

20. Volume and Surface Area of Solids

- i.) A field is 80 m long and 50 m broad. In one corner of the field, a pit which is 10 m long, 7.5 m, broad and 8 m deep has been dug out. The earth taken out of it is evenly spread over the remaining part of the field. Find the rise in the level of the field.

$$\begin{aligned} \text{Area of field} &= (80 \times 50) \text{ m}^2 \\ &= 4000 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of pit} &= (10 \times 7.5) \text{ m}^2 \\ &= 75 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area over the earth spread out} &= (4000 - 75) \text{ m}^2 \\ &= 3925 \text{ m}^2 \end{aligned}$$

$$\text{Now, Vol. of field} = \text{Vol. of earth.}$$

$$3925 \times h = 600 \text{ m}^3$$

$$h = \left(\frac{600}{3925} \right) \text{ m}$$

$$h = \frac{600}{3925} \times 100 \text{ cm}$$

$$h = 15.28 \text{ cm} = 15.3 \text{ cm}$$

- ii.) The volume of a reservoir is 108 m³. Water is poured into it at the rate of 60 litres per minute. How many hours will it take to fill the reservoir?

$$\text{Vol. of reservoir} = (108 \times 1000) \text{ litres}$$

$$\text{Rate of flow water} = 60 \text{ litres per min}$$

Time taken fill the reservoir

$$= \left(\frac{\text{V. of reservoir lit}}{\text{Rate of flow in lit}} \right)$$

$$= \left(\frac{60 \times 1000}{60} \right) \text{ min} \left(\frac{108 \times 1000}{60 \times 60} \right) \text{ hrs}$$

$$\text{time} = 30 \text{ hrs.}$$

- iii.) A wall 15m long, 30cm wide and 4m high is made of bricks, each measuring 22cm x 12.5cm x 7.5cm. If $\frac{1}{12}$ of the total volume of the wall consists of mortar, how many bricks are there in the wall?

$$= \text{No. of brick} = \frac{11}{12} \times \frac{\text{V. of wall}}{\text{V. of brick.}}$$

$$\frac{11}{12} \times \frac{1500 \times 30 \times 100 \times 100 \times 4}{22 \times 125 \times 75}$$

$$= 800 \text{ bricks}$$

iv.) A beam of wood is 5m long and 36cm thick. It is made of 1.35 m^3 of wood. What is the width of the beam?

$$= b = \frac{V}{l \times h} = \frac{1.35 \text{ m}^3}{5 \text{ m} \times 0.36 \text{ m}} = \frac{1.35}{1.8}$$

width = 0.75 m Ans.

v.) If the length of each edge of a cube is doubled, how many times does its volume become? How many times does its surface area become?

$$= \frac{V \text{ of cube}}{V \text{ of new cube}} \Rightarrow \frac{2^3}{(2 \times 2)^3} \Rightarrow \frac{2^3}{8 \times 2^3} = \frac{1}{8}$$

Volume of new = $8 \times V$ of original cubes

$$\frac{\text{S.A. of original cube}}{\text{S.A. of new cube}} \Rightarrow \frac{6 \times 2^2}{6 \times (2 \times 2)^2} = \frac{6 \times 2^2}{6 \times 4 \times 2^2}$$

$$= \frac{1}{4}$$

S.A. of new cube = $4 \times$ S.A. of original cube.

vi) A solid cubical block of fine wood costs ₹ 256 at ₹ 500 per m^3 . Find its volume and the length of each side.

$$= \text{Volume of cube} = \frac{\text{Total Volume}}{\text{Rate}}$$

$$s^3 = \frac{256}{500}$$

$$s^3 = \left(\frac{4}{5}\right)^3$$

$$s = \frac{4}{5} \text{ m}$$

$$\frac{4}{5} \times 20$$

$$= 80 \text{ cm}$$

$$V = s^3$$

$$80 \times 80 \times 80$$

$$= 512000 \text{ cm}^3 \quad \text{Ans.}$$

vii) A solid cube of side 16cm is cut into eight equal part. find the side of each new cubes

= let side of new cube = x .

V. of original cube = $8 \times$ V. of new cube

$$16 \times 16 \times 16 = 8 \times x \times x \times x$$

$$\frac{16 \times 16 \times 16}{8} = x^3$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = x^3$$

$$(8)^3 = x^3 = x = 8$$

\therefore Side of new cube = 8cm.

viii.) A river 3m deep and 40m wide is flowing km/h. How much water will flow into the sea in 1 minutes.

= Speed of flowing water = 2km/h.

$$\frac{2 \times 100}{60} \times \frac{100}{3} \text{ km/min}$$

$$l = \frac{100}{3} \text{ m}, \quad b = 40 \text{ m}, \quad h = 3 \text{ m}.$$

$$\begin{aligned} \text{V. of water flow in 1 min} &= l \times b \times h \\ &= \left(\frac{100}{3} \times 40 \times 3 \right) \text{ m}^3 \\ &= 4000 \text{ m}^3 \end{aligned}$$

vii.) The area of the adjacent faces of the cuboid are 15 cm^2 , 20 cm^2 and 12 cm^2 . Find its volume.

$$lb = 15 \text{ cm}^2, bh = 20 \text{ cm}^2, hl = 12 \text{ cm}^2$$

$$lb \times bh \times hl = 15 \times 20 \times 12$$

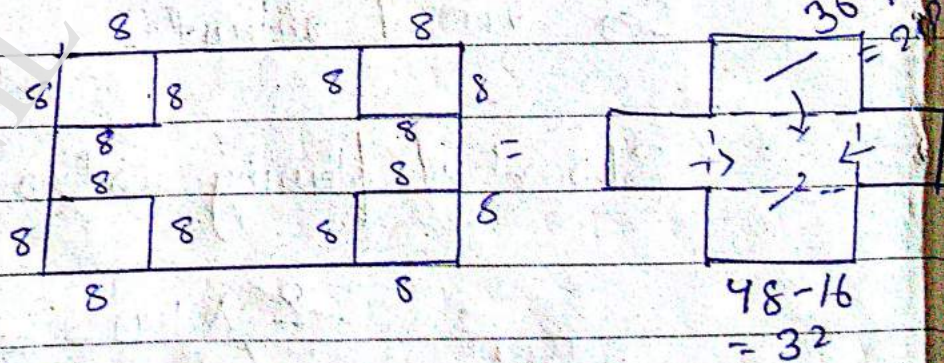
$$(l \times b \times h)^2 = 15 \times 20 \times 12$$

$$l \times b \times h = \sqrt{15 \times 20 \times 12}$$

$$= \sqrt{3 \times 5 \times 5 \times 4 \times 3 \times 4}$$

$$= 3 \times 5 \times 4 = 60 \text{ cm}^3$$

viii.) A rectangular sheet has dimensions $48 \text{ cm} \times 36 \text{ cm}$. From each of its corners a square side of 8 cm is cut off to make an open box. Find the volume of its box.



$$l = 32 \text{ cm}$$

$$b = 20 \text{ cm}$$

$$h = 8 \text{ cm}$$

$$V \text{ of open box} = l \times b \times h$$

$$= 32 \times 20 \times 8$$

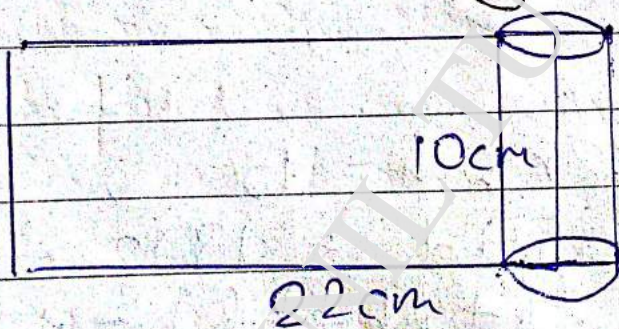
$$= 5120$$

xi.) If the volume of cuboid is 880 cm^3 and the area of base is 88 cm^2 . find its height.

$$h = \frac{\text{Volume}}{A}$$

$$= \left(\frac{880}{88} \right) \text{ cm} = 10 \text{ cm}$$

xii.) A rectangular piece of paper is 22 cm long and 10 wide. A cylinder is formed by rolling the paper along its length - find the volume of cylinder.



Circumference of base
= 22, $h = 10 \text{ cm}$.

$$= 2 \times \frac{22}{2} \times r = 22$$

$$= r = \frac{22 \times 2}{2 \times 22} = 3.5 \text{ cm}$$

$$V. \text{ of cylinder} = \pi r^2 h$$

$$= \frac{22}{7} \times 3.5 \times \frac{3.5 \times 10}{10}$$

$$= (22 \times 0.5 \times 3.5) \text{ cm}^3$$

$$= 385 \dots$$

(iii.) If the length of diagonal of the cube find the length of the edge of the cube.

$$\text{length of diagonal} = \sqrt{3a}$$

$$6\sqrt{2} = \sqrt{3a}$$

$$a = 6$$

(iv.) The surface area of cuboid is 1372 cm^2 if ratio of side are $4:2:1$. Find its lengths.

$$l = 4x, b = 2x, h = 1x$$

$$\text{S.A of cuboid} = 1372 \text{ cm}^2$$

$$= 2(lb + bh + hl) = 1372$$

$$= 2(4x \times 2x + 2x \times 1x + 1x \times 4x) = 1372$$

$$2 \times 14x^2 = 1372$$

$$x^2 = \frac{1372}{2 \times 14} = \frac{1372}{28} = 49$$

$$\therefore x = \sqrt{49} = 7$$

$$l = 28 \text{ cm}$$

$$b = 14 \text{ cm}$$

$$h = 7 \text{ cm}$$

xv.) The Circumference of the base of cylinder vessel is 132 cm and height is 25 cm. How much water can it hold.

$$\text{Circumference of base} = 132$$

$$2\pi r = 132$$

$$2 \times \frac{22}{7} \times r = 132$$

$$r = \frac{132 \times 7}{2 \times 22}$$

$$r = 21$$

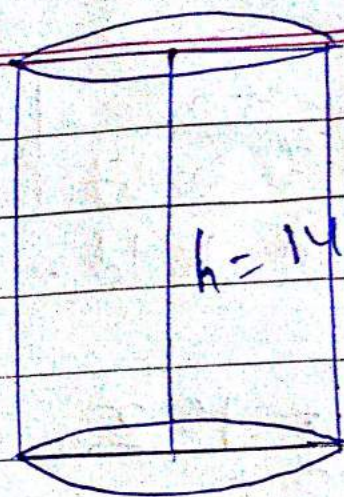
$$h = 25$$

$$V \text{ of cylinder} = \pi r^2 h$$

$$\frac{22}{7} \times 21 \times 21 \times 25$$

$$= \left(\frac{34650}{1000} \right) \text{ l} = 34.65 \text{ l.}$$

xvi.) A well of diameter 10 cm is dug upto 14 m deep. Earth taken out of it is spread all around it. To width 5 m to form an embankment. Find the height of embankment.



$$d = 10 \text{ cm}$$

$$r = 5 \text{ cm}$$

$$h = 14 \text{ cm}$$

$$V. \text{ of earth taken out} = \pi r^2 h$$

$$= \frac{22}{7} \times 5 \times 5 \times 14$$

$$= 1100 \text{ cm}^3$$

$$\text{width of embankment} = 5 \text{ m}$$

$$= R = 5 + 5 = 10 \text{ m}$$

$$V. \text{ of ring} = V. \text{ of earth}$$

$$= \pi (R^2 - r^2) \times h = 1100$$

$$= \frac{22}{7} \times (10^2 - 5^2) \times h = 1100$$

$$= \frac{22}{7} \times 75 \times h = 1100$$

$$h = \frac{1100 \times 7}{22 \times 75} = \frac{14}{3} \text{ m}$$