Class-X Maths Introduction to Trigonometry Solved Problems

1. Q. Prove that: $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

$\sin^2\theta)^3 + (\cos^2\theta)^3 = (\sin^2\theta + \cos^2\theta)^3 - 3(\sin^2\theta\cos^2\theta)(\sin^2\theta + \cos^2\theta)$ [since $a + b = (a+b)^3 - 3ab(a+b)$]
$= 1 - 3\sin^2\theta\cos^2\theta$ [Since $\sin^2\theta + \cos^2\theta = 1$]

2. Q. If sin A+cosA=x , prove that $\sin^6 A + \cos^6 A = [4 - 3(x^2 - 1)]/4$

$\sin A + \cos A = x$

Squaring on both sides, we get $\sin^2 A + \cos^2 A + 2\sin A\cos A = x^2$

$\sin A\cos A = (x^2-1)/2$ ----(i)

$\sin^2\theta)^3 + (\cos^2\theta)^3 = (\sin^2\theta + \cos^2\theta)^3 - 3(\sin^2\theta\cos^2\theta)(\sin^2\theta + \cos^2\theta)$ [since $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$]
$= 1 - 3\sin^2\theta\cos^2\theta$ [Since $\sin^2\theta + \cos^2\theta = 1$]

$= 1 - 3\{(x^2 - 1)/2\}^2$

$= 1 - [3(x^2 - 1)^2]/4$

$= [4 - 3(x^2 - 1)^2]/4$

3. Q. if cosec A - sin A=a^3 , sec A - cos A=b^3 ,prove that $a^2b^2(a^2+b^2) = 1$

$a^3 = \cos^2 A/\sin A$

$\sin A = \cos^2 A/a^3$

$b^3 = \sin^2 A/\cos A = \cos^4 A/a^6 \times 1/\cos A$

so $\cos^3 A = b^3 a^6$

$\Rightarrow \cos A = b.a^2$

similarly .... $\sin A = b^2 a$

so $\sin^2 A + \cos^2 A = 1$

$= b^4a^2 + b^2a^4 > a^2b^2(a^2 + b^2) = 1$

4. If sec A+ tan A = p, then find the value of sec A – tan A

Given secA + tan A = p

Recall the identity, $\sec^2 A - \tan^2 A = 1$

$\Rightarrow (\sec A + \tan A)(\sec A - \tan A) = 1$

$\Rightarrow p \times (\sec A - \tan A) = 1$

$\therefore (\sec A - \tan A) = 1/p$

5. f (sec A + tan A)(sec B + tan B)(sec C + tan C) = (secA - tanA)(secB - tanB)(secC - tanC), prove that each of the sides is equal to 1 or -1.

Solution: (secA + tanA)(secB + tanB)(secC + tan C) = (secA - tanA)(secB - tanB)(secC - tanC)
Mulitply both sides with "(secA + tanA)(secB + tanB)(secC + tan C)",

we get $(\sec A + \tan A)^2(\sec B + \tan B)^2(\sec C + \tan C)^2$

= $(\sec^2A - \tan^2A)(\sec^2B - \tan^2B)(\sec^2C - \tan^2C)$ = (1)(1)(1) = 1

⇒ $[(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C)]^2 = 1$

∴ $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1$

Similarly, we get $(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm 1$

6. Prove (1+cotA-cosecA)(1+tanA+secA) = 2

Solution: (1+cotA-cosecA)(1+tanA+secA)

$$= \left(1 + \frac{1}{\tan A} - \operatorname{cosec} A\right)(1 + \tan A + \sec A)$$

$$= \left(\frac{\dfrac{\tan A + 1 - \dfrac{1}{\sin A} \times \operatorname{Sin} A}{\operatorname{Cos} A}}{\tan A}\right)(1 + \tan A + \operatorname{Sec} A)$$

$$= \frac{1}{\tan A}(1 + \tan A - \sec A)(1 + \tan A + \sec A)$$

$$= \frac{1}{\tan A}[(1 + \tan A)^2 - \sec^2 A] \qquad = \frac{1}{\tan A}[1 + 2\tan A + \tan^2 A - \sec^2 A]$$

$$= \frac{1}{\tan A}[1 + 2\tan A - 1] \text{ [Since } \sec^2 A - \tan^2 A = 1] \qquad = \frac{1}{\tan A} \times 2\tan A = 2$$

7. If $\sin A + \sin^2 A + \sin^3 A = 1$, prove that $\cos^6 A - 4\cos^4 A + 8\cos^2 A = 4$.

Solution:

Given $\sin A + \sin^2 A + \sin^3 A = 1$

$\Rightarrow \sin A + \sin^3 A = 1 - \sin^2 A$

$\Rightarrow \sin A + \sin^3 A = \cos^2 A$

Let's square on both the sides of the above equation, we get

$(\sin A + \sin^3 A)^2 = (\cos^2 A)^2$

$\Rightarrow \sin^2 A + 2 \times \sin A \times \sin^3 A + \sin^6 A = \cos^4 A$

$\Rightarrow \sin^2 A + 2\sin^4 A + \sin^6 A = \cos^4 A$

$\Rightarrow (1 - \cos^2 A) + 2(\sin^2 A)^2 + (\sin^2 A)^3 = \cos^4 A$

$\Rightarrow (1 - \cos^2 A) + 2(1 - \cos^2 A)^2 + (1 - \cos^2 A)^3 = \cos^4 A$

$\Rightarrow (1 - \cos^2 A) + (2 - 4\cos^2 A + 2\cos^4 A) + (1 - 3\cos^2 A + 3\cos^4 A - \cos^6 A) = \cos^4 A$

$\Rightarrow 4 - 8\cos^2 A + 5\cos^4 A - \cos^6 A = \cos^4 A$

$\Rightarrow 4 - 8\cos^2 A + 4\cos^4 A - \cos^6 A = 0$

$\therefore \cos^6 A - 4\cos^4 A + 8\cos^2 A = 4$

8. **Show that (1-sinA+cosA) ^{2}=2(1+cosA)(1-sinA)**

Solution: $(1-\sin A+\cos A)^2 = [(1-\sin A) + \cos A]^2$

$= (1-\sin A)^2 + \cos^2 A + 2(1-\sin A)\cos A$ $= 1 + \sin^2 A - 2\sin A + \cos^2 A + 2(1-\sin A)\cos A$

$= 1 + (\sin^2 A + \cos^2 A) - 2\sin A + 2(1-\sin A)\cos A$ $= 1 + 1 - 2\sin A + 2(1-\sin A)\cos A$ [Since, $\sin^2 A + \cos^2 A = 1$]

$= 2 - 2\sin A + 2(1-\sin A)\cos A$ $= 2(1 - \sin A) + 2(1-\sin A)\cos A$ $= 2(1 - \sin A)(1 + \cos A)$

9. Prove that sinθ - cosθ + 1/ sinθ + cosθ - 1 = 1/ secθ - tanθ, using the identity $\sec^2\theta = 1 + \tan^2\theta$

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{\frac{\sin\theta - \cos\theta + 1}{\cos\theta}}{\frac{\sin\theta + \cos\theta - 1}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} - 1 + \frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}}$$

$$= \frac{\tan\theta - 1 + \sec\theta}{\tan\theta + 1 - \sec\theta} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

Put $1 = \sec^2\theta - \tan^2\theta$ in the numerator

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - (\sec\theta - \tan\theta)]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)[1 - \sec\theta + \tan\theta]}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\sec\theta + \tan\theta)[\tan\theta - \sec\theta + 1]}{\tan\theta - \sec\theta + 1}$$

$$= \sec\theta + \tan\theta$$

10. write all other trignometrical identities in form of cot A?

Consider, $\operatorname{cosec}^2 A - \cot^2 A = 1 \Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A \Rightarrow \operatorname{cosec} A = \sqrt{1 + \cot^2 A}$

We know that $\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1+\cot^2 A}}$

$\cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{1}{1+\cot^2 A}} \therefore \cos A = \sqrt{\frac{\cot^2 A}{1+\cot^2 A}}$

We know that $\sec A = \frac{1}{\cos A}$ $\therefore \sec A = \sqrt{\frac{1+\cot^2 A}{\cot^2 A}}$ $\tan A = \frac{1}{\cot A}$

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