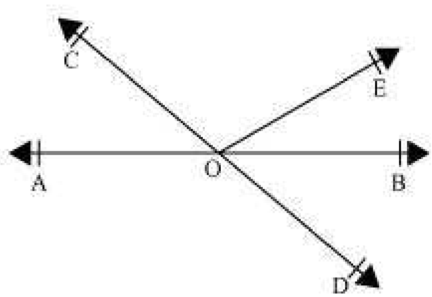
Exercise 6.1

Question 1:

In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle$BOE and reflex $\angle$COE.



Answer:

AB is a straight line, rays OC and OE stand on it.

$\therefore \angle AOC + \angle COE + \angle BOE = 180^\circ$

$\Rightarrow (\angle AOC + \angle BOE) + \angle COE = 180^\circ$

$\Rightarrow 70^\circ + \angle COE = 180^\circ$

$\Rightarrow \angle COE = 180^\circ - 70^\circ = 110^\circ$

Reflex $\angle COE = 360^\circ - 110^\circ = 250^\circ$

CD is a straight line, rays OE and OB stand on it.

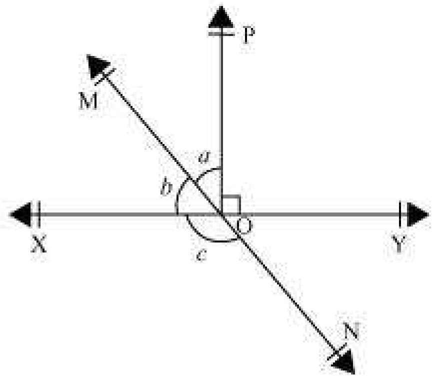
$\therefore \angle COE + \angle BOE + \angle BOD = 180^\circ$

$\Rightarrow 110^\circ + \angle BOE + 40^\circ = 180^\circ$

$\Rightarrow \angle BOE = 180^\circ - 150^\circ = 30^\circ$

Question 2:

In the given figure, lines XY and MN intersect at O. If $\angle$POY = 90° and a:b = 2 : 3, find c.



Answer:

Let the common ratio between *a* and *b* be *x.*

∴ $a = 2x$, and $b = 3x$

XY is a straight line, rays OM and OP stand on it.

∴ $\angle XOM + \angle MOP + \angle POY = 180^\circ$

$b + a + \angle POY = 180^\circ$

$3x + 2x + 90^\circ = 180^\circ$

$5x = 90^\circ$

$x = 18^\circ$

$a = 2x = 2 \times 18 = 36^\circ$

$b = 3x = 3 \times 18 = 54^\circ$

MN is a straight line. Ray OX stands on it.

∴ $b + c = 180^\circ$ (Linear Pair)

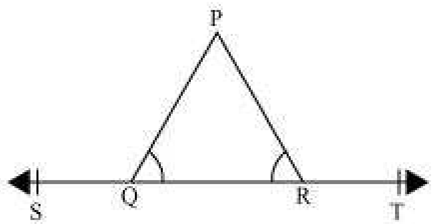
$54^\circ + c = 180^\circ$

$c = 180^\circ - 54^\circ = 126^\circ$

∴ $c = 126^\circ$

Question 3:

In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Answer:

In the given figure, ST is a straight line and ray QP stands on it.

$\therefore$ $\angle PQS + \angle PQR = 180^\circ$ (Linear Pair)

$\angle PQR = 180^\circ - \angle PQS$ (1)

$\angle PRT + \angle PRQ = 180^\circ$ (Linear Pair)

$\angle PRQ = 180^\circ - \angle PRT$ (2)

It is given that $\angle PQR = \angle PRQ$.

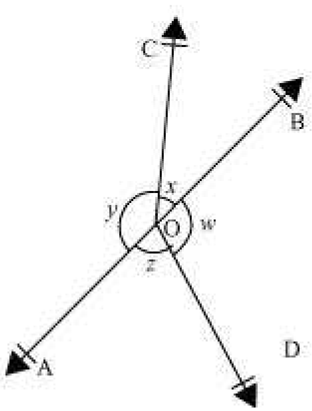
Equating equations (1) and (2), we obtain

$180^\circ - \angle PQS = 180^\circ - \angle PRT$

$\angle PQS = \angle PRT$

Question 4:

In the given figure, if $x + y = w + z$, then prove that AOB is a line.



Answer:

It can be observed that,

$x + y + z + w = 360^\circ$ (Complete angle)

It is given that,

$x + y = z + w$

$\therefore x + y + x + y = 360^\circ$

$2(x + y) = 360^\circ$

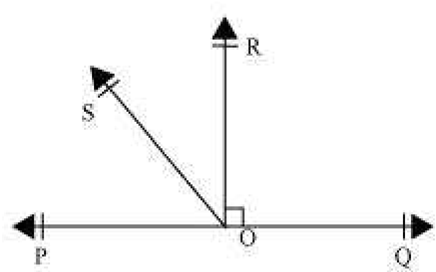
$x + y = 180^\circ$

Since x and y form a linear pair, AOB is a line.

Question 5:

In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$$



Answer:

It is given that OR ⊥ PQ

☐ ☐POR = 90º

☐ ☐POS + ☐SOR = 90º

☐ROS = 90º − ☐POS … (1)

☐QOR = 90º (As OR ☐ PQ)

☐QOS − ☐ROS = 90º

☐ROS = ☐QOS − 90º … (2)

On adding equations (1) and (2), we obtain

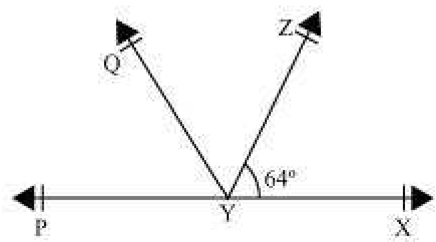
2 ☐ROS = ☐QOS − ☐POS

$\square$ROS = $\frac{1}{2}$ ($\square$QOS − $\square$POS)

Question 6:

It is given that $\square$XYZ = $64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\square$ZYP, find $\square$XYQ and reflex $\square$QYP.

Answer:



It is given that line YQ bisects $\square$PYZ.

Hence, $\square$QYP = $\square$ZYQ

It can be observed that PX is a line. Rays YQ and YZ stand on it.

$\square$ $\square$XYZ + $\square$ZYQ + $\square$QYP = 180º

$\square$ 64º + 2$\square$QYP = 180º

$\square$ 2$\square$QYP = 180º − 64º = 116º

$\square$ $\square$QYP = 58º

Also, $\square$ZYQ = $\square$QYP = 58º

Reflex $\square$QYP = 360º − 58º = 302º

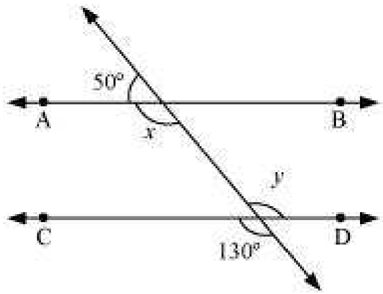
$\square$XYQ = $\square$XYZ + $\square$ZYQ

= 64º + 58º = 122º

Exercise 6.2

Question 1:

In the given figure, find the values of *x* and *y* and then show that AB || CD.



Answer:

It can be observed that,

$50^\circ + x = 180^\circ$ (Linear pair)

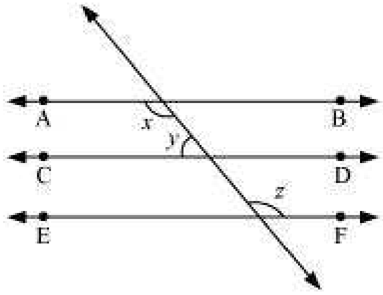
$x = 130^\circ$ … (1)

Also, $y = 130^\circ$ (Vertically opposite angles)

As *x* and *y* are alternate interior angles for lines AB and CD and also measures of these angles are equal to each other, therefore, line AB || CD.

Question 2:

In the given figure, if AB || CD, CD || EF and *y*: z = 3: 7, find *x*.



Answer:

It is given that AB || CD and CD || EF

☐ AB || CD || EF (Lines parallel to the same line are parallel to each other)

It can be observed that

$x = z$ (Alternate interior angles) … (1)

It is given that y: z = 3: 7

Let the common ratio between y and z be a.

☐ $y = 3a$ and $z = 7a$

Also, $x + y = 180^{\circ}$ (Co-interior angles on the same side of the transversal)

$z + y = 180^{\circ}$ [Using equation (1)]

$7a + 3a = 180^{\circ}$

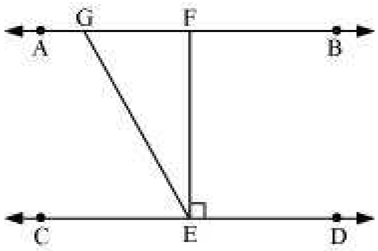
$10a = 180^{\circ}$

$a = 18^{\circ}$

☐ $x = 7a = 7 \times 18^{\circ} = 126^{\circ}$

Question 3:

In the given figure, If AB || CD, EF ☐ CD and ☐GED = 126º, find ☐AGE, ☐GEF and ☐FGE.



Answer:

It is given that,

AB || CD

EF ☐ CD

☐GED = 126º

☐ ☐GEF + ☐FED = 126º

☐ ☐GEF + 90º = 126º

☐ ☐GEF = 36º

☐AGE and ☐GED are alternate interior angles.

☐ ☐AGE = ☐GED = 126º

However, ☐AGE + ☐FGE = 180º (Linear pair)

☐ 126º + ☐FGE = 180º

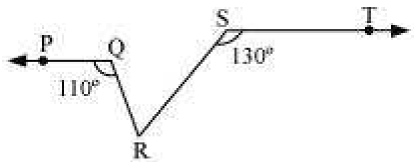
☐ ☐FGE = 180º − 126º = 54º

☐ ☐AGE = 126º, ☐GEF = 36º, ☐FGE = 54º

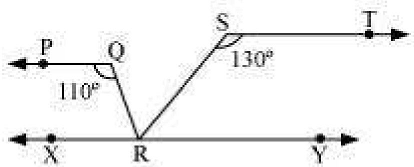
Question 4:

In the given figure, if PQ || ST, ☐PQR = 110º and ☐RST = 130º, find ☐QRS.

[**Hint**: Draw a line parallel to ST through point R.]



Answer:



Let us draw a line XY parallel to ST and passing through point R.

☐PQR + ☐QRX = 180º (Co-interior angles on the same side of transversal QR)

☐ 110º + ☐QRX = 180º

☐ ☐QRX = 70º

Also,

☐RST + ☐SRY = 180º (Co-interior angles on the same side of transversal SR)

130º + ☐SRY = 180º

☐SRY = 50º

XY is a straight line. RQ and RS stand on it.

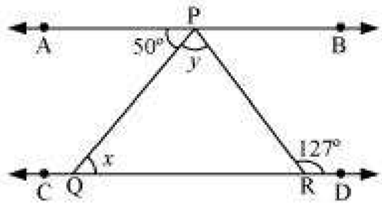
☐ ☐QRX + ☐QRS + ☐SRY = 180º

70º + ☐QRS + 50º = 180º

☐QRS = 180º − 120º = 60º

Question 5:

In the given figure, if AB || CD, ☐APQ = 50º and ☐PRD = 127º, find *x* and *y*.



Answer:

□APR = □PRD (Alternate interior angles)

50º + *y* = 127º

y = 127º − 50º

y = 77º

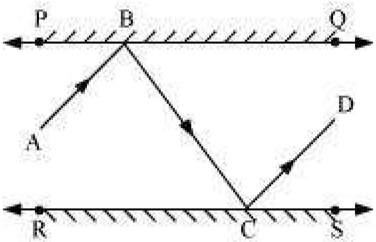
Also, □APQ = □PQR (Alternate interior angles)

50º = *x*

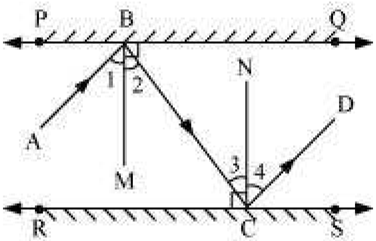
□ *x* = 50º and *y* = 77º

Question 6:

In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



Answer:



Let us draw BM □ PQ and CN □ RS.

As PQ || RS,

Therefore, BM || CN

Thus, BM and CN are two parallel lines and a transversal line BC cuts them at B and C respectively.

☐☐2 = ☐3 (Alternate interior angles)

However, ☐1 = ☐2 and ☐3 = ☐4 (By laws of reflection)

☐ ☐1 = ☐2 = ☐3 = ☐4

Also, ☐1 + ☐2 = ☐3 + ☐4

☐ABC = ☐DCB

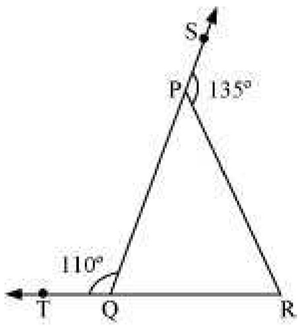
However, these are alternate interior angles.

☐ AB || CD

Exercise 6.3

Question 1:

In the given figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If □SPR = 135º and □PQT = 110º, find □PRQ.



Answer:

It is given that,

□SPR = 135º and □PQT = 110º

□SPR + □QPR = 180º (Linear pair angles)

□ 135º + □QPR = 180º

□ □QPR = 45º

Also, □PQT + □PQR = 180º (Linear pair angles)

□ 110º + □PQR = 180º

□ □PQR = 70º

As the sum of all interior angles of a triangle is 180º, therefore, for ΔPQR,

□QPR + □PQR + □PRQ = 180º

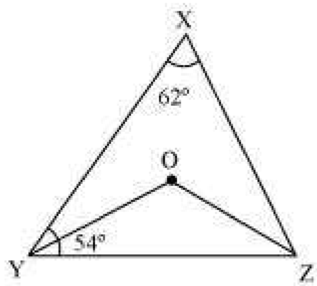
□ 45º + 70º + □PRQ = 180º

□ □PRQ = 180º − 115º

□ □PRQ = 65º

Question 2:

In the given figure, □X = 62º, □XYZ = 54º. If YO and ZO are the bisectors of □XYZ and □XZY respectively of ΔXYZ, find □OZY and □YOZ.



Answer:

As the sum of all interior angles of a triangle is 180º, therefore, for ΔXYZ,

☐X + ☐XYZ + ☐XZY = 180º

62º + 54º + ☐XZY = 180º

☐XZY = 180º − 116º

☐XZY = 64º

☐OZY = $\frac{64}{2}$ = 32º (OZ is the angle bisector of ☐XZY)

Similarly, ☐OYZ = $\frac{54}{2}$ = 27º

Using angle sum property for ΔOYZ, we obtain

☐OYZ + ☐YOZ + ☐OZY = 180º

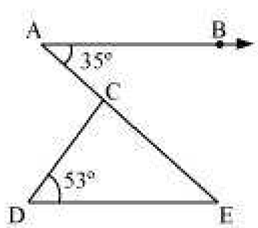
27º + ☐YOZ + 32º = 180º

☐YOZ = 180º − 59º

☐YOZ = 121º

Question 3:

In the given figure, if AB || DE, ☐BAC = 35º and ☐CDE = 53º, find ☐DCE.



Answer:

AB || DE and AE is a transversal.

☐BAC = ☐CED (Alternate interior angles)

☐ ☐CED = 35º

In ΔCDE,

☐CDE + ☐CED + ☐DCE = 180º (Angle sum property of a triangle)

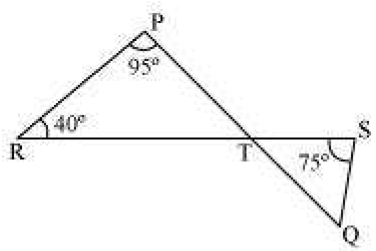
53º + 35º + ☐DCE = 180º

☐DCE = 180º − 88º

☐DCE = 92º

Question 4:

In the given figure, if lines PQ and RS intersect at point T, such that ☐PRT = 40º, ☐RPT = 95º and ☐TSQ = 75º, find ☐SQT.



Answer:

Using angle sum property for ΔPRT, we obtain

☐PRT + ☐RPT + ☐PTR = 180º

40º + 95º + ☐PTR = 180º

☐PTR = 180º − 135º

☐PTR = 45º

☐STQ = ☐PTR = 45º (Vertically opposite angles)

☐STQ = 45º

By using angle sum property for ΔSTQ, we obtain

☐STQ + ☐SQT + ☐QST = 180º

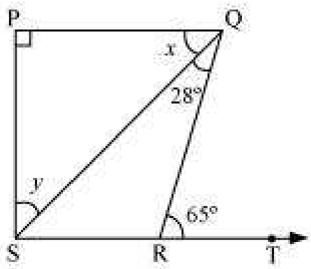
45º + ☐SQT + 75º = 180º

☐SQT = 180º − 120º

☐SQT = 60º

Question 5:

In the given figure, if PQ ☐ PS, PQ || SR, ☐SQR = 28º and ☐QRT = 65º, then find the values of *x* and *y*.



Answer:

It is given that PQ || SR and QR is a transversal line.

☐PQR = ☐QRT (Alternate interior angles)

$x + 28º = 65º$

$x = 65º - 28º$

$x = 37º$

By using the angle sum property for ΔSPQ, we obtain

☐SPQ + $x + y = 180º$

$90º + 37º + y = 180º$

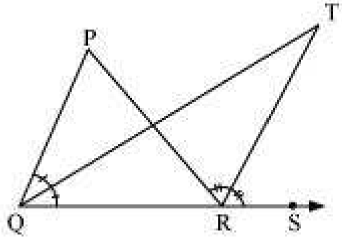
$y = 180º - 127º$

$y = 53º$

$\therefore x = 37º$ and $y = 53º$

Question 6:

In the given figure, the side QR of ΔPQR is produced to a point S. If the bisectors of ☐PQR and ☐PRS meet at point T, then prove that ☐QTR= $\frac{1}{2}$ ☐QPR.



Answer:

In ΔQTR, ☐TRS is an exterior angle.

∴ ☐QTR + ☐TQR = ☐TRS

☐QTR = ☐TRS − ☐TQR (1)

For ΔPQR, ☐PRS is an external angle.

∴ ☐QPR + ☐PQR = ☐PRS

☐QPR + 2☐TQR = 2☐TRS (As QT and RT are angle bisectors)

☐QPR = 2(☐TRS − ☐TQR)

☐QPR = 2☐QTR [By using equation (1)]

☐QTR = $\frac{1}{2}$ ☐QPR