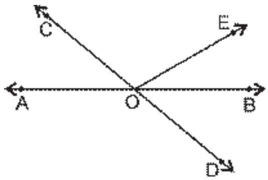
6 LINES AND ANGLES

EXERCISE 6.1

Q.1. *In the figure lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.*

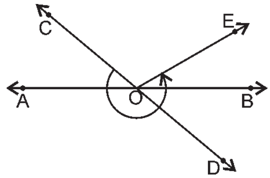


Sol. Lines AB and CD intersect at O.

∠AOC + ∠BOE = 70° (Given) ...(1)

∠BOD = 40° (Given) ...(2)

Since, ∠AOC = ∠BOD (Vertically opposite angles)



Therefore, ∠AOC = 40° [From (2)]

and 40° + ∠BOE = 70° [From (1)]

⇒ ∠BOE = 70° – 40° = 30°

Also, ∠AOC + ∠BOE + ∠COE = 180° (∵ AOB is a straight line)

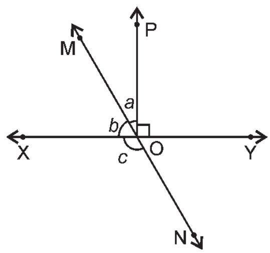
⇒ 70° + ∠COE = 180° [Form (1)]

⇒ ∠COE = 180° – 70° = 110°

Now, reflex ∠COE = 360° – 110° = 250°

Hence, **∠BOE = 30°** and **reflex ∠COE = 250° Ans.**

Q.2. *In the figure, lines XY and MN intersect at O. If ∠POY = 90° and a : b = 2 : 3, find c.*



Sol. In the figure, lines XY and MN intersect at O and ∠POY = 90°.

Also, given $a : b = 2 : 3$

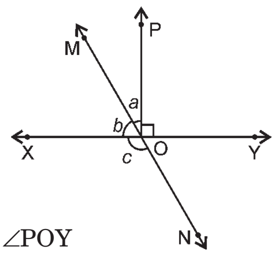
Let $a = 2x$ and $b = 3x$.

Since, ∠XOM + ∠POM + ∠POY = 180° (Linear pair axiom)

⇒ $3x + 2x + 90° = 180°$

⇒ $5x = 180° – 90°$

⇒ $x = \frac{90°}{5} = 18°$



∴ ∠XOM = $b = 3x = 3 \times 18° = 54°$

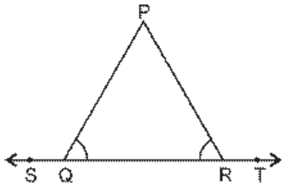
and ∠POM = $a = 2x = 2 \times 18° = 36°$

Now, ∠XON = c = ∠MOY = ∠POM + ∠POY (Vertically opposite angles)

= 36° + 90° = 126°

Hence, **c = 126° Ans.**

Q.3. *In the figure, ∠PQR = ∠PRQ, then prove that ∠PQS = ∠PRT.*



Sol. ∠PQS + ∠PQR = 180° ...(1)

(Linear pair axiom)

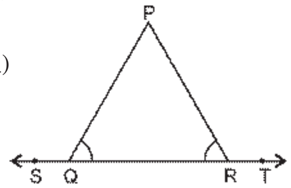
∠PRQ + ∠PRT = 180° ...(2)

(Linear pair axiom)

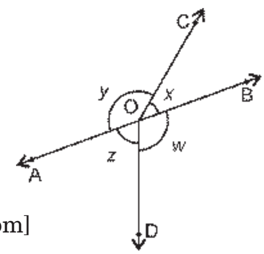
But, ∠PQR = ∠PRQ (Given)

∴ From (1) and (2)

∠PQS = ∠PRT **Proved.**



Q.4. *In the figure, if $x + y = w + z$, then prove that AOB is a line.*



Sol. Assume AOB is a line.

Therefore, $x + y = 180°$ ...(1)

[Linear pair axiom]

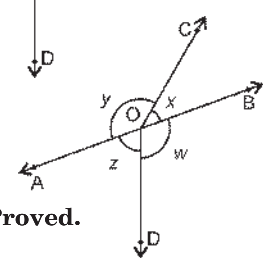
$w + z = 180°$ ...(2)

[Linear pair axiom]

Now, from (1) and (2)

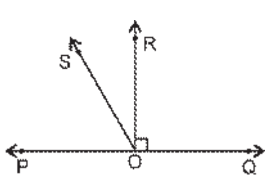
$x + y = w + z$

Hence, our assumption is correct, AOB is a line **Proved.**



Q.5. *In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that*

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$$



Sol. ∠ROS = ∠ROP – ∠POS ...(1)

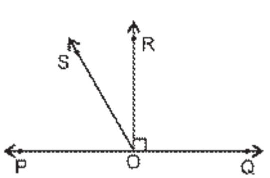
and ∠ROS = ∠QOS – ∠QOR ...(2)

Adding (1) and (2),

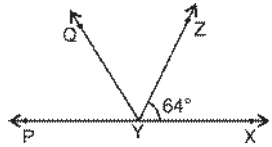
∠ROS + ∠ROS = ∠QOS – ∠QOR + ∠ROP – ∠POS

⇒ 2∠ROS = ∠QOS – ∠POS (∵ ∠QOR = ∠ROP = 90°)

⇒ $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$ **Proved.**



Q.6. *It is given that $\angle XYZ = 64°$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.*



Sol. From figure,

$\angle XYZ = 64°$ (Given)

Now, $\angle ZYP + \angle XYZ = 180°$ (Linear pair axiom)

$\Rightarrow \angle ZYP + 64° = 180°$

$\Rightarrow \angle ZYP = 180° - 64° - 116°$

Also, given that ray YQ bisects $\angle ZYP$.

But, $\angle ZYP = \angle QYP \ \angle QYZ = 116°$

Therefore, $\angle QYP = 58°$ and $\angle QYZ = 58°$

Also, $\angle XYQ = \angle XYZ + \angle QYZ$

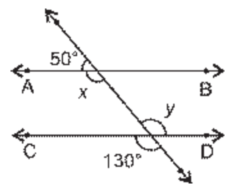
$\Rightarrow \angle XYQ = 64° + 58° = 122°$

and reflex $\angle QYP = 360° - \angle QYP = 360° - 58° = 302°$ $(\because \angle QYP = 58°)$

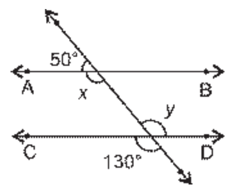
Hence, $\boldsymbol{\angle XYQ = 122°}$ **and reflex** $\boldsymbol{\angle QYP = 302°}$ **Ans.**

EXERCISE 6.2

Q.1. *In the figure, find the values of x and y and then show that AB || CD.*



Sol. In the given figure, a transversal intersects two lines AB and CD such that



$x + 50° = 180°$ (Linear pair axiom)

$\Rightarrow x = 180° - 50°$

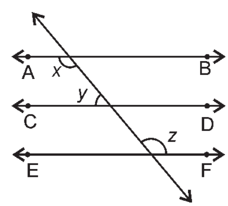
$= 130°$

$y = 130°$ (Vertically opposite angles)

Therefore, $\angle x = \angle y = 130°$ (Alternate angles)

$\therefore$ AB || CD (Converse of alternate angles axiom) **Proved.**

Q.2. *In the figure, if AB || CD, CD || EF and $y : z = 3 : 7$, find x.*



Sol. In the given figure, AB || CD, CD || EF and $y : z = 3 : 7$.

Let $y = 3a$ and $z = 7a$

$\angle DHI = y$ (vertically opposite angles)

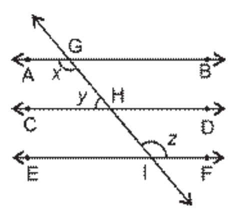
$\angle DHI + \angle FIH = 180°$ (Interior angles on the same side of the transversal)

$\Rightarrow y + z = 180°$

$\Rightarrow 3a + 7a = 180°$

$\Rightarrow 10a = 180° \Rightarrow a = 18°$

$\therefore y = 3 \times 18° = 54°$ and $z = 18° \times 7 = 126°$



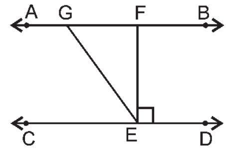
Also, $x + y = 180°$

$\Rightarrow x + 54° = 180°$

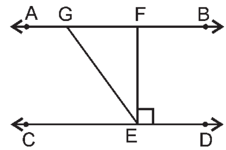
$\therefore x = 180° - 54° = 126°$

Hence, $\boldsymbol{x = 126°}$ **Ans.**

Q.3. *In the figure, if AB || CD, EF ⊥ CD and ∠GED = 126°. Find ∠AGE, ∠GEF and ∠FGE.*



Sol. In the given figure, AB || CD, EF ⊥ CD and ∠GED = 126°



∠AGE = ∠LGE (Alternate angle)

∴ ∠AGE = 126°

Now, ∠GEF = ∠GED – ∠DEF

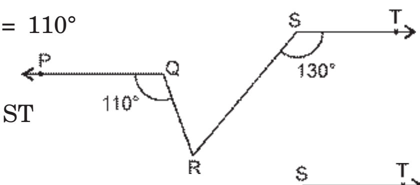
= 126° – 90° = 36° (∵ ∠DEF = 90°)

Also, ∠AGE + ∠FGE = 180° (Linear pair axiom)

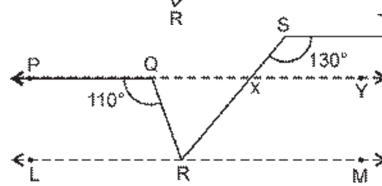
⇒ 126° + FGE = 180°

⇒ ∠FGE = 180° – 126° = 54°

Q.4. *In the figure, if PQ || ST, ∠PQR = 110° and ∠RST = 130°, find ∠QRS.*



Sol. Extend PQ to Y and draw LM || ST through R.



∠TSX = ∠QXS

[Alternate angles]

⇒ ∠QXS = 130°

∠QXS + ∠RXQ = 180°

[Linear pair axiom]

⇒ ∠RXQ = 180° – 130° = 50° ...(1)

∠PQR = ∠QRM [Alternate angles]

⇒ ∠QRM = 110° ...(2)

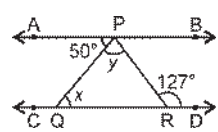
∠RXQ = ∠XRM [Alternate angles]

⇒ ∠XRM = 50° [By (1)]

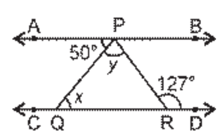
∠QRS = ∠QRM – ∠XRM

= 110° – 50° = **60°** **Ans.**

Q.5. *In the figure, if AB || CD, ∠APQ = 50° and ∠PRD = 127°, find x* and *y.*



Sol. In the given figure, AB || CD, ∠APQ = 50° and ∠PRD = 127°



∠APQ + ∠PQC = 180°

[Pair of consecutive interior angles are supplementary]

⇒ 50° + ∠PQC = 180°

⇒ ∠PQC = 180° – 50° = 130°

Now, ∠PQC + ∠PQR = 180° [Linear pair axiom]

⇒ 130° + x = 180°

⇒ x = 180° – 130° = 50°

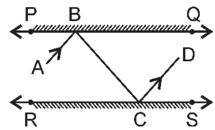
Also, x + y = 127° [Exterior angle of a triangle is equal to the sum of the two interior opposite angles]

⇒ 50° + y = 127°

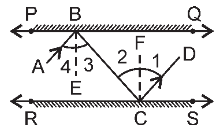
⇒ y = 127° – 50° = 77°

Hence, **x = 50° and y = 77°** **Ans.**

Q.6. *In the figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.*



Sol. At point B, draw BE ⊥ PQ and at point C, draw CF ⊥ RS.



$\angle 1 = \angle 2$ ...(i) (Angle of incidence is equal to angle of reflection)

$\angle 3 = \angle 4$ ...(ii) [Same reason]

Also, $\angle 2 = \angle 3$ ... (iii) [Alternate angles]

$\Rightarrow \angle 1 = \angle 4$ [From (i), (ii), and (iii)]

$\Rightarrow 2\angle 1 = 2\angle 4$

$\Rightarrow \angle 1 + \angle 1 = \angle 4 + \angle 4$

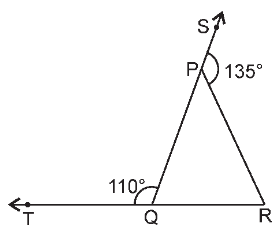
$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$ [From (i) and (ii)]

$\Rightarrow \angle BCD = \angle ABC$

Hence, AB || CD. [Alternate angles are equal] **Proved.**

EXERCISE 6.3

Q.1. *In the figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If $\angle SPR = 135°$ and $\angle PQT = 110°$, find $\angle PRQ$.*



Sol. In the given figure, $\angle SPR = 135°$ and $\angle PQT = 110°$.

$\angle PQT + \angle PQR = 180°$ [Linear pair axiom]

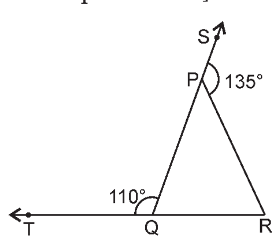
$\Rightarrow 110° + \angle PQR = 180°$

$\Rightarrow \angle PQR = 180° - 110° = 70°$

Also, $\angle SPR + \angle QPR = 180°$ [Linear pair axiom]

$\Rightarrow 135° + \angle QPR = 180°$

$\Rightarrow \angle QPS = 180° - 135° = 45°$



Now, in the triangle PQR

$\angle PQR + \angle PRQ + \angle QPR = 180°$

[Angle sum property of a triangle]

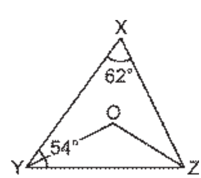
$\Rightarrow 70° + \angle PRQ + 45° = 180°$

$\Rightarrow \angle PRQ + 115° = 180°$

$\Rightarrow \angle PRQ = 180° - 115° = 65°$

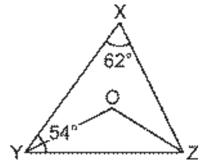
Hence, $\angle$ **PRQ = 65°** **Ans.**

Q.2. *In the figure, $\angle X = 62°$, $\angle XYZ = 54°$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of ΔXYZ, find $\angle OZY$ and $\angle YOZ$.*



Sol. In the given figure,

$\angle X = 62°$ and $\angle XYZ = 54°$.



$\angle XYZ + \angle XZY + \angle YXZ = 180°$ ...(i)

[Angle sum property of a triangle]

$\Rightarrow$ $54° + \angle XZY + 62° = 180°$

$\Rightarrow$ $\angle XZY + 116° = 180°$

$\Rightarrow$ $\angle XZY = 180° - 116° = 64°$

Now, $\angle OZY = \frac{1}{2} \times \angle XZY$ [$\because$ ZO is bisector of $\angle XZY$]

$= \frac{1}{2} \times 64° = 32°$

Similarly, $\angle OYZ = \frac{1}{2} \times 54° = 27°$

Now, in ΔOYZ, we have

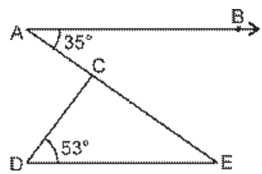
$\angle OYZ + \angle OZY + \angle YOZ = 180°$ Angle sum property of a triangle]

$\Rightarrow 27° + 32° + \angle YOZ = 180°$

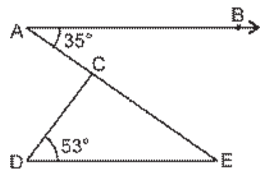
$\Rightarrow \angle YOZ = 180° - 59° = 121°$

Hence, **$\angle OZY = 32°$ and $\angle YOZ = 121°$ Ans.**

Q.3. *In the figure, if AB || DE, $\angle BAC = 35°$ and $\angle CDE = 53°$, find $\angle DCE$.*



Sol. In the given figure



$\angle BAC = \angle CED$

[Alternate angles]

$\Rightarrow$ $\angle CED = 35°$

In ΔCDE,

$\angle CDE + \angle DCE + \angle CED = 180°$ [Angle sum property of a triangle]

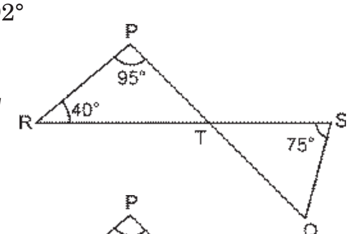
$\Rightarrow$ $53° + \angle DCE + 35° = 180°$

$\Rightarrow$ $\angle DCE + 88° = 180°$

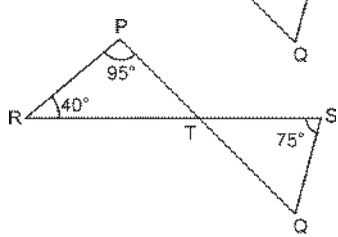
$\Rightarrow$ $\angle DCE = 180° - 88° = 92°$

Hence, **$\angle DCE = 92°$ Ans.**

Q.4. *In the figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$, find $\angle SQT$.*



Sol. In the given figure, lines PQ and RS intersect at point T, such that $\angle PRT = 40°$, $\angle RPT = 95°$ and $\angle TSQ = 75°$.



In ΔPRT

$\angle PRT + \angle RPT + \angle PTR = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $40° + 95° + \angle PTR = 180°$

$\Rightarrow$ $135° + \angle PTR = 180°$

$\Rightarrow$ $\angle PTR = 180° - 135° = 45°$

Also, $\angle PTR = \angle STQ$ [Vertical opposite angles]

$\therefore$ $\angle STQ = 45°$

Now, in ΔSTQ,

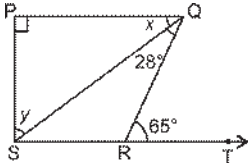
$\angle STQ + \angle TSQ + \angle SQT = 180°$ [Angle sum property of a triangle]

$\Rightarrow 45° + 75° + \angle SQT = 180°$

$\Rightarrow 120° + \angle SQT = 180°$

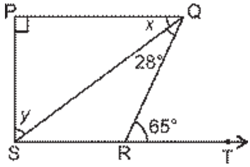
$\Rightarrow \angle SQT = 180° - 120° = 60°$

Hence, **$\angle SQT = 60°$ Ans.**



Q.5. *In the figure, if $PT \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$, then find the values of x and y.*

Sol. In the given figure, lines $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28°$ and $\angle QRT = 65°$



$\angle PQR = \angle QRT$ [Alternate angles]

$\Rightarrow x + 28° = 65°$

$\Rightarrow x = 65° - 28° = 37°$

In ΔPQS,

$\angle SPQ + \angle PQS + \angle QSP = 180°$ [Angle sum property of a triangle]

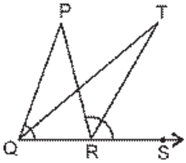
$\Rightarrow 90° + 37° + y = 180°$

[$\because PQ \perp PS$, $\angle PQS = x = 37°$ and $\angle QSP = y$)

$\Rightarrow 127° + y = 180°$

$\Rightarrow y = 180° - 127° = 53°$

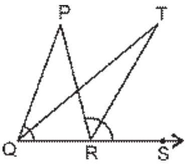
Hence, **$x = 37°$ and $y = 53°$ Ans.**



Q.6. *In the figure, the side QR of ΔPQR is produced to a point S. If the bisectors of $\angle PQR$ and $\angle PRS$ meet at point T, then prove that $\angle QTR = \frac{1}{2}\angle QPR$.*

Sol. Exterior $\angle PRS = \angle PQR + \angle QPR$

[Exterior angle property]



Therefore, $\frac{1}{2}\angle PRS = \frac{1}{2}\angle PQR + \frac{1}{2}\angle QPR$

$\Rightarrow \angle TRS = \angle TQR + \frac{1}{2}\angle QPR$ ...(i)

But in ΔQTR,

Exterior $\angle TRS = \angle TQR + \angle QTR$ ...(ii)

[Exterior angles property]

Therefore, from (i) and (ii)

$\angle TQR + \angle QTR = \angle TQR + \frac{1}{2}\angle QPR$

$\Rightarrow \angle QTR = \frac{1}{2}\angle QPR$ **Proved.**