

UNIT-6

TRIGONOMETRY

"The mathematician is fascinated with the marvelous beauty of the forms he constructs, and in their beauty he finds everlasting truth."

1. If $x\cos\theta - y\sin\theta = a$, $x\sin\theta + y\cos\theta = b$, prove that $x^2+y^2=a^2+b^2$.

Ans: $x\cos\theta - y\sin\theta = a$
$x\sin\theta + y\cos\theta = b$
Squaring and adding
$x^2+y^2=a^2+b^2$.

2. Prove that $\sec^2\theta+\text{cosec}^2\theta$ can never be less than 2.

Ans: S.T $\text{Sec}^2\theta + \text{Cosec}^2\theta$ can never be less than 2.
If possible let it be less than 2.
$1 + \text{Tan}^2\theta + 1 + \text{Cot}^2\theta < 2.$
$\Rightarrow \quad 2 + \text{Tan}^2\theta + \text{Cot}^2\theta$
$\Rightarrow \quad (\text{Tan}\theta + \text{Cot}\theta)^2 < 2.$
Which is not possible.

3. If $\sin\varphi = \dfrac{1}{2}$, show that $3\cos\varphi - 4\cos^3\varphi = 0$.

Ans: $\text{Sin}\ \varphi = ½$
$\Rightarrow \varphi = 30^o$
Substituting in place of $\varphi = 30^o$. We get 0.

4. If $7\sin^2\varphi+3\cos^2\varphi = 4$, show that $\tan\varphi = \dfrac{1}{\sqrt{3}}$.

Ans: If $7\ \text{Sin}^2\varphi + 3\ \text{Cos}^2\varphi = 4$ S.T. $\text{Tan}\varphi\ \frac{1}{\sqrt{3}}$

$7\ \text{Sin}^2\varphi + 3\ \text{Cos}^2\varphi = 4\ (\text{Sin}^2\varphi + \text{Cos}^2\varphi)$

$\Rightarrow 3\ \text{Sin}^2\varphi = \text{Cos}^2\varphi$

$$\Rightarrow \frac{Sin^2\varphi}{Cos^2\varphi} = \frac{1}{3}$$

$\Rightarrow \text{Tan}^2\varphi = \frac{1}{3}$

$\text{Tan}\varphi = \frac{1}{\sqrt{3}}$

5. If $\cos\varphi+\sin\varphi = \sqrt{2}\cos\varphi$, prove that $\cos\varphi - \sin\varphi = \sqrt{2}\sin\varphi$.

Ans: $\text{Cos}\varphi + \text{Sin}\varphi = \sqrt{2}\,\text{Cos}\varphi$

$\Rightarrow (\text{Cos}\varphi + \text{Sin}\varphi)^2 = 2\text{Cos}^2\varphi$

$\Rightarrow \text{Cos}^2\varphi + \text{Sin}^2\varphi + 2\text{Cos}\varphi\,\text{Sin}\varphi = 2\text{Cos}^2\varphi$

$\Rightarrow \text{Cos}^2\varphi - 2\text{Cos}\varphi\,\text{Sin}\varphi + \text{Sin}^2\varphi = 2\text{Sin}^2\varphi$ $\left[\therefore 2\text{Sin}^2\varphi = 2 - 2\text{Cos}^2\varphi;\ 1 - \text{Cos}^2\varphi = \text{Sin}^2\varphi \ \&\ 1 - \text{Sin}^2\varphi = \text{Cos}^2\varphi\right]$

$\Rightarrow (\text{Cos}\varphi - \text{Sin}\varphi)^2 = 2\text{Sin}^2\varphi$

or $\text{Cos}\varphi - \text{Sin}\varphi = \sqrt{2}\,\text{Sin}\varphi$.

6. If tanA+sinA=m and tanA-sinA=n, show that $m^2-n^2 = 4\sqrt{mn}$

Ans: TanA + SinA = m TanA – SinA = n.

$m^2-n^2=4\sqrt{mn}$.

$m^2-n^2= (\text{TanA} + \text{SinA})^2-(\text{TanA} - \text{SinA})^2$

$= 4\ \text{TanA SinA}$

RHS $4\sqrt{mn} = 4\sqrt{(TanA + SinA)(TanA - SinA)}$

$= 4\sqrt{Tan^2A - Sin^2A}$

$= 4\sqrt{\frac{Sin^2A - Sin^2ACos^2A}{Cos^2A}}$

$= 4\sqrt{\frac{Sin^4A}{Cos^2A}}$

$= 4\frac{Sin^2A}{Cos^2A} = 4\ \text{TanA SinA}$

$\therefore m^2 - n^2 = 4\sqrt{mn}$

7. If secA$= x + \frac{1}{4x}$, prove that secA+tanA=2x or $\frac{1}{2x}$.

Ans: $\text{Sec}\varphi = x + \frac{1}{4x}$

$\Rightarrow \text{Sec}^2\varphi = (x + \frac{1}{4x})^2$ $(\text{Sec}^2\varphi = 1 + \text{Tan}^2\varphi)$

$\text{Tan}^2\varphi = (x + \frac{1}{4x})^2 - 1$

$\text{Tan}^2\varphi = (x - \frac{1}{4x})^2$

$\text{Tan}\varphi = \underline{+}\ x - \frac{1}{4x}$

$\text{Sec}\varphi + \text{Tan}\varphi = x + \frac{1}{4x} \underline{+}\ x - \frac{1}{4x}$

$= 2x \text{ or } \frac{1}{2x}$

8. If A, B are acute angles and sinA= cosB, then find the value of A+B.

Ans: $A + B = 90^o$

9. a)Solve for φ, if tan5φ = 1.

Ans: $\text{Tan } 5\varphi = 1 \quad \Rightarrow \varphi = \frac{45}{5} \Rightarrow \varphi = 9^o$.

b)Solve for φ if $\frac{Sin\,\varphi}{1+Cos\,\varphi} + \frac{1+Cos\,\varphi}{Sin\,\varphi} = 4$.

Ans: $\frac{Sin\,\varphi}{1+Cos\,\varphi} + \frac{1+Cos\,\varphi}{Sin\,\varphi} = 4$

$\frac{\text{Sin}^2\varphi + 1(\text{Cos}\,\phi)^2}{\text{Sin}\,\varphi(1+\text{Cos}\,\varphi)} = 4$

$\frac{\text{Sin}^2\varphi + 1 + \text{Cos}^2\varphi + 2\text{Cos}\,\varphi}{\text{Sin}\,\varphi + \text{Sin}\,\varphi\text{Cos}\,\varphi} = 4$

$\frac{2 + 2Cos\,\varphi}{Sin\,\varphi(1+Cos\,\varphi)} = 4$

$\Rightarrow \quad \frac{2+(1+Cos\ \varphi)}{Sin\ \varphi(1+Cos\ \varphi)} = 4$

$\Rightarrow \quad \frac{2}{Sin\ \varphi} = 4$

$\Rightarrow \quad Sin\varphi = \frac{1}{2}$

$\Rightarrow \quad Sin\varphi = Sin30$

$\varphi = 30^o$

10. If $\frac{cos\alpha}{cos\beta} = m\ and\ \frac{cos\alpha}{sin\beta} = n, show\ that\ (m^2+n^2)cos^2\beta = n^2$

Ans: $\frac{Cos\ \alpha}{Cos\ \beta} = m \qquad \frac{Cos\ \alpha}{Sin\ \beta} = n$

$\Rightarrow m^2 = \frac{Cos^2\alpha}{Cos^2\beta} \qquad n^2 = \frac{Cos^2\alpha}{Sin^2\beta}$

LHS = $(m^2+n^2)\ Cos^2\beta$

$\left[\frac{Cos^2\alpha}{Cos^2\beta} + \frac{Cos^2\alpha}{Sin^2\beta}\right] Cos^2\beta$

$= Cos^2\alpha\left(\frac{1}{Cos^2\beta Sin^2\beta}\right) Cos^2\beta$

$= \frac{Cos^2\alpha}{Sin^2\beta} = n^2$

$\Rightarrow (m^2+n^2)\ Cos^2\beta = n^2$

11. If 7 cosecφ-3cotφ = 7, prove that 7cotφ - 3cosecφ = 3.

Ans: 7 Cosecφ-2Cotφ=7

P.T 7Cotφ - 3 Cosecφ=3

7 Cosecφ-3Cotφ=7

$\Rightarrow$7Cosecφ-7=3Cotφ

$\Rightarrow$7(Cosecφ-1)=3Cotφ

$\Rightarrow 7(\text{Cosec}\varphi-1)(\text{Cosec}\varphi+1)=3\text{Cot}\varphi(\text{Cosec}\varphi+1)$
$\Rightarrow 7(\text{Cosec}^2\varphi-1)=3\text{Cot}\varphi(\text{Cosec}\varphi+1)$
$\Rightarrow 7\text{Cot}^2\varphi=3\text{ Cot}\varphi\ (\text{Cosec}\varphi+1)$
$\Rightarrow 7\text{Cot}\varphi= 3(\text{Cosec}\varphi+1)$
$7\text{Cot}\varphi-3\text{ Cosec}\varphi=3$

12. $2(\sin^6\varphi+\cos^6\varphi) - 3(\sin^4\varphi+\cos^4\varphi)+1 = 0$

Ans: $(\text{Sin}^2\varphi)^3 + (\text{Cos}^2\varphi)^3-3\ (\text{Sin}^4\varphi+(\text{Cos}^4\varphi)+1=0$
Consider $(\text{Sin}^2\varphi)^3 +(\text{Cos}^2\varphi)^3$
$\Rightarrow(\text{Sin}^2\varphi+\text{Cos}^2\varphi)^3-3\ \text{Sin}^2\varphi\text{Cos}^2\varphi(\ \text{Sin}^2\varphi+\text{Cos}^2\varphi)$
$= 1- 3\text{Sin}^2\varphi\ \text{Cos}^2\varphi$
$\text{Sin}^4\varphi+\text{Cos}^4\varphi(\text{Sin}^2\varphi)^2+(\text{Cos}^2\varphi)^2$
$= (\text{Sin}^2\varphi+\text{Cos}^2\varphi)^2-2\ \text{Sin}^2\varphi\ \text{Cos}^2\varphi$
$= 1- 2\ \text{Sin}^2\varphi\ \text{Cos}^2\varphi$
$= 2(\text{Sin}^6\varphi+\text{Cos}^6\varphi)-3(\text{Sin}^4\varphi+\text{Cos}^4\varphi) +1$
$= 2\ (1-3\ \text{Sin}^2\varphi\ \text{Cos}^2\varphi)-3\ (1-2\ \text{Sin}^2\varphi+\text{Cos}^2\varphi)+1$

13. $5(\sin^8 A- \cos^8 A) = (2\sin^2 A - 1)\ (1- 2\sin^2 A \cos^2 A)$

Ans: Proceed as in Question No.12

14. If $\tan\theta = \frac{5}{6}$ & $\theta + \phi = 90^o$ what is the value of $\cot\phi$.

Ans: $\text{Tan}\theta = {}^5\!/_6$ i.e. $\text{Cot}\phi = {}^5\!/_6$ Since $\varphi + \theta = 90^o$.

15. What is the value of $\tan\varphi$ in terms of $\sin\varphi$.

Ans: $\text{Tan}\ \varphi = \frac{Sin\ \varphi}{Cos\ \varphi}$

$\text{Tan}\ \varphi = \frac{Sin\ \varphi}{\sqrt{1 - Sin^2\varphi}}$

16. If $\text{Sec}\varphi+\text{Tan}\varphi=4$ find $\sin\varphi$, $\cos\varphi$

Ans: $\text{Sec}\ \varphi + \text{Tan}\ \varphi = 4$

$\frac{1}{Cos\ \varphi} + \frac{Sin\ \varphi}{Cos\ \varphi} = 4$

$\frac{1 + Sin\ \varphi}{Cos\ \varphi} = 4$

$$\Rightarrow \frac{(1+Sin\varphi)^2}{Cos^2\varphi} = 16$$

$\Rightarrow$ apply (C & D)

$$= \frac{(1+Sin\phi)^2 + Cos^2\phi}{(1+Sin\phi)^2 - Cos^2\phi} = \frac{16+1}{16-1}$$

$$\Rightarrow \frac{2(1+Sin\phi)}{2Sin\phi(1+Sin\phi)} = \frac{17}{15}$$

$$\Rightarrow \frac{1}{Sin\phi} = \frac{17}{15}$$

$$\Rightarrow Sin\varphi = \frac{15}{17}$$

$$Cos\varphi = \sqrt{1 - Sin^2\varphi}$$

$$\sqrt{1 - \left(\frac{15}{17}\right)^2} = \frac{8}{17}$$

17. Secφ+Tanφ=p, prove that sinφ = $\frac{p^2-1}{p^2+1}$

Ans: Secφ + Tanφ= P. P.T Sinφ= $\frac{P^2-1}{P^2+1}$

Proceed as in Question No.15

18. Prove geometrically the value of Sin 60^o

Ans: Exercise for practice.

19. If $\frac{1-\tan\theta}{1+\tan\theta} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$,show that $\frac{\sin\theta}{\cos 2\theta}$ =1

Ans: Exercise for practice.

20. If 2x=secθ and $\frac{2}{x}$ = tanθ ,then find the value of $2\left(x^2 - \frac{1}{x^2}\right)$. (Ans:1)

Ans: Exercise for practice.