# TRIGNOMETRY

## KEY POINTS

1. **Trignometrical Ratios :** In $\Delta ABC$, $\angle B = 90°$ for angle '$A$'

$$\sin A = \frac{Perpendicular}{Hypotenuse}$$

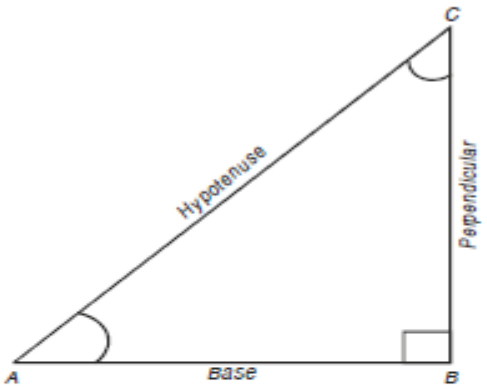
$$\cos A = \frac{Base}{Hypotenuse}$$

$$\tan A = \frac{Perpendicular}{Base}$$

$$\cot A = \frac{Base}{Perpendicular}$$

$$\sec . A = \frac{Hypotenuse}{Base}$$

$$\text{cosec } A = \frac{Hypotenuse}{Perpendicular}$$




2. **Reciprocal Relations :**

$$\sin\theta = \frac{1}{\text{cosec }\theta} \quad , \quad \text{cosec }\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad , \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad , \quad \cot\theta = \frac{1}{\tan\theta}$$

3. **Quotient Relations :**

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad , \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

4. **Indentities :**

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

6. **Trignometric Ratios of Complementary Angles**

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \text{cosec}\,\theta$

$\text{cosec}(90° - \theta) = \sec\theta$

5. **Trignometric Ratios of Some Specific Angles :**

| $\angle A$ | 0° | 30° | 45° | 60° | 90° |
|---|---|---|---|---|---|
| sin $A$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos $A$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| tan $A$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | Not defined |
| cosec $A$ | Not defined | 2 | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | 1 |
| sec $A$ | 1 | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | 2 | Not defined |
| cot $A$ | Not defined | $\sqrt{3}$ | 1 | $\frac{1}{\sqrt{3}}$ | 0 |

## MULTIPLE CHOICE QUESTIONS

**Note :** In the following questions $0° \leq \theta \leq 90°$

1. If $x = a \sin\theta$ and $y = a \cos\theta$ then the value of $x^2 + y^2$ is _____
   (a) $a$ (b) $a^2$
   (c) 1 (d) $\frac{1}{a}$
2. The value of cosec 70° − sec 20° is _____
   (a) 0 (b) 1
   (c) 70° (d) 20°
3. If 3 sec $\theta$ − 5 = 0 then cot $\theta$ = _____
   (a) $\frac{5}{3}$ (b) $\frac{4}{5}$
   (c) $\frac{3}{4}$ (d) $\frac{3}{5}$
4. If $\theta$ = 45° then sec $\theta$ cot $\theta$ − cosec $\theta$ tan $\theta$ is
   (a) 0 (b) 1
   (c) $\sqrt{2}$ (d) $2\sqrt{2}$
5. If sin (90 − $\theta$) cos $\theta$ = 1 and $\theta$ is an acute angle then $\theta$ = _____
   (a) 90° (b) 60°
   (c) 30° (d) 0°
6. The value of (1 + cos $\theta$) (1 − cos $\theta$) $\text{cosec}^2\theta$ = _____
   (a) 0 (b) 1
   (c) $\cos^2\theta$ (d) $\sin^2\theta$
7. $\Delta TRY$ is a right-angled isosceles triangle then cos $T$ + cos $R$ + cos $Y$ is _____
   (a) $\sqrt{2}$ (b) $2\sqrt{2}$
   (c) $1 + \sqrt{2}$ (d) $1 + \frac{1}{\sqrt{2}}$

8. If $K + 7\sec^2 62° - 7\cot^2 28° = 7\sec 0°$ then the value of $K$ is _____

(a) 1 (b) 0

(c) 7 (d) $\frac{1}{7}$

9. The value of $\cot\theta - \sin\left(\frac{\pi}{2} - \theta\right)\cos\left(\frac{\pi}{2} - \theta\right)$ is _____

(a) $\cot\theta\cos^2\theta$ (b) $\cot^2\theta$

(c) $\cos^2\theta$ (d) $\tan^2\theta$

10. If $\sin\theta - \cos\theta = 0$, $0 \le \theta \le 90°$ then the value of $\theta$ is _____

(a) $\cos\theta$ (b) 45°

(c) 90° (d) $\sin\theta$

11. $\frac{\sin\theta}{\sqrt{1 - \sin^2\theta}}$ can be written as

(a) $\cot\theta$ (b) $\sqrt{\sin\theta}$

(c) $\frac{\sin\theta}{\sqrt{\cos\theta}}$ (d) $\tan\theta$

12. If $\sin\theta = \frac{1}{2}$ then the value of $\sin\theta + \text{cosec}\,\theta$ is _____

(a) 0 (b) 1

(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

13. In an isosceles right-angled $\Delta ABC$, $\angle B = 90°$. The value of $2\sin A\cos A$ is _____

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

14. If $\frac{\sin^2 20° + \sin^2 70°}{2\left(\cos^2 69° + \cos^2 21°\right)} = \frac{\sec 60°}{K}$ then $K$ is _____

(a) 1 (b) 2

(c) 3 (d) 4

15. $\Delta ABC \sim \Delta PRT$ and $\angle C = \angle R = 90°$. If $\frac{AC}{AB} = \frac{3}{5}$ then $\sin T$ is _____

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$

(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

## SHORT ANSWER TYPE QUESTIONS

16. In $\Delta PQR$, $\angle Q = 90°$ and $\sin R = \frac{3}{5}$, write the value of $\cos P$.

17. If $A$ and $B$ are acute angles and $\sin A = \cos B$ then write the value of $A + B$.

18. If $4 \cot \theta = 3$ then write the value of $\tan \theta + \cot \theta$.

19. Write the value of $\cot^2 30° + \sec^2 45°$.

20. Write the value of $\sin (90 - \theta) \cos \theta + \cos (90 - \theta) \sin \theta$.

21. If $\theta = 30°$ then write the value of $\sin \theta + \cos^2 \theta$.

22. If $1 - \tan^2 \theta = \frac{2}{3}$ then what is the value of $\theta$.

23. What is the value of $2 \operatorname{cosec}^2 \theta + 3 \sec^2 \theta - 10$ if $\theta = 45°$.

24. If $\theta$ and $\phi$ are complementary angles then what is the value of

$$\operatorname{cosec} \theta \sec \phi - \cot \theta \tan \phi$$

25. If $\tan (3x - 15°) = 1$ then what is the value of $x$.

26. If $8 \cot \theta - 15 = 0$ then what is the value of $\frac{1 + \sin \theta}{\cos \theta}$.

## LONG ANSWER TYPE QUESTIONS

27. Simplify :

$$\tan^2 60° + 4 \cos^2 45° + 3 (\sec^2 30° + \cos^2 90°)$$

28. Find the value of

$$\frac{4 \sin 65°}{} - \frac{13 \cos 53° . \operatorname{cosec} 37°}{}$$

29. Prove that

$$\operatorname{cosec}^4 \theta - \operatorname{cosec}^2 \theta = \cot^2 \theta + \cot^4 \theta.$$

30. If $\sin \theta + \sin^2 \theta = 1$ then find the value of $\cos^2 \theta + \cos^4 \theta$.

31. If $\sin 2\theta = \cos (\theta - 36°)$, $2\theta$ and $\theta - 26°$ are acute angles then find the value of $\theta$.

32. If $\sin (3x + 2y) = 1$ and $\cos (3x - 2y) = \frac{\sqrt{3}}{2}$, where $0 \leq (3x + 2y) \leq 90°$ then find the value of $x$ and $y$.

33. If $\sin (A + B) = \sin A \cos B + \cos A \sin B$ then find the value of

(a) $\sin 75°$

(b) $\cos 15°$

34. Prove that $\frac{\cos A}{1 - \tan A} + \frac{\cos A}{1 - \cot A} = \cos A$, $A \neq 45°$.

35. Prove that $\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta$

36. Find the value of

$$\sin^2 5° + \sin^2 10° + \sin^2 15° + .... + \sin^2 85°$$

37. Prove that

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{1 - \sin\theta}.$$

38. If $2\sin(2x - 15) = \sqrt{3}$ then find the value of $\sin^2(2x + 10) + \tan^2(x + 5)$.

39. Find the value of sin 60° geometrically.

40. Let $p = \tan\theta + \sec\theta$ then find the value of $P + \frac{1}{p}$.

41. In right angled $\Delta OPQ$, right angle at $P$, $OP = 7$ cm and $\angle Q = \alpha$. If $\sec(90 - \alpha) - \tan(90 - \alpha) = \frac{1}{7}$ then what is the value of $OQ - PQ$.

42. If $\sin\alpha = a\sin\beta$ and $\tan\alpha = b\tan\beta$ then prove that $\cos^2\alpha = \frac{a^2 - 1}{b^2 - 1}$.

43. $\pi\theta$ is acute angle and $5\sin^2\theta + \cos^2\theta = 4$ then find the value of $\theta$.

44. In an acute angled $\Delta ABC$, if $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$ then find angles $A$, $B$ and $C$.

45. If $A$, $B$, $C$ are the interior angles of a triangle $ABC$, show that

$$\sin\left(\frac{B + C}{2}\right)\cos\frac{A}{2} + \cos\left(\frac{B + C}{2}\right)\sin\frac{A}{2} = 1.$$

## ANSWERS

1. *b*
2. *a*
3. *c*
4. *a*
5. *d*
6. *b*
7. *a*
8. *b*
9. *a*
10. *b*
11. *d*
12. *d*
13. *a*
14. *d*
15. *a*
16. cos *P* =
17. 90°
18. $\frac{25}{12}$
19. 5
20. 1
21. $\frac{5}{4}$
22. 30°
23. 0
24. 1
25. $x = 20$.
26. $\frac{5}{3}$
27. 9
28. $\frac{3}{7}$
30. 1
31. 42°
32. $x = 20$, $y = 15$
33. $\frac{\sqrt{3}+1}{2\sqrt{2}}, \frac{\sqrt{3}+1}{2\sqrt{2}}$, take $A = 45°$, $B = 30°$
34. –
35. –
36. $\frac{17}{2}$
37. –
38. $\frac{13}{12}$
39. –
40. 2 sec θ
41. 1
42. –
43. 60°
44. $\angle A = 67.5°$, $\angle B = 37.5°$, $\angle C = 75°$