10th Trigonometry Identities

1. Simplify: $(1+\tan^2\theta)(1-\sin\theta)(1+\cos\theta)$

2. Evaluate: $(\sec^2\theta-1)(1-\operatorname{cosec}^2\theta)$

3. Evaluate: $4(\sin^4 30^0+\cos^4 60^0)-\frac{2}{3}(\sin^2 60^0-\cos^2 45^0)+\frac{1}{2}\sin^2 60^0$

4. Evaluate: $\frac{2}{3}(\cos^4 30^0-\sin^4 45^0)-3(\sin^2 60^0-\sec^2 45^0)+\frac{1}{4}\cot^2 30^0$

Prove the following indentities :

5. $\frac{\sin\theta}{1-\cos\theta}=\operatorname{cosec}\theta+\cot\theta$

6. $\frac{\tan\theta+\sin\theta}{\tan\theta-\sin\theta}=\frac{\sec\theta+1}{\sec\theta-1}=\frac{1+\cos\theta}{1-\cos\theta}$

7. $\frac{1-\cos\theta}{1+\cos\theta}=(\operatorname{cosec}\theta-\cot\theta)^2$

8. $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\cot^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}$

9. $(\sin\theta+\operatorname{cosec})^2+(\cos\theta+\sec\theta)^2=7+\tan^2\theta+\cot^2\theta$

10. $\sec^4\theta-\sec^2\theta=\tan^4\theta+\tan^2\theta=\sec^2\theta.\tan^2\theta$

11. $\cot\theta-\tan\theta=\frac{2\cos^2\theta-1}{\sin\theta\cos\theta}$

12. $\frac{1-\sin\theta}{\cos\theta}+\frac{\cos\theta}{1-\sin\theta}=2\sec\theta$

13. $\sec^2\theta+\operatorname{cosec}^2\theta=\sec^2\theta.\operatorname{cosec}^2\theta$

14. $\frac{1+\cos\theta}{1-\cos\theta}=(\operatorname{cosec}\theta+\cot\theta)^2$

15. $\frac{1+\sin\theta}{1-\sin\theta}=(\sec\theta+\tan\theta)^2$

16. $(\operatorname{cosec}\theta-\sin\theta)(\sec\theta-\cos\theta)=\frac{1}{\tan\theta+\cot\theta}$

17. $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

18. $2\sec^2\theta-\sec^4\theta-2\operatorname{cosec}^2\theta+\operatorname{cosec}^4\theta=\cot^4\theta-\tan^4\theta$

19. $\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\sin\theta+\cos\theta$

20. $\frac{\sin\theta}{\cot\theta+\operatorname{cosec}\theta}=2+\frac{\sin\theta}{\cot\theta-\operatorname{cosec}\theta}$

21. $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\operatorname{cosec}\theta$

22. $(\sin\theta+\sec\theta)^2+(\cos\theta+\operatorname{cosec}\theta)^2=(1+\sec\theta\operatorname{cosec}\theta)^2$

23. $\frac{\cos^2\theta}{1-\tan\theta}+\frac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\cos\theta$

24. $\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}+\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=\frac{2}{2\sin^2\theta-1}$

25. $\frac{\sin\theta+1-\cos\theta}{\cos\theta-1+\sin\theta}=\frac{1-\sin\theta}{\cos\theta}$

26. $(1-\sin\theta+\cos\theta)^2=2(1+\cos\theta)(1-\sin\theta)$

27. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}+\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=2\operatorname{cosec}\theta$

28. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}+\sqrt{\frac{1-\sin\theta}{1+\sin\theta}}=2\sec\theta$

29. $\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=2\operatorname{cosec}\theta$

30. $\frac{1}{\operatorname{cosec}\theta+\cot\theta}-\frac{1}{\sin\theta}=\frac{1}{\sin\theta}-\frac{1}{\operatorname{cosec}\theta-\cot A}$

31. $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$

32. $\frac{\cos A}{1-\tan A}-\frac{\sin^2 A}{\cos A-\sin A}=\sin A+\cos A$

33. $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)}=\cot\theta$

34. $\frac{\operatorname{cosec}A}{\operatorname{cosec}A-1}+\frac{\operatorname{cosec}A}{\operatorname{cosec}A+1}=2+2\tan^2 A$

35. If $\sin\theta+\cos\theta=\sqrt{2}\cos(90^0-\theta)$, determine $\cot\theta$

36. If $\cos\theta + \cos\theta = \sqrt{2}\cos\theta,$ Show taht $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

37. If $\tan\theta + \sin\theta = m \text{ and } \tan\theta - \sin\theta = n,$ Show that $m^2 - n^2 = 4\sqrt{mn}$

38. If $\sin\theta + \cos\theta = p \text{ and } \sec\theta + \operatorname{cosec}\theta = q$ Show that $q(p^2 - 1) = 2p$

39. If $3\tan\theta = 4,$ find the value of $\dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$

40. If $3\cot\theta = 4,$ find the value of $\dfrac{5\cos\theta - 2\sin\theta}{5\cos\theta + 3\sin\theta}$

41. If $\dfrac{\cos\alpha}{\cos\beta} = m \text{ and } \dfrac{\cos\alpha}{\sin\beta} = n,$ show that $(m^2 + n^2)\cos^2\beta = n^2$

42. $a\cos\theta - b\sin\theta = x \text{ and } a\sin\theta + b\cos\theta = y,$ prove that $a^2 + b^2 = x^2 + y^4$

43. If $\sec\theta = x + \dfrac{1}{4x},$ prove that $\sec\theta + \tan\theta = 2x \text{ or } \dfrac{1}{2x}$

44. If $x = p\sec\theta + q\tan\theta$ and $y = p\tan\theta + q\sec\theta,$ prove that $x^2 - y^2 = p^2 - q^2$

45. If $\sin\theta + \sin^2\theta = 1$ prove that $\cos^2\theta + \cos^4\theta = 1$