

## Class 10 Arithmetic progression CBSE Test Paper- 2

Q.1. Prove that  $S_n - S_{n-1} = t_n$

Solution:  $S_n = \frac{n}{2} [2a + (n-1)d]$

$$S_{n-1} = \frac{n-1}{2} [2a + (n-1-1)d] \Rightarrow \frac{n-1}{2} [2a + (n-2)d]$$

$$\begin{aligned} \text{Now, } S_n - S_{n-1} &= \frac{n}{2} [2a + (n-1)d] - \frac{n-1}{2} [2a + (n-2)d] \\ &= \frac{n[2a + (n-1)d] - (n-1)[2a + (n-2)d]}{2} \\ &= \frac{2an + n(n-1)d - 2an - n(n-2)d + 2a + (n-2)d}{2} \\ &\Rightarrow \frac{2a + 2(n-1)d}{2} \\ &= a + (n-1)d \\ &= t_n \end{aligned}$$

Q.2. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29<sup>th</sup> term.

Solution: Given,  $n = 50$ ;  $t_n = 106$

$$t_3 = a + (3-1)d \Rightarrow 12 = a + 2d \text{ -----(i)}$$

$$t_n = 106 \Rightarrow 106 = a + 49d \text{ -----(ii)}$$

Subtract (i) from (ii),  $106 - 12 = 49d - 2d \Rightarrow 94 = 47d \Rightarrow \text{Or, } d = 2$

Put this value in (i),  $12 = a + 2 \times 2 \Rightarrow a = 12 - 4$  Hence,  $a = 8$

And,  $T_{29} = a + 28d \Rightarrow 8 + 28 \times 2 = 64$

Q. 3. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Solution: the 3rd term of an AP = 4  $\Rightarrow 4 = a + 2d$  -----(i)

The 9th term of an AP = -8  $\Rightarrow -8 = a + 8d$  -(ii)

Subtract (i) from (ii),  $-8 - 4 = 8d - 2d \Rightarrow -12 = 6d \Rightarrow d = -2$

Put this value in (i),  $-8 = a + 8 \times -2 \Rightarrow a = -8 + 16 = 8$

Let nth term of this AP is zero

$$t_n = 0 \Rightarrow a + (n - 1)d \Rightarrow 0 = 8 + (n - 1) \times -2 \Rightarrow 8 = 2(n - 1) \Rightarrow n - 1 = 4 \Rightarrow n = 5$$

So, 5th term of this AP is zero

**Q. 4. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.**

Solution: Let first term = a and common difference = d

$$t_{17} = a + 16d \text{ and } t_{10} = a + 9d$$

The 17th term of an AP exceeds its 10th term by 7

$$\Rightarrow a + 16d = a + 9d + 7 \Rightarrow a + 16d - a - 9d = 7 \Rightarrow 7d = 7 \Rightarrow d = 1$$

**Q.5. Which term of the AP: 3, 15, 27, 39, ..... will be 132 more than its 54th term?**

Solution: a = 3, d = 12,

$$54\text{th term of AP} = a + 53d = 3 + 53 \times 12 = 3 + 636 = 639$$

Let, an term will be 132 more than the 54th term

$$\Rightarrow a + (n - 1)d = 639 + 132$$

$$\Rightarrow 3 + (n - 1) \times 12 = 771$$

$$\Rightarrow (n - 1) \times 12 = 771 - 3 \Rightarrow n - 1 = \frac{768}{12} = 64 \Rightarrow n = 64 + 1 = 65$$

**Q. 6. How many three digit numbers are divisible by 7?**

Solution: Smallest three digit number divisible by 7 is 105

Greatest three digit number divisible by 7 is 994

$$a = 105 \text{ and } a_n = 994, d = 7$$

$$a_n = a + (n - 1)d \Rightarrow a_n = 994 = 105 + (n - 1)7 \Rightarrow \frac{889}{7} = n - 1 \Rightarrow n = 128$$

Q.7. How many multiples of 4 lie between 10 and 250?

Solution: Smallest number divisible by 4 after 10 is 12,

The greatest number below 250 which is divisible by 4 is 248

$$a = 12, d = 4 \text{ and } a_n = 248$$

$$\Rightarrow a + (n - 1) d = 248 \Rightarrow 12 + (n - 1) \times 4 = 248 \Rightarrow \frac{248-12}{4} = n - 1 \Rightarrow 59 + 1 = n$$

$n = 60$  Thus, 60 multiples of 4 lie between 10 and 250.

Q.8. For what value of  $n$ , are the  $n$ th terms of two APs: 63, 65, 67,... and 3, 10, 17,... equal?

Solution: in the first AP:  $a = 63$  and  $d = 2$  and in the second AP:  $a = 3$  and  $d = 7$

As per question,

$$63 + (n - 1) \times 2 = 3 + (n - 1) \times 7 \Rightarrow 63 + 2n - 2 = 3 + 7n - 7 \Rightarrow 61 + 2n = -4 + 7n$$

$$\Rightarrow 61 + 4 = 7n - 2n \Rightarrow 65 = 5n \Rightarrow n = 13$$

Therefore, 13 terms of both these A.P.s are equal to each other.

Q.9. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Solution: 7th term exceeds the 5th term by 12,

$$\Rightarrow a + 6d = a + 4d + 12 \Rightarrow 2d = 12 \Rightarrow d = 6$$

$$3^{\text{rd}} \text{ term of Ap} = 16 \Rightarrow a + 2d = 16 \Rightarrow a + 2 \times 6 = 16 - 12 = a \Rightarrow a = 4$$

AP: 4, 10, 16, 22, 28, 34, 40, 46, .....

Q. 10. Find the 20th term from the last term of the AP: 3, 8, 13, ....., 253.

$$\text{Solution: } a = 3, d = 5 \text{ and } a_n = 253 \Rightarrow 253 = 3 + (n - 1) \times 5 \Rightarrow \frac{253-3}{5} + 1 = n \Rightarrow n = 51$$

From the last:  $a = 253, d = -3, n = 20$

$$\text{So, the 20th term from the last term} = a + 19d = 253 + 19 \times -3 = 196$$

Q. 11. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and the 10th terms is 44. Find the first three terms of the AP.

Solution: The sum of the 4th and 8th terms of an AP is 24

$$\Rightarrow a + 3d + a + 7d = 24 \Rightarrow 2a + 10d = 24 \Rightarrow a + 5d = 12 \text{ -----(i)}$$

Similarly, the sum of the 6th and the 10th terms is 44  $\Rightarrow a + 5d + a + 9d = 44$

$$\Rightarrow 2a + 14d = 44 \Rightarrow a + 7d = 22 \text{ -----(ii)}$$

From (ii) - (i) we get,  $a + 7d - a - 5d = 22 - 12 \Rightarrow 2d = 10 \Rightarrow d = 5$

Put,  $d = 5$  in (i) ,  $a + 5 \times 5 = 12 \Rightarrow a = 12 - 25 = - 13$

So, first three terms of AP : -13, - 8, - 3

Q. 12. Subba Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reached Rs. 7000.?

Solution:  $a_n = a + (n - 1)d \Rightarrow 7000 = 5000 + 200 (n - 1) \Rightarrow 200 (n - 1) = 2000$

$$\Rightarrow n - 1 = 10 \Rightarrow n = 11$$

Q. 13. Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her savings become Rs. 20.75, find n.

Solution:  $a_n = a + (n - 1)d \Rightarrow 20.75 = 5 + 1.75 (n - 1) \Rightarrow 1.75 (n - 1) = 15.75$

$$\Rightarrow n - 1 = 9 \Rightarrow n = 10$$

Q.14. Given  $a_{12} = 37$ ,  $d = 3$ , find  $a$  and  $S_{12}$ .

Solution:  $a_{12} = 37 \Rightarrow a + 11d = 37 \Rightarrow a + 11 \times 3 = 37 \Rightarrow a = 37 - 33 = 4$

Or,  $S_n = \frac{n}{2}[a + l] \Rightarrow S_{12} = \frac{12}{2}[4 + 37] = 6 \times 41 = 246$

Q.15. Given  $a = 5$ ,  $d = 3$ ,  $a_n = 50$ , find  $n$  and  $S_n$ .

Solution:  $a_n = 50 \Rightarrow 5 + (n - 1) \times 3 = 50 \Rightarrow n = 16$

$$S_{16} = \frac{n}{2}[a + l] = 8 [5 + 50] = 440$$