

(1.)

$$A^2 = A.$$

Only possible when  $A = I$ .

$$A/9 \quad 7A - (I+A)^3.$$

$$= 7I - (I+I)^3 \quad (A=I)$$

$$= 7I - 8I$$

$$= -I$$

(2.)

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}.$$

$$x-y = -1 \quad \text{--- (1)}$$

$$2x-y = 0 \quad \text{--- (2)}$$

From (2)

$$2x = y$$

$$y = 2$$

$$\therefore x+y = -1+2 = 1.$$

In eqn (1)

$$x-2x = -1$$

$$x = 1.$$

(3.)

$$\tan^{-1}x + \tan^{-1}y = \pi/4 \quad \text{--- } xy < 1.$$

taking tan on both sides.

$$\tan(\tan^{-1}x + \tan^{-1}y) = \tan \pi/4$$

$$\frac{x+y}{1-xy} = 1.$$

$$x+y = 1-xy$$

$$x+y+xy = 1.$$

$$(4.) \quad \begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$$

$$3x \times 4 - 7 \times (-2) = 8 \times 4 - 7 \times 6.$$

$$\circ 12x + 14 = 32 - 42.$$

$$12x = -10 - 14$$

$$12x = -24$$

$$x = -2.$$

$$(5.) \quad f(x) = \int_0^x t \sin t \, dt.$$

Using Integration By Parts.

$$= (-t \cdot \cos t) \Big|_0^x + \int_0^x 1 \cdot \cos t \, dt.$$

$$= \left\{ (-x \cos x) - (-0 \times \cos 0) \right\} + (\sin t) \Big|_0^x.$$

$$= -x \cos x - (\sin x \sin 0)$$

$$= -x \cos x - \sin x.$$

(6.)

$$\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$$

$$\vec{b} = \hat{i} - 2p\hat{j} + 3\hat{k}$$

for being Parallel.

$$\frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$$

$$\frac{3}{1} = \frac{-1}{p} = \frac{3}{1}$$

$$p = -1/3$$

(8) Cartesian Eq<sup>n</sup>.

$$\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4} = \lambda$$

$$x = 3 - 5\lambda \quad y = 7\lambda - 4 \quad z = \frac{4\lambda + 6}{2}$$
$$z = 2\lambda + 3$$

∴ vector eq<sup>n</sup>.

$$(x, y, z) = (3, -4, 3) + \lambda(-5, 7, 2).$$

7. ~~1.8.1~~

$$R = \{(x, y) : x + 2y = 8\}$$

Domain = R.

$$f(x) = \frac{8-x}{2} = 4 - \frac{x}{2}$$

Given No. are Natural

But here outcomes can be a Natural No, Whole No, or Fractional No, Integers, ...

∴ Range of the f(x) = R

(9.)  
=

$$\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$$

$$= \left( \frac{1}{2} \tan^{-1} \frac{x}{2} \right) \Big|_0^a = \frac{\pi}{8}$$

$$\therefore \left( \tan^{-1} \frac{x}{2} \right) \Big|_0^a = \frac{\pi}{4}$$

$$\tan^{-1} \frac{a}{2} = \frac{\pi}{4}$$

$$\frac{a}{2} = 1$$

(10.)  
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$$|\vec{a} + \vec{b}| = 13.$$

$$|\vec{a}| = 5$$

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta.$$

$$(13)^2 = 25 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos 90$$

$$169 - 25 = |\vec{b}|^2$$

$$|\vec{b}| = 12.$$

Sec B

(11.)  
=

$$(1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1}x}.$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}.$$

$$P(x) = \frac{1}{1+x^2}$$

$$Q(x) = \frac{e^{\tan^{-1}x}}{1+x^2}$$

$$IF = e^{\int P(x) dx}$$

$$= e^{\int \frac{1}{1+x^2} dx}.$$

$$= e^{\tan^{-1}x} + C.$$

$$\therefore y \cdot e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x} dx.$$

$$\text{Let } \tan^{-1}x = t$$

$$\frac{1}{x^2+1} dx = dt.$$

$$y \cdot e^{\tan^{-1}x} = \int \frac{e^{2t}}{1} dt$$

$$= \frac{e^{2t}}{2} + k$$



$$y \cdot e^{\tan^{-1} x} = \frac{2 \tan^{-1} x}{2} + c$$

$$y = \frac{e^{\tan^{-1} x}}{2} + c \cdot e^{-\tan^{-1} x}$$

(12.)

OR

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{d} = \vec{b} + \vec{c} = (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\hat{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + 36 + 4}}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

$$\vec{a} \cdot \hat{d} = \left( \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}} \right) \cdot (\hat{i} + \hat{j} + \hat{k})$$

$$1 = \frac{(2+\lambda) + (6) - 2}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

$$\lambda^2 + 4\lambda + 44 = \lambda^2 + 36 + 12\lambda$$

$$8\lambda = 8$$

$$\lambda = 1$$

$$\therefore \hat{d} = \frac{\vec{b} + \vec{c}}{|\vec{b} + \vec{c}|} = \frac{3\hat{i}}{7} + \frac{6\hat{j}}{7} - \frac{2\hat{k}}{7}$$

$$(13.) I = \int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x} dx.$$

Using King's

$$= \int_0^{\pi} \frac{4(\pi-x) \sin(\pi-x)}{1 + \cos^2(\pi-x)} dx.$$

$$= 4\pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx - \int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x} dx$$

$$I = 4\pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx - I.$$

~~4x~~

Let  $\cos x = t$   
 $-\sin x dx = dt$

$$\therefore 2I = -4\pi \int_1^{-1} \frac{dt}{1+t^2}$$

$$= -4\pi \left( \tan^{-1} t \right) \Big|_1^{-1}$$

Putting the value of  $t$

$$= -4\pi \left( \tan^{-1}(\cos x) \right) \Big|_0^{\pi}$$

$$= -4\pi \left( \tan^{-1}(\cos \pi) - \tan^{-1}(\cos 0) \right)$$

$$I = -2\pi \left( \tan^{-1}(-1) - \tan^{-1}(1) \right)$$

$$= -2\pi \left( -\frac{\pi}{4} - \frac{\pi}{4} \right)$$

$$= \pi^2.$$

(14.)  
=.

$$y = [x(x-2)]^2$$
$$= [x^2 - 2x]^2$$

$$y = x^4 + 4x^2 - 4x^3.$$

$$\frac{dy}{dx} = 4x^3 + 8x - 12x^2.$$
$$= x(4x^2 - 12x + 8).$$

$$\text{If, } \frac{dy}{dx} = 0,$$

$$x(4x^2 - 12x + 8) = 0.$$

Can only be possible when  $x=0$ .

Because  $D < 0$  of  $(4x^2 - 12x + 8)$

$$\therefore \frac{dy}{dx} > 0$$

As the given quadratic is always +ve  
than the fu. is always  $\uparrow$ .

(15.)  
=.

$$f: \mathbb{R} \rightarrow \mathbb{R}$$
$$f(x) = x^2 + 2$$

$$g: \mathbb{R} \rightarrow \mathbb{R}$$
$$g(x) = \frac{x}{x-1}, \quad x \neq 1$$

$$f \circ g(x) = \left(\frac{x}{x-1}\right)^2 + 2.$$

$$g \circ f(x) = \frac{x^2 + 2}{(x^2 + 2) - 1}$$

$$f \circ g(2) = \left(\frac{2}{2-1}\right)^2 + 2$$

$$g \circ f(2) = \frac{2+2}{2+2-1}$$

$$= 4 + 2$$

$$= \frac{11}{10}.$$

$$= 6.$$



(16.)  
=

$$\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$$

taking tan on Both side.

$$\frac{\left(\frac{x-2}{x-4}\right) + \left(\frac{x+2}{x+4}\right)}{1 - \left(\frac{x-2}{x+4}\right) \cdot \left(\frac{x+2}{x+4}\right)} = 1.$$

$$\frac{(x-2)(x+4) + (x+2)(x-4)}{(x+4)(x-4) - (x-2)(x+2)} = 1$$

$$\frac{x^2 + 2x - 8 + x^2 - 2x + 8 - 8}{x^2 - 16 - x^2 + 4} = 1$$

$$\frac{2x^2 - 16 - x^2 + 4}{x^2 - 16 - x^2 + 4} = 1$$

$$2x^2 - 16 = -12$$

$$2x^2 = 4$$

$$x^2 = 2$$

$$x = \pm \sqrt{2}$$

$$x = \pm \sqrt{2}$$



$$(18.) \quad y = Pe^{ax} + Qe^{bx}$$

$$\frac{dy}{dx} = Pa e^{ax} + Qb e^{bx}$$

$$\frac{d^2y}{dx^2} = Pa^2 e^{ax} + Qb^2 e^{bx}$$

$$\frac{d^2y}{dx^2} - (a+b) \frac{dy}{dx} + aby = 0.$$

$$\begin{aligned} \text{LHS} &= Pa^2 e^{ax} + Qb^2 e^{bx} - Pa^2 e^{ax} - Qab e^{bx} \\ &\quad - Pa b e^{ax} - Qb^2 e^{bx} + Pa b e^{ax} + Qab e^{bx} \\ &= 0 = \text{RHS.} \end{aligned}$$

(19.)  
=

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc + bc + ca + ab.$$

$$= \begin{vmatrix} a & 0 & 1 \\ -b & b & 1 \\ 0 & -c & 1+c \end{vmatrix}$$

$$= a(b+bc+c) + 1(bc-0)$$

$$= ab + abc + ac + bc$$

$$= ab + bc + ca + abc.$$

=

(20.)

$$x = \cos t (3 - 2 \cos^2 t)$$
$$= 3 \cos t - 2 \cos^3 t$$

$$\frac{dx}{dt} = -3 \sin t + 6 \cos^2 t \cdot \sin t$$
$$= -3 \sin t + 3 \cos t \cdot \sin 2t$$

$$y = \sin t (3 - 2 \sin^2 t)$$

$$= 3 \sin t - 2 \sin^3 t$$

$$\frac{dy}{dt} = 3 \cos t - 6 \sin^2 t \cdot \cos t$$
$$= 3 \cos t - 3 \sin t \cdot \sin 2t$$

$$\therefore \frac{dy}{dx} = \frac{3 \cos t - 3 \sin t \cdot \sin 2t}{-3 \sin t + 3 \cos t \cdot \sin 2t}$$

$$\left(\frac{dy}{dx}\right)_{t=\pi/4} = \frac{3 \cdot \frac{1}{\sqrt{2}} - 3 \cdot \frac{1}{\sqrt{2}} \cdot 1}{-3 \cdot \frac{1}{\sqrt{2}} + 3 \cdot \frac{1}{\sqrt{2}} \cdot 1}$$

$$\left(\frac{dy}{dx}\right)_{t=\pi/4} = \frac{3 \cdot \frac{1}{\sqrt{2}} - 3 \cdot \frac{1}{\sqrt{2}} \cdot 1}{-3 \cdot \frac{1}{\sqrt{2}} + 3 \cdot \frac{1}{\sqrt{2}} \cdot 1}$$
$$= - \left\{ \frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} \right\}$$

$$= -1$$

(21.)

$$\log \left(\frac{dy}{dx}\right) = 3u + 4y$$

$$\frac{dy}{dx} = e^{3u + 4y}$$

$$\frac{dy}{dx} = e^{3u} \cdot e^{4y}$$

$$\frac{dy}{e^{4y}} = e^{3u} dx$$

$$\int e^{-4y} dy = \int e^{3u} du.$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3u}}{3} + c.$$

$$3e^{-4y} = -4e^{3u} - 4c$$

$$3e^{-4y} + 4e^{3u} = -4c.$$

Putting the value of  $x$  &  $y$ .

$$x=0 \quad y=0.$$

$$3 \times 1 + 4 \times 1 = -4 \times c$$

$$7 = -4 \times c$$

$$c = -\frac{7}{4}$$

$\therefore$  Particular Soln:

$$\frac{3e^{-4y}}{3}$$

$$3e^{-4y} + 4e^{3u} = 7$$

$$\frac{4e^{3u}}{4} + 3 = 7e^{4y}$$

$$4e^{3u} \cdot e^{4y} + 3 = 7e^{4y}$$

$$4e^{3u+4y} + 3 = 7e^{4y}$$

(22.)  
2

$$l_1 = \frac{1-7}{3} = \frac{7y-14}{9p} = \frac{z-3}{2}$$

$$l_2 = \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

⊙ Given Point,  
P(3, 2, -4).

Dir. cosines,

$$\text{line } l_1 = (3, p, 2)$$

$$\text{line } l_2 = (3p, 1, 5).$$

Given that Both of the lines are  $\perp$ .

$$\therefore (\text{Dir. cosine})_{l_1} \cdot (\text{Dir cosine})_{l_2} = 0.$$

$$(3, p, 2) \cdot (3p, 1, 5) = 0$$

$$9p + p + 10 = 0$$

$$9p + p + 10 = 0$$

$$10p = -10$$

$$p = -1$$

$\therefore$  dir cosine of line  $l_1 = (3, -1, 2)$ .

dir. cosine of line  $l_2 = (-3, 1, 5)$ .

Req. eq<sup>n</sup> of line =  $(3, 2, -4) + \lambda(3, -1, 2)$ .

SEC-3.



$$m = \frac{4-3}{3-1} = \frac{-1}{2}$$

17.

As given in question.

$$P(S) = 3P(F)$$

Let  $P(F)$  be  $1/4$

$$\therefore P(S) = 3/4$$

Using Bernoulli's Theorem;

$${}^n C_r (P(F))^r (P(S))^{n-r}$$

$$P(\text{Getting } \geq 3S) = {}^5 C_3 (P(F))^3 (P(S))^2 + {}^5 C_4 (P(F))^4 (P(S))^1 + {}^5 C_5 (P(F))^5$$

$$= {}^5 C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 + {}^5 C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^1 + {}^5 C_5 \left(\frac{1}{4}\right)^5$$

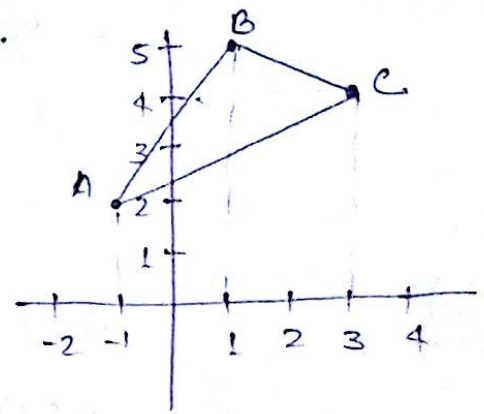
expand

$$= (90 + 15 + 1) \frac{1}{4^5}$$

$$= \frac{106}{1024} = \frac{53}{512}$$

(24.)  
=.

$(-1, 2) ; (1, 5) ; (3, 4)$ .



Eq. of AB.

$$m = \frac{5-2}{1-(-1)} = \frac{3}{2}$$

$$(y-5) = \frac{3}{2}(x-1)$$

$$y = \frac{3x}{2} + \frac{7}{2}$$

Eq. of BC.

$$m = \frac{4-5}{3-1} = -\frac{1}{2}$$

$$(y-4) = -\frac{1}{2}(x-3)$$

$$y = -\frac{x}{2} + \frac{5}{2}$$

Eq. of CA.

$$m = \frac{2-4}{-1-3} = \frac{1}{2}$$

$$(y-2) = \frac{1}{2}(x+1)$$

$$y = \frac{x}{2} + \frac{3}{2}$$

$$= \int_{-1}^1 \left(\frac{3x}{2} + \frac{7}{2}\right) dx + \int_1^3 \left(-\frac{x}{2} + \frac{5}{2}\right) dx - \int_{-1}^3 \left(\frac{x}{2} + \frac{3}{2}\right) dx$$

$$= \frac{3}{4}(1+1) + \frac{7}{2}(1+1) + \frac{1}{4}(9-1) + \frac{5}{2}(3-1)$$

$$- \frac{1}{4}(9-1) - \frac{3}{2}(3+1)$$

$$= 7 + 2 + 5 - 2 - 6$$

$$= 14 - 8 = \underline{\underline{6 \text{ sq. unit.}}}$$

25.)

	<u>fab.</u>	<u>fin.</u>	<u>Profit.</u>
A (x)	9	1.	₹ 80
B (y)	12	3.	₹ 120.
	<u>180</u>	<u>30.</u>	

$$9x + 12y = 180$$

$$x + 3y = 30.$$

$$R = 80x + 120y$$

$$9x + 12y = 180$$

$$9x + 12y = 180$$

$$\hline 15y = 90$$

$$15y = 90$$

$$y = \frac{90}{15} = 6.$$

max profit = 80x + 120y

$$x = 30 - 6 \times 3.$$

$$= 30 - 18.$$

$$= \underline{\underline{12}}$$

x	12	0	30	0	20.
y	6	10	0	15	0

Max. Profit.

30 +



$$(0, 10) \Rightarrow$$

$$R = .80 \times 0 + 120 \times 10 \\ = 1200.$$

$$\swarrow (12, 6)$$

$$R = 80 \times 12 + 120 \times 6 \\ = .960 + 720 = 1680$$

$$(20, 0)$$

$$R = 80 \times 20 + 120 \times 0 \\ = 1600$$

$$12 \rightarrow A$$

$$6 \rightarrow B$$

(26.)

$C_{01}$

$C_{02}$

$C_{03}$

H T

H H

T H

↑

↑

↑

75% H

$P(H) = 1$

40% T

$$P(H) = \frac{75}{100} \\ = 3/4$$

$$P(T) = \frac{40}{100} \\ P(H) = 3/5$$

$$P(\text{Coin}) = 1/3$$

$$1/3 \times 1$$

$$P(H_2) = \frac{3/4 \times 1/3 + 1/3 \times 1 + 3/5 \times 1/3}{1/3}$$

$$= \frac{1/3}{15 + 20 + 12} \\ = \frac{1/3}{47}$$

$$= \frac{20}{47}$$



(27.)  
=.

$$A \begin{matrix} \text{Sin.} & \text{Dry} & \text{Help} \\ \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 900 \\ 1600 \\ 2300 \end{bmatrix} \end{matrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } |A|$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{vmatrix} = 1(6-1) - 1(9-4) + 1(3-8)$$

engus

$$= 5 - 5 + (-5) = \underline{-5}$$

$$\text{adj } |A| = \begin{bmatrix} 5 & -2 & 1 \\ -5 & -1 & 4 \\ -5 & 4 & -1 \end{bmatrix}$$

$$C_{(1,1)} = 6-1 = 5$$

$$C_{(2,1)} = 9-4 = 5$$

$$C_{(3,1)} = 3-8 = -5$$

$$C_{(1,2)} = 3-1 = 2$$

$$C_{(2,2)} = (8-4) = -4$$

$$C_{(3,2)} = 1-4 = -3$$

$$C_{(1,3)} = 1-2 = -1$$

$$C_{(2,3)} = 1-3 = -2$$

$$C_{(3,3)} = (2-8) = -6$$

$$A^{-1} = \begin{bmatrix} -1 & +2/5 & 1/5 \\ \cancel{\#}L & 1/5 & -2/5 \\ \cancel{\#}L & -3/5 & 1/5 \end{bmatrix}$$

Now

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 2/5 & 1/5 \\ \cancel{\#}L & 1/5 & -2/5 \\ \cancel{\#}L & -3/5 & 1/5 \end{bmatrix} \begin{bmatrix} 900 \\ 1600 \\ 2300 \end{bmatrix}$$

$$\begin{bmatrix} -900 + 640 + 460 \\ 900 + 320 - 920 \\ 900 - 960 + 460 \end{bmatrix}$$

$$\begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

$$x = ₹ 200$$

$$y = ₹ 300$$

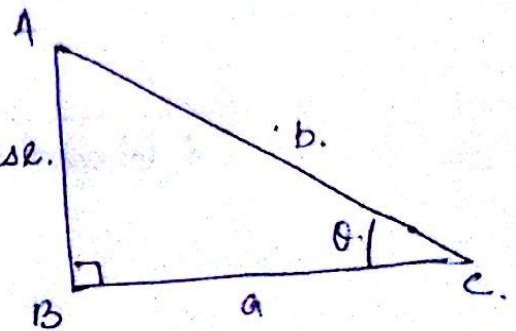
$$z = ₹ 400$$

Another value should be discipline.

28.

Given is sum of a side and the hypotenuse.

To Prove is Max. Area of  $\Delta$  will be only when Angle Between them is  $60^\circ$



Proof:  $A = \frac{1}{2} ab \sin \theta$ .

$$\left[ \frac{dA}{d\theta} \right]_{\theta = \frac{\pi}{3}} = \frac{1}{2} ab \cos \theta$$

$$\frac{d^2A}{d\theta^2} = -\frac{1}{2} ab \sin \theta$$

$$\left. \frac{d^2A}{d\theta^2} \right|_{\theta = \frac{\pi}{3}} = -\frac{1}{2} ab \frac{\sqrt{3}}{2}$$

$$\therefore \text{At } \theta = \frac{\pi}{3} \quad \frac{d^2A}{d\theta^2} = \frac{-\sqrt{3} \cdot ab}{4} < 0.$$

area is maximum.



(23)  
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Given Planes;

$$x + y + z = 1.$$

$$2x + 3y + 4z = 5$$

Eq. of Plane passing through line of int.

~~(1+2λ)~~

$$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z = 1+5\lambda.$$

It is  $\perp$  to Plane.

$$x - y + z = 0.$$

$\therefore$  Scalar Product of the dir cosine

$$(1+2\lambda) \cdot 1 + (1+3\lambda)(-1) + (1+4\lambda) \cdot 1 = 0$$

$$1+2\lambda + (-1) - 3\lambda + 1+4\lambda = 0$$

$$3\lambda + 1 = 0$$

$$\lambda = -\frac{1}{3}$$

$\therefore$  Eq. of Plane =

$$\left(1 - \frac{2}{3}\right)x + \left(1 - \frac{3}{3}\right)y + \left(1 + 4 \cdot -\frac{1}{3}\right)z = 1 + 5 \cdot -\frac{1}{3}$$

$$\frac{1}{3}x + 0y - \frac{1}{3}z = -\frac{2}{3}$$

$\therefore$  Distance Between the Plane & Origin.

$$d = \left| \frac{0 + 0 + 0 + \frac{2}{3}}{\sqrt{\frac{1}{9} + 0 + \frac{1}{9}}} \right|$$

$$= \left| \frac{\frac{2}{3}}{\frac{\sqrt{2}}{3}} \right| = \underline{\underline{\sqrt{2} \text{ units.}}}$$



29

$$\int \frac{1}{\sin^4 x + \sin^2 x \cdot \cos^2 x + \cos^4 x} dx$$

$$= \int \frac{\sec^4 x}{\tan^4 x + \tan^2 x + 1} dx$$

Let  $\tan x = t$

$$= \int \frac{\sec^2 x dt}{t^4 + t^2 + 1}$$

$$= \sec^2 x \int \frac{1}{t^4 + t^2 + 1} dt$$

$$= \sec^2 x \int \frac{1}{(t^2 + \frac{1}{2})^2 + \frac{3}{4}} dt$$

Using  $\int \frac{1 dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$$= \sec^2 x \left\{ \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{t^2 + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \right\} + c$$

$$= \frac{2 \sec^2 x}{\sqrt{3}} \tan^{-1} \left( \frac{2t^2 + 1}{\sqrt{3}} \right) + c$$

Putting the value of  $t$ .

$$= \frac{2 \sec^2 x}{\sqrt{3}} \tan^{-1} \left( \frac{2 \tan^2 x + 1}{\sqrt{3}} \right) + c$$

$$= \frac{2 \sec^2 x}{\sqrt{3}} \tan^{-1} \left( \frac{\tan^2 x + \sec^2 x}{\sqrt{3}} \right) + c$$