(1) Determine k so that k+2 , 4k-6 and 3k-2 are the three consecutive terms of an AP.

a_1= k+2

a_2= 4k-6

a_3= 3k-2

a_2-a_1= a_3- a_2

4k-6-(k+2) = 3k-2-(4k-6)

4k -6-k-2 = 3k-2-4k+6

3k-8 = -k+4

3k+k = 4+8

4k = 12

k = 3

(2) if 7 times the 7th term of an AP is equal to 11 times the 11th term , show that the 18th term is zero.

Given: 7 times the 7th term of an AP is equal to 11 times the 11th term

7(a+6d) =11(a+10d)

7a+42d=11a+110d

42d-110d=11a-7a

68d = 4a

a =-17d

Now, the 18th term = a+17d=-17d-17d=0

(3) If the nth term of an A.P is 7n-5. Find 100th term

Given that the n th term of the A.P. is 7n-5

So 100 th term will be 7 (100) -5 =695

(4) if m times the mth term of an AP is equal to n times the nth term . Show that (m+n)th term of the AP is zero

We know :- $a_n = a + (n-1)d$

$a_{(m+n)} = a + (m+n-1)d$ (just put m+n in place of n) -----------------------------(1)

Let the first term and common difference of the A.P. be '*a*' and '*d*' respectively.

Then, m^{th} term = $a + (m – 1) d$ and n^{th} term = $a + (n – 1) d$

By the given condition,

$m \times a_m = n \times a_n$

$m [a + (m – 1) d] = n [a + (n – 1) d]$

$\Rightarrow ma + m (m – 1) d = na + n (n – 1) d$

=> $ma + (m^2 - m)d - na - (n^2 - n)d = 0$ (taking the Left Hand Side to the other side)

=> $ma - na + (m^2 - m)d - (n^2 - n)d = 0$ (re-ordering the terms)

=> $a (m-n) + d (m^2 - n^2 - m + n) = 0$ (taking 'a ' and 'd ' common)

=> $a (m-n) + d \{(m+n)(m-n)-(m-n)\} = 0$ ($a^2 - b^2$ identity)

Now divide both sides by (m-n)

=> $a (1) + d \{(m+n)(1)-(1)\} = 0$

=>$a + d (m+n-1) = 0$ ---------------(ii)

From equation number 1 and 2 ,

$a_{(m+n)} = a + (m+n-1)d$

And we have shown ,

$a + d (m+n-1) = 0$

So, $a_{(m+n)} = 0$

(5). Prove that the nth term of an AP cannot be $n^2 + 1$. Justify your answer.

Common difference of an A.P. must always be a constant.

∴ d cannot be $n - 1$. Here, d varies when n takes different values.

For $n = 1$, $d = 1 - 1 = 0$

For $n = 2$, $d = 2 - 1 = 1$

For $n = 3$, $d = 3 - 1 = 2$

∴ d is not constant.

Thus, d cannot be taken as $n - 1$.

a_n is the n^{th} term of an A.P. if $a_n - a_{n-1}$ = constant

Given, $a_n = n^2 + 1$

$a_n - a_{n-1} = (n^2 + 1) - [(n - 1)^2 + 1]$

$= (n^2 + 1) - (n^2 - 2n + 2)$

$= 2n - 1$

∴ $a_n - a_{n-1} \neq$ constant

Thus, $a_n = n^2 + 1$ cannot be the n^{th} term of A.P.

(6) Find the sum of the first k terms of a series whose n^{th} term is $2an+b$

The n^{th} term of the AP is given by 2an+b

a1=2a+b

a2 =4a+b

a3=6a+b

Common difference = d=(4a + b) - (2a + b) = 2a

Therefore, sum of first *k* terms =k/2[(2a+(k-1)d]= k/2[(2(2a+b)+(k-1)2a]= k/2 x 2 (2a+b+k-a)=k(a+b+ak)

(7) If S_n denotes the sum of n terms of an AP whose common difference is d and the first term is a the find -S_n - 2S_n-1 + S_n-2

iven, *a* and *d* are the first term and common difference of the A.P.

Sum of *n* term of the A.P, $S_n = \frac{n}{2}[2a+(n-1)d]$

$$S_n - 2\,S_{n-1} + S_{n+2}$$

$$= \frac{n}{2}\left[2a+(n-1)d\right] - 2\frac{(n-1)}{2}\left[2a+(n-1-1)d\right] + \frac{(n+2)}{2}\left[2a+(n+2-1)d\right]$$

$$= \frac{n}{2}\left[2a+(n-1)d\right] - \frac{2(n-1)}{2}\left[2a+(n-2)d\right] + \frac{(n+2)}{2}\left[2a+(n+1)d\right]$$

$$= \frac{1}{2}\left[2an + n(n-1)d - 4a(n-1) - 2(n-1)(n-2)d + 2a(n+2) + (n+2)(n+1)d\right]$$

$$= \frac{1}{2}\left[2a(n-2n+2+n+2) + d\left(n^2 - n - 2n^2 + 6n - 4 + n^2 + 3n + 2\right)\right]$$

$$= \frac{1}{2}\left[2a(4) + d(8n-2)\right]$$

$$= 4a + (4n-1)d$$

(8) How many terms of the arithmetic series 24 + 21 + 18 + 15 +g, be taken continuously so that their sum is – 351.

In the given arithmetic series, a = 24, d =- 3.

Let us find n such that Sn = – 351

Now, S_n = n/2[(2a + (n-1)d]

– 351 = n/2[(48 + (n-1)x(-3)]

on solving we get, n^2 - 17n -234 = 0

⇒ (*n* - 26h)(*n* + 9) = 0

⇒ *n* = 26 or *n* = - 9

Here *n,* being the number of terms needed, cannot be negative

Thus, 26 terms are needed to get the sum -351.

(9) Find the sum of the first 2*n* terms of the following series. $1^2 - 2^2 + 3^2 - 4^2$ + ...

We want to find $1^2 - 2^2 + 3^2 - 4^2$ +..... to 2*n* terms

= 1 - 4 + 9 - 16 + 25 ------------2*n* terms

= (1 – 4) +(9 – 16)+(25 – 36) + ----------- to *n* terms. (after grouping)

= -3 +(-7)+(-11)+ -------------- *n* terms

Now, the above series is in an A.P. with first term *a* = - 3 and common difference *d* = - 4

Now, S_n = n/2[(2a + (n-1)d] = = n/2[(2 x -3) + (n-1)(-4)] = -*n*(2*n* + 1).

(10) A circle is completely divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. If the smallest-of these angles is 8° and the largest 72°, calculate n and the angle in the fourth sector.

Let the common difference of the A.P. be *x*

Given: The smallest angle = 8°

⇒ *a* = 8

And the largest is 72°

⇒ a_n = 72

⇒ *a* + (*n* – 1)*d* = 72

⇒ 8 + (*n* – 1)*d* = 72

⇒ (*n* – 1) *d* = 72 – 8 = 64 ... (1)

We know that sum of all the angles of a circle is 360°

S_n = n/2[(2a + (n-1)d] = 360

⇒ S_n = n/2[(2x8 + 64] = 360

⇒ n= 9

Putting the value of *n* in equation (1) we get

(9 – 1) *d* = 64

d = 8

Now angle in fourth sector = a_4 = *a* + (4 – 1) *d*

= *a* + 3*d* = 8 + 3 × 8 = 8 + 24 = 32

∴ The value of *n* = 9 and angle in fourth sector is 32°

(11) If the sum of *n* terms of an A.P. is $3n^2 - 5n$, then which term of the A.P. is 130?

$S_n = 3n^2 - 5n$ $Put\ n = 1$ $S_1 = T_1 = 3(1)^2 - 5(1) = -2$ $Put\ n = 2$ $S_2 = 3(2)^2 - 5(2) = 2$ $T_2 = S_2 - S_1 = 2 - (-2) = 4$ $Put\ n = 3$ $S_3 = 3(3)^2 - 5(3) = 12$ $T_3 = S_3 - S_2 = 12 - 2 = 10$	Thus, we have First term, *a = -2* Common difference, *d = 4 - (-2) = 6* Let $T_n = 130$ $\Rightarrow a + (n-1)d = 130$ $\Rightarrow -2 + (n-1)6 = 130$ $\Rightarrow (n-1)6 = 132$ $\Rightarrow n - 1 = 22$ $\therefore n = 23$ Thus, 23rd term is 130.

(12) Which term of the AP, 3,10,17 will be 84 more than its 13th term?

Let the nth term be 84 more than the 13th term.

Now a/q,

a=3, d=10-3=7

So, 13th term= a+12d =3+12x7=87

Then nth term=84+87=171

171=a + (n-1)d

171=3 + (n-1)x7

171-3/7+1=n

168/7+1=n

24+1=25=n

Therefore 25th term of the ap will be 84 more than 13th term

(13) A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. if each prize is Rs 20 less than its preceding prize, find the value of each prize.

Let AP be x , (x-20) , (x-40) ,(x-60) ,(x-80),(x-100),(x-120)

$S_n = 700$, n = 7

then , $S_n = n/2(a+a_n)$

700 = 7/2 (x + x-120)

700 = 7/2(2x-120)

700 = 7x - 420

x = 160

Then the AP --- 160 , 140 , 120 , 100 , 80 , 60 , 40

(14) there are 25 trees at equal distances of 5 metres in a line with a well, the distance of the well from the nearest tree being 10 metres.a gardener waters all the trees separately starting from the well and he returns to well after watering each tree to get water for the next.find the tatal distance the gardener will cover in order to water all the trees.

Gardner is standing near the well initially and he did not return to the well after watering the last tree.

Distance covered by Gardner to water 1st tree and return to the initial position = 10 m + 10 m = 20 m

Distance covered by Gardner to water 2nd tree and return to the initial position = 15 m + 15 m = 30 m

Distance covered by Gardner to water 3^{rd} tree and return to the initial position = 20 m + 20 m = 40 m

∴Distances covered by the Gardner to water the plants are in A.P.

Here a = 20, d = 10

Distance covered to water 25^{th} tree = 20 + (25 – 1) x 10 = 20 + 240 =260

Total distance covered by the Gardner=25/2[(2x20+(25-1)x10] -260 =2470

Thus, the total distance covered by the Gardner is 2740m.

(15) if 9th term of an A.P.is zero prove that its 29th term is double the 19th term.

Let a and d be the first term and common difference of the given A.P.

n th term of A.P= a + (n– 1) d = 0

Given, 9th term of A.P =0 ⇒ a + (9 – 1) d = 0⇒ a + 8d = 0

19th term of A.P.= a + (19 – 1) d = a + 18d = a + 8d + 10d = 0 + 10d (from (1))

= = 10d(2)

∴ 29^{th} term of AP = a + (29 – 1)d = a + 28d = a + 8d + 10d+10d

= 0 + 2x10d = 20d

= 2 x 10d

=2x 19th term of A.P (from-2)

Thus, 29th term of the given A.P. is double the 19th term of the given A.P.

(16) Find a, b such that 27, a, b - 6 are in A.P.

27, a, b - 6 are in A.P.

d= t_2-t_1=t_3-t_2= a -27 =b-6-a

⇒ a+a=b-6+27

⇒2a=b+21

⇒2a-b=21

(17) For what value of n, the nth terms of the sequences 3, 10, 17,... and 63, 65, 67,... are equal.

Given, the nth terms of the sequences 3, 10, 17,... and 63, 65, 67,... are equal.

since, nth term of A.P.= a + (n – 1) d

⇒ 3+(n-1)7=63+(n-1)2

⇒ 3 + 7n-7=63 + 2n-2

7n-2n= 61+4

5*n* = 65

n = 13

Therefore, 13th terms of both these A.P.s are equal to each other.

(18)Find the sum of n terms of an A.P.whose nth terms is given by an= 5 - 6n.

We have, a_n = 5-6n

⇒ a_1 = 5 - 6 x 1 = -1

So, the given sequence is an A.P with first term a = a_1 = -1 and last term *l* = a_n = 5 - 6n

Therefore the sum of n terms is given by: S_n= n/2 (a+l)= n/2 (-1+5-6n)=2n-3n^2

(19) the digits of a positive integer, having three digits are in A.P. and their sum is 15.the number obtained by reversing the digits is 594 less than the original number. Find the number.

Let digits of the number be (*a* – *d*), *a* and (*a* + *d*) respectively.

∴ The required number is 100 (*a* – *d*) + 10*a* + (*a* + *d*) .

Given : The sum of the digits = 15

⇒ (a-d) + a + (a+d) = 15

3a=15⇒ a = 5

Now, the number on reversing the digits is 100 (*a* + *d*) + 10*a* + (*a* – *d*) .

According to the question

100(*a* – *d*) + 10*a* + *a* + *d* = 100 (*a* + *d*) + 10*a* + (*a* – *d*) + 594

on solving we get, d = -3

The digits of the number are (5 – (–3)), 5, (5 + (–3) = 8, 5, 2

And the required number is 8 × 100 + 5 × 10 + 2 = 852