster of a local post office wishes to hire extra helpers during the Deepawali season, because of a large increase in the volume of mail handling and delivery. Because of the limited office space and the budgetary conditions, the number of temporary helpers must not exceed 10. According to past experience, a man can handle 300 letters and 80 packages per day, on the average, and a woman can handle 400 letters and 50 packets per day. The postmaster believes that the daily volume of extra mail and packages will be no less than 3400 and 680 respectively. A man receives ₹ 225 a day and a woman receives ₹ 200 a day. How many men and women helpers should be hired to keep the pay-roll at a minimum ? Formulate an LPP and solve it graphically.

Mathematics Class 12 Syllabus 2016-17

Course Structure

Unit	Topic	Marks
I.	Relations and Functions	10
II.	Algebra	13
III.	Calculus	44
IV.	Vectors and 3-D Geometry	17
V.	Linear Programming	6
VI.	Probability	10
	Total	**100**

Unit I: Relations and Functions

1. Relations and Functions

Types of relations: reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

Unit II: Algebra

1. Matrices

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Noncommutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2).Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. Determinants

Determinant of a square matrix (up to 3 x 3 matrices), properties of determinants, minors, co-factors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit III: Calculus

1. Continuity and Differentiability

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit functions. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretation.

2. Applications of Derivatives

Applications of derivatives: rate of change of bodies, increasing/decreasing functions, tangents and normals, use of derivatives in approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. Integrals

Integration as inverse process of differentiation.Integration of a variety of functions by substitution, by partial fractions and by parts, Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx,\ \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx,\ \int \sqrt{a^2 \pm x^2}\, dx, \int \sqrt{x^2 - a^2}\, dx$$

$$\int \sqrt{ax^2 + bx + c}\, dx, \int (px + q)\sqrt{ax^2 + bx + c}\, dx$$

Definite integrals as a limit of a sum, Fundamental Theorem of Calculus (without proof). Basic propertiesof definite integrals and evaluation of definite integrals.

4. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, circles/parabolas/ellipses (in standard form only), Area between any of the two above said curves (the region should be clearly identifiable).

5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation.Formation of differential equation whose general solution is given.Solution of differential equations by method of separation of variables solutions of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$dy/dx + py = q$, where p and q are functions of x or constants.

$dx/dy + px = q$, where p and q are functions of y or constants.

Unit IV: Vectors and Three-Dimensional Geometry

1. Vectors

Vectors and scalars, magnitude and direction of a vector.Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors, scalar triple product of vectors.

2. Three - dimensional Geometry

Direction cosines and direction ratios of a line joining two points.Cartesian equation and vector equation of a line, coplanar and skew lines, shortest distance between two lines.Cartesian and vector equation of a plane.Angle between (i) two lines, (ii) two planes, (iii) a line and a plane.Distance of a point from a plane.

Unit V: Linear Programming

1. Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded and unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

Unit VI: Probability

1. Probability

Conditional probability, multiplication theorem on probability. independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

Review of Study Material

Master Card, Chapter : 07 (Integrals)

Q1: $\boldsymbol{Evaluate.} \int \frac{sec^2 x}{3+\tan x}$**dx.**

Q. 2.Evaluate: $\int \frac{2x}{(x^2+1)(x^2+2)} dx$

Q 3. Evaluate: $\int \frac{1-x^2}{x(1-2x)} dx$

Q4. EVALUATE : $\int e^{2x}. sinx\, dx.$

Q5. Evaluate: $\int \frac{xe^x}{(x+1)^2} dx$

Q 6. Evaluate $\int \frac{1}{3x^2+13x-10} dx$

Q7. Evaluate $\int \frac{x^2+4}{x^4+x^2+16}dx.$

Q .08.Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$

Q09. Evaluate: $\int (3x-2)\sqrt{x^2+x+1}\,dx.$

Q10. Evaluate: $\int \frac{1}{cos^4x+sin^4x}dx$

Q11. Evaluate : $\int_0^{\pi/4} \log(1+\tan x)\ dx$

Q12. Evaluate: $\int_0^{\pi} \frac{x\sin x}{1+cos^2x}dx$

Q13. $Evaluate$: $\int_1^3 \{|x-1|+|x-2|+|x-3|\}\,dx$

Ans1: $\log(3+\tan x)$+ C

Ans2. $3\log|\sin x-2|$- $\frac{4}{|sinx-2|}$ + C

Ans4: $e^{2x}/5$(2sinx- cosx) +C

Ans5. $e^x f(x)+c = e^x\left(\frac{1}{x+1}\right)+c$

Ans6. $= \frac{1}{17}\log\left|\frac{3x-2}{3x+15}\right|+c$

$= \frac{1}{3}\tan^{-1}\frac{x^2-4}{3x}+c$ ————————————(1)

Ans7:

Ans8:.

= $\sqrt{x^2+5x+6}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|$+C

ANS09:

=$(X^2$+X +1)$^{3/2}$ – 7/2(X +1/2)$\sqrt{X^2+X+1}$ - 21/16LOG $\left[\left(X+\frac{1}{2}\right)+\sqrt{X^2+X+1}\right]$ +C

Ans10.

=$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{tan^2x-1}{\sqrt{2}\ tanx}\right)+C$

ANS11. $\pi/8log2$

ANS12. $\pi^2/4$

ANS13. **5 sq units**

Board Question ,**Chapter : 07 (Integrals)**

2013

1. Evaluate : $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$**;** Evaluate : $\int \frac{x+2}{\sqrt{x^2+2x+3}} dx$.; Evaluate: $\int \frac{\tan\theta + \tan^3\theta}{1+\tan^3\theta}$ d$\boldsymbol{\theta}$.

2. Evaluate: $\int \frac{1}{x(x^3+8)} dx$; Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$; Evaluate: $\int \frac{dx}{x(x^5+3)}$

3. Evaluate: $\int \frac{x^3+x+1}{x^2-1}$dx **;**Evaluate: $\int e^x \frac{(1-\sin x)}{(1-\cos x)} dx;$ Evaluate: $\int \frac{2x}{(x^2+1)(x^2+2)}$ dx

*4.*Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$, using properties of definite integral

5. $\boldsymbol{Evaluate}$**:** $\int_1^3 \{|x-1| + |x-2| + |x-3|\}\, dx$

2014

1. Evaluate: $\int \frac{1}{\cos^4 x + \sin^4 x}$ dx; Evaluate: $\int_0^{\pi} \frac{4x \sin x}{1+\cos^2 x} dx$.; Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$

2. Evaluate: $\int (\sqrt{\cot x} + \sqrt{\tan x})\, dx$; *Evaluate*: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}$dx

3. Evaluate: $\int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}$ dx.Evaluate: $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x} dx$

4. Evaluate: $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$

2015

1. Evaluate: $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$; $\int_0^{3/2} |x\cos(\pi x)|\, dx$; $\int \frac{x^3 - 1}{x^3 + x} dx$

2. Evaluate: $\int_0^{\pi/2} \frac{\cos^2 X}{1+\sin^2 x} dx$. ; $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}$dx ; $\int (3-2x)\sqrt{2+x-x^2}\, dx$

3. Evaluate : $\int_0^2 (x^2 + e^{2x+1}) dx$ as a limit of sum.

4. Evaluate : $\int_{-1}^2 (e^{3x} + 7x - 5) dx$ as a limit of sum.

5. Evaluate: $\int \frac{(x+3)}{(x+5)^3} e^x dx$; Evaluate: $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$; Find: $\int \frac{dx}{x^3(x^5+1)^{3/5}}$

6. $\int \frac{3x+1}{\sqrt{5-2x-x^2}}$dx; $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}$dx; $\int (\sqrt{\cot x} + \sqrt{\tan x})\, dx$.

7. *Evaluate*: $\int_2^4 \{|x-2| + |x-3| + |x-4|\}\, dx$

2016

Q1. $\int_0^{3/2} |x\cos(\pi x)|\, dx$;

Q2. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}$dx

Q3. Evaluate: $\int (3x+1)\sqrt{4-3x-2x^2}$ **dx.**

Q4.Evaluate: $\int \frac{x^2}{x^4 + x^2 - 2}$dx

TEST PAPER **Chapter : 07 (Integrals)**

Q1. Evaluate: $\int \frac{x^2}{1+x^3} dx$

Q2. . Evaluate: $\int_{-\pi/2}^{\pi/2} sin^5\, x\, dx$

Q3. Evaluate: $\int \frac{2}{(1-x)(1+x^2)} dx$

Q4. Evaluate: $\int_0^{\pi} \frac{x \cos x}{1+cos^2 x} dx$

Q5. Evaluate: $\int_0^{\pi/4} \log(1+tanx)\, dx$, using properties of definite integrals.

Q6.Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$

Q7.Evaluate: $\int (\sqrt{cotx} + \sqrt{tanx})\, dx$;

$Q8.\ Evaluate$: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x}$dx

Q9. Evaluate: $\int_0^{\pi/4} \frac{sinx+cosx}{9+16sin2x} dx$

Q10. Evaluate: $\int e^x \left(\frac{2+sin2x}{1+cos2x}\right) dx$

Master Card , Chapter : 08 (Application of the Integrals)

1. Find the area of the region $\{(x, y): x^2 + y^2 \leq 4, x + y \geq 2\}$ using integration.
2. Find the area of the region in the first quadrant enclosed by x-axis, the line x $= \sqrt{3}$ y and the circle $x^2 + y^2 = 4$.
3. *Sketch the graph of* $y = |x+3|$ *and evaluate the area under the curve* $y = |x+3|$ *above x*-axis
   and between x=-6 to x=0.
4. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$and the line $\frac{x}{3} + \frac{y}{2} = 1$
5. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices

   are $\mathrm{A}(-1,0), \mathrm{B}(1,3)$ and $\mathrm{C}(3,2)$.
6. Using the method of integration, find the area of the region bounded by the parabola$y^2 = 4\,x$ and the circle $4x^2 + 4y^2 = 9$.
7. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2 + y^2 = 1$.
8. Find the area bounded by the curves $x^2 = 4y$ and the straight line $x = 4y - 2$.

9. Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where a> 0.
10. Find the area of the region $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$.

ANS: Q1. $\pi - 2\ sq\ units$. Q2. $\frac{\pi}{3} sq. units$, Q3. 9 sq. unitsQ4. $3\pi/2 - 3\ sq\ units$

Q5. 4 sq units, Q6. $\frac{1}{3\sqrt{2}} + \frac{9\pi}{8} - \frac{9}{8\sin^{-1}\frac{1}{3} sq} units,$ Q7. $2\pi/3 - \frac{\sqrt{3}}{2}\ sq\ units$

Q8. 9/8 sq. units., Q9. $16a^2/3$ sq units, Q10. $\pi/4 - 1/2\ sq\ units$

Board Question ,**Chapter : 08 (Application of the Integrals)**

2013

1. Find the area of the region $\{(x , y): y^2 \le 6ax and x^2 + y^2 \le 16a^2\}$using the method of integration.
2. Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2 + y^2 = 1$.

3.Using the method of integration, find the area of the region bounded by the parabola$y^2 = 4$ x and the circle $4x^2 + 4y^2 = 9$.

2014

1. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices

are A$(-1,2)$, B$(1,5)$ and C$(3,4)$.

2. Find the area of the smaller region bounded by the ellipse$\frac{x^2}{4} + \frac{y^2}{9} = 1$and the line $\frac{x}{3} + \frac{y}{2} = 1$

2015

1. Sketch the region bounded by the curvesy =$\sqrt{5 - x^2}$ and y =$|x - 1|$ and find its area using integration.
2. Using integration find the area of the region bounded by the lines y = 2+ x, y = 2 –x and x = 2.

2016

1. Prove that the curves$y^2 = 4x$ and$x^2 = 4y$ divides the area of square bounded by x =0, x= 4, and

y=0 into three equal parts.

TEST PAPER Chapter : 07 (Application of the Integrals)

Q1. Find the area of the region enclosed by the parabola $x^2 = y$, the line y = x +2 and x- axis.

Q2. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(-1,2), B(1,5)$ and $C(3,4)$.

Q3.Compute the area bounded by the lines $x + 2y = 2$; $y - x = 1$ and $2x + y = 7$.

Q4. Find the area of the smaller region bounded by the ellipse ;

$4x^2 + 9y^2 = 36$ and the line $2x + 3y = 6$

Q5. Find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}\,y$ and the circle $x^2 + y^2 = 4$.

Q6..Find the area of the region enclosed between the two circles $x^2+y^2=1$ and $(x-1)^2 + y^2 = 1$.

MASTER CARD ON

3D GEOMETRY

Q1.Find the point on the $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ line at a distance $3\sqrt{2}$ form at Point (1,2,3). ans.(1,2,3)or (56/17,43/17,111/17)

Q2.Find the shortest distance between the line: ans. $\frac{3\sqrt{2}}{2}$ units

$\vec{r}=(1+2k)\hat{i}+(2-\lambda)\hat{j}+(\lambda+1)\hat{k}$ and

$\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$.

Q3.Find the distance between the planes ans. $\frac{7}{\sqrt{56}}$ *units*

$\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})+7=0$ and $\vec{r}.(2\hat{i}+4\hat{j}+6\hat{k})+7=0$

Q4.Find the equation of the plane determined by the point A(3,-1,2), B(5,2,4),and C(-1,-1,6). Also find the distance of the point P(6,5,9) from the plane. ans. $\frac{3\sqrt{34}}{17}$ units.

Q5.(i)Find the equation of the plane passing through the point (1,-1,2) and perpendicular to each of the planes ans.(i) 5x-4y-z=7

2x+3y-2z=5 and x+2y-3z=8.

(ii)Find the vector equation of the plane through the point (2,1,-1) and (-1,3,4)and perpendicular to the plane x-2y+4z=10.

Q6.Find Cartesian and vector equation of the plane passing through the point (3,4,1) and (0,1,0) and parallel to the line $\vec{r} = -3\hat{\imath} + 3j + 2\hat{k} + \lambda(2\hat{\imath} + 7\hat{\jmath} + 5\hat{k})$.

ans. 8x-13y+15z+13=10

Q7.Find the vector and Cartesian equation of the line passing through the point (1,2,-4) and perpendicular to the two lines

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

ans. $\vec{r} = \hat{\imath} + 2j - 4\hat{k} + (2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$.

Q8.Find the equation of the line passing through the point (3,0,1) and parallel to the plane x+2y=0 and 3y-z=0

ans. $\frac{x-3}{2} = \frac{y}{-1} = \frac{z-1}{-3}$

Q9.Find the equation of plane through the intersection of the planes $\vec{r}.(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 4$ and $\vec{r}.(2\hat{\imath} + \hat{\jmath} - \hat{k}) = 5$ and which is perpendicular to $\vec{r}.(5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 8 = 0$

ans. 33x+45y+50z=41

Q10.Find the equation the plane through the intersection of the planes $\vec{r}.(\hat{\imath} + 3\hat{\jmath}) + 6 = 0$ and $\vec{r}.(3\hat{\imath} - \hat{\jmath} + 4\hat{k}) = 0$ whose perpendicular distance from the origin is unity.

ans. 2x+y+2z+3=0 or x-2y+2z-3=0

Q11.Find the equation of the plane passing the line of intersection of the plane $\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 1$ and $\vec{}.(2 + 3\hat{\jmath} - 6\hat{k}) + 4 = 0$. and is parallel is x-ais

ans. y-3z+6=0

Q12.(i)Find the image of the point (1,3,4)in the plane x-y+z-5=0.

ans.(i)(3,1,6)

(ii)Find the coordinates at the foot of the perpendicular and the perpendicular distance of the point P(3,2,1) from the plane 2x-y+z+1=0.find also the image of the point in the plane.

Q13.(i)Find the image of the point (1,6,3) in the line $\frac{x}{1}\ \frac{y-1}{2}\ \frac{z-2}{3}$ ans.(i)(1,0,7);

Also write the equation the line joining the given point and its image and find the length of the segment joining the given point and its image.

$2\sqrt{13}$units; eq. of line $\frac{x-1}{0}=\frac{y-6}{-6}=\frac{z-3}{4}$

(ii)Find the of the foot of the perpendicular from a point (1,2,-3) with respect to the line $\frac{x+1}{2}=\frac{3-y}{2}=\frac{z}{-1}$. ans. (1,1,-1)

Q14.If the line $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{-k}=\frac{y-2}{1}=\frac{z-3}{5}$ ans. 22x-19y+31=0

Are perpendicular to each other, find the value of K and hence find the equation of the plane containing these line.

Q15.Find the coordinate of the point where the line through the points A(3,4,1)and B(5,1,6) crosses the XY-plane. ans. ($\frac{13}{5}$, $\frac{23}{5}$, 0)

Q16. Find the distance of the point (1,-2,3) from from the plane x-y+z=5 measured parallel to the line $\frac{x-1}{2}=\frac{y-3}{3}=\frac{z+2}{-6}$. ans. 1unit

Q17.Find the distance of the point (-2,3,-4)from the line $\frac{x+2}{3}\ \frac{2y+3}{4}\ \frac{3z+4}{5}$ measured parallel to the plane 4x+12y-3z+1=0. ans. $\frac{17}{2}\, units$

Q18.Check whether the following lines are intersected Or not . If intersected find the of their intersection ans. (-1,3,2) or $-\hat{\imath}+3\hat{\jmath}+2\hat{k}$

$\vec{r}=5\hat{\imath}+7\hat{\jmath}-3\hat{k}+(4\hat{\imath}+4\hat{\jmath}-5\hat{k})$.

$\vec{r}=8\hat{\imath}+4\hat{\jmath}+5\hat{k}+\mu(7\hat{\imath}+\hat{\jmath}+3\hat{k})$.

Chapter -15

Points to be remembered

1. E and F are independent events iff p (E $\cap$F) =P (E).P (F)
2. P(A/B)=P(A$\cap$B)/P(B)
3. P(A)+P(B)=1 IF A and B are complement events
4. If $E_{1,}E_{2,}............E_n$ are mutually exclusive and exhaustive events associative with event E then

   $P(E_i / E) = \frac{P(E / E_i).P(E_i)}{\sum_{i=1}^{n} P(E / E_i).P(E_i)}$ (Bayes' theorem)
5. Mean of a probability distribution-=$\sum_{i=1}^{n} p_i x_i$
6. Variance=$\sum_{i=1}^{n} X_i^2 p_i - (mean)^2$
7. Standard deviation= $\sqrt{\text{var}\, iance}$
8. prob. of r success out of n trials= $n_{c_r}.p^r q^r$ where p+q=1 and p=prob of a success in one trial

   q=prob of failure in one trial

9. Mean of binomial distribution =np

10. Variance of binomial distribution=npq,where p=prob. Of success, q=prob of failure=number of trials.

BOARD EXAMINATION

SUBJECT-MATHEMATICS M.MARKS=100

TIME-3HOURS

CLASS-XII

General instructions:
1. All questions are compulsory

2. The question paper consists of 26 questions divided into three sections A,B and C. Section A comprises 6 questions of one mark each, section B comprises 13 questions of four marks each and section C comprises 7 question of six marks each.

3. All questions in section A are to be answered in one word, one sentence or as per exact requirement of the question.

4. Use of calculators is not permitted. You may ask for logarithm tables, if required.

Section-A

1. IF $\tan^{-1}\alpha + \tan^{-1}\beta = \frac{\pi}{4}$ then write the value of $\alpha+\beta+\alpha\beta$

2. *Let* $\bullet$ be a binary operation on N given by a $\bullet$ b =H.C.F(a,b), a,b $\in$ N,Write $44\bullet 6$

3. *If* $\begin{vmatrix} \sin\alpha & -\cos\alpha \\ \sin\beta & \cos\beta \end{vmatrix} = \frac{\sqrt{3}}{2}$,where α ,β are acute angles, then write the value of $\alpha + \beta$

4. Find the value of x and y, if $3\begin{vmatrix} 2 & 3 \\ 1 & x \end{vmatrix} - \begin{vmatrix} y & 0 \\ -1 & 2 \end{vmatrix} = \begin{vmatrix} 5 & 9 \\ 4 & 10 \end{vmatrix}$

5. If A= $\begin{vmatrix} 2 & 7 \\ -4 & -2 \end{vmatrix}$, write A^{-1} in terms of A

6. *If* $\vec{x}$ is a unit vector such that $\vec{x} \times \vec{i} = \vec{k}$ find $\vec{x}.\vec{j}$

Section-B

7. A binary operation * on the set R--$\{-1\}$ is defined as

$a * b = a + b + ab$, $\forall$ a,b $\in$ R-$\{-1\}$ prove that $*$ is commutative and associative.

also find the identity element for a $\in$ R-$\{-1\}$, *if* exists.

8. Prove that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}(\frac{1}{3}) = \frac{9}{4}\tan^{-1}(2\sqrt{2})$.

9. Prove that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$ =(a-b) (b-c) (c-a) (a+b+c)

10. The surface area of a spherical bubble is increasing at the rate of $2cm^2/\sec$ .Find the rate at which the volume of bubble is increasing at the instant,when radius is 6cm.

11. *If* $y = e^{a\cos^{-1}x}$,$-1 \le x \le 1$,show that $(1-x^2)y_2 - xy_1 - a^2y = 0$.

12. Show that the function $f(x) = |x-3|$,is continuous but not differentiable at x=3.

13. Evaluate $\int x\,(\log x)^2\,dx$.

14. *show* that the equation (x+y)*dy* +(x-y)*dx* $dx = 0$ is homogeneous.Also find the particular solution, given that x=1 when y=1

15. *If* $\vec{a} = 3\vec{i} + 4\vec{j} + 5\vec{k}$ and $\vec{b} = 2\vec{i} + \vec{j} - 4\vec{k}$ then express $\vec{b}$ in the form $\vec{b} = b_1 + b_2$ where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$.

16. *Find* the equation of the plane containing the lines $\vec{r} = \vec{i} + 2\vec{j} - \vec{k} + \lambda(2\vec{i} - \vec{j} + \vec{k})$ and $\vec{r} = \vec{i} + 2\vec{j} - \vec{k} + \mu(\vec{i} - \vec{j} + 2\vec{k})$.Also find the distance of this plane from origin.

17. *Find* the distance of the point (2,2,-1) from the plane x+2y-z=1 measured parallel to the line $\frac{x+1}{2} = \frac{y+1}{2} = \frac{z}{3}$

18. *An* experiment succeeds thrice as often as it fails.Find the probability that in the next five trials there will be 3 successes.
19. *For* non zero vectors $\vec{a}$,$\vec{b}$,and $\vec{c}$, show that $\vec{a}$ -$\vec{b}$, $\vec{b}$ -$\vec{c}$, $\vec{c}$ -$\vec{a}$ are always coplanar .

Section -c

20. A,B,and C are three friends .During lunch time they went to buy three plates of chhole-bhature.Each one has amount but not equal to the price of per plate.They decided to buy together. The sum of all the amount is Rs.90.The sum of the money with A and C is added to the twice of the amount of B which results in Rs.115.A has the amount equal to the sum of the amount of B and C .Find the amount that each one has ,using the matrix method.

21. Show that of all rectangles of given area, the square has the least perimeter.

22. *Find* the area of the circle $x^2 + y^2 = 6x$, lying above x-axis and bounded by the parabola $y^2 = 3x$

23. *Show* that $\int_0^a f(x).g(x).dx = 2\int_0^a f(x).dx$ if f and g are defined as f(x)=f(a-x) and g(x)+g(a-x)=4

24. Solve the differential equation $x\frac{dy}{dx} - y = (x-1)e^x$

25. A gardener has a supply of fertilizers of type I which consists of 10% nitrogen and 6% phosphoric acid and type II which consists of 5% nitrogen and 10% phosphoric acid. After testing the soil condition, he finds that he needs at least 14 kg of nitrogen and 14kg of phosphoric acid for his crop. If the type I fertilizer costs 600 paisa per kg and type II fertilizer costs 400 paisa per kg, find how many kilograms of each fertilizer should be used so that nutrient requirements are met at a minimum cost? What is the minimum cost?

26. In a group of 900 students, 200 attend extra classes, 300 attend school regularly and 400 students study themselves with the help of peers. The probability that the student will succeed in competition who attend extra classes, attend school regularly and study themselves with the help of peers is 0.3, 0.4 and 0.2 respectively. One student is selected, who succeeded in the competition. What is the probability that he attend school regularly?

MASTER CARD QUESTIONS

Chapter-probability

1. A bag contains 5 white, 7 red and 3 black balls. If three balls are drawn one by one without replacement,find what is the probability that none is red.
2. Two integers are selected from integers 1 through 11 .if the sum is even find the prob. That both the numbers are odd.
3. A card from a pack of 52 cards is lost, from the remaining cards of pack ,two cards are drawn and found to be both hearts. Find the probability of the lost card being a heart.
4. Find the prob. Of drawing a one rupee coin from a purse with two compartments one of which contains 3 fifty-paise coins and 2 one rupee coins and other contains 2 fifty paise coins and 3 one rupee coins.

5. A letter is known to have come either from LONDON or CLIFTON. On the envelope just has two consecutive letters ON are visible.what is the prob that the letter has come from LONDON.
6. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards.find the probability distribution of number of spades.
7. A pair of dice is thrown 200 times. If getting a sum 9 nis considered a success find the mean and variance of the number of success.
8. A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third sixes in the sixth throw of the die.
9. A random variable X is specified by the following distribution

X	2	3	4
P(x)	0.3	0.4	0.3

Find the variance of the distribution.