EXAMPLE 1 : Multiple Choice Question

Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must be satisfied as

(a) $1 < r < b$ (b) $0 < r \le b$ (c) $0 \le r < b$ (d) $0 < r < b$

[NCERT EXEMPLAR]

SOLUTION : r is the smallest non-negative integer satisfying $a = bq + r$ such that $r < b$.
Therefore, $0 \le r < b$.
So, the option **(c)** is correct, which is the required answer, *i.e.* answer **(c)**.

EXAMPLE 2 : Multiple Choice Question

For some integer m, every even integer is of the form

(a) m (b) $m + 1$ (c) $2m$ (d) $2m + 1$

[NCERT EXEMPLAR]

SOLUTION : Every even integer is a multiple of 2.
So, the option **(c)** is correct, which is the required answer, *i.e.* answer **(c)**.

EXAMPLE 3 : Multiple Choice Question

Euclid's division algorithm is based on

(a) *Euclid's division lemma* (b) *rational number*
(c) *fundamental theorem of arithmetic*
(d) *irrational number*

SOLUTION : Euclid's division algorithm is based on Euclid's division lemma.
So, the option **(a)** is correct, which is the required answer, *i.e.* answer **(a)**.

EXAMPLE 4 : Multiple Choice Question

For some integer q, every odd integer is of the form

(a) $2q$ Even (b) q Even or odd
(c) $2q + 1$ Even (d) $q + 1$ Evenor odd

SOLUTION : We know that every odd integer is either not divisible by 2 or on dividing by 2 leaves 1 as remainder.

EXAMPLE 5 : *Are 0 and 1 only the values of the remainder r, when a positive integer 'a' is divided by 3 ?*

Multiple Choice Question

(a) *yes* (b) *no* (c) *can't say* (d) *none of these*

SOLUTION : According to Euclid's division lemma, $a = 3q + r$, where $0 \leq r < 3$ and r is an integer. Therefore, the values of r can be 0, 1 or 2.
So, the option **(b)** is correct, which is the required answer, *i.e.* answer **(b)**.

EXAMPLE 6 : *Show that any positive integer is of the form 3q or 3q + 1 or 3q + 2 for some integer q.*

SOLUTION : Let a be any positive integer and $b = 3$.
Using division Lemma, we have

$$a = 3q + r, \text{ where } r = 0, 1, 2 \text{ and } q \text{ is an integer.}$$

$$\Rightarrow \quad a = 3q + 0 \quad \text{or} \quad a = 3q + 1 \quad \text{or} \quad a = 3q + 2$$

$$\Rightarrow \quad a = 3q \quad \text{or} \quad a = 3q + 1 \quad \text{or} \quad a = 3q + 2$$

Hence proved.

EXAMPLE 07 : *Prove that an odd positive integer should be of the form 8k + 1 to be a perfect square.*

SOLUTION : Let $a = 2p + 1$ [$\because$ *a is an odd number*]

Now, $a^2 = (2p + 1)^2 = 4p^2 + 4p + 1 = 4p(p + 1) + 1$

$= 4 \times 2k + 1$ [$\because$ *Product of two consecutive numbers is always even*]

$= 8k + 1$ **Hence proved.**

EXAMPLE 08 : *Show that only one of the numbers $(n + 2)$, n and $(n + 4)$ is divisible by 3.*

SOLUTION : Let n be any positive integer.

On dividing n by 3, let q be the quotient and r be the remainder.

Then, by Euclid's division lemma, we have

$n = 3q + r$, where $0 \le r < 3$ $\boxed{r = 0, 1 \text{ or } 2}$

$\Rightarrow$ $n = 3q$ (where $r = 0$), $n = 3q + 1$ (when $r = 1$) and $n = 3q + 2$(when $r = 2$)

Then following table shows the divisibility of n, $(n + 2)$ and $(n + 4)$ by 3.

	Positive Integer (n)	n	$n + 2$	$n + 4$
Case 1	When $n = 3q$	$3q$	$(3q) + 2$	$(3q) + 4 = 3(q + 1) + 1$
	Division by 3	**Divisible**	Leaves remainder 2 $\therefore$ not divisible	Leaves remaidner 1 $\therefore$ not divisible
Case 2	When $n = 3q + 1$	$3q + 1$	$(3q + 1) + 2 = 3(q + 1)$	$(3q + 1) + 4 = 3(q + 1) + 2$
	Division by 3	Leaves remainder 1 $\therefore$ not divisible	**Divisible**	Leaves remaidner 2 $\therefore$ not divisible
Case 3	When $n = 3q + 2$	$3q + 2$	$(3q + 2) + 2 = 3(q + 1) + 1$	$(3q + 2) + 4 = 3(q + 2)$
	Division by 3	Leaves remainder 2 $\therefore$ not divisible	Leaves remaidner 1 $\therefore$ not divisible	**Divisible**

EXAMPLE 09 : *Prove that the product of two odd numbers of the form $4n + 1$ is of the form $4n + 1$.*

SOLUTION : Let us assume two numbers of the form $4n + 1$,

i.e., $a = 4h + 1$ and $b = 4k + 1$.

Now, $a \times b = (4h + 1)(4k + 1) = 16hk + 4h + 4k + 1$

$= 4(4hk + h + k) + 1 = 4 \times n + 1$, where $n = 4hk + h + k$

Clearly, $a \times b$ is of the form $4n + 1$. **Hence proved.**

EXAMPLE 10 : *If a and b are two odd prime numbers, show that $a^2 - b^2$ is composite.*

SOLUTION : Let a and b be odd numbers *i.e.,* $a = 2k + 1$ and $b = 2h + 1$

We have $a^2 - b^2 = (a - b)(a + b)$

Now,
$$a - b = (2k + 1) - (2h + 1) = 2(k - h)$$
$$a + b = (2k + 1) + (2h + 1) = 2(k + h + 1)$$

Since, neither of the two divisors $(a - b)$ or $(a + b)$ is equal to 1, therefore $a^2 - b^2$ is composite. **Hence proved.**

EXAMPLE 11 : *If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$, then the value of y is*

Multiple Choice Question

(a) 5 (b) – 5 (c) 19 (d) – 19

SOLUTION : Applying Euclid's division lemma on 210 and 55, we get

$$210 = 55 \times 3 + 45 \qquad ...(i)$$
$$55 = 45 \times 1 + 10 \qquad ...(ii)$$
$$45 = 4 \times 10 + 5 \qquad ...(iii)$$
$$10 = 5 \times 2 + 0$$

$$\left[\begin{array}{r} 3 \\ 55\overline{)\,210} \\ \underline{165} \\ \underline{45} \end{array}\right] \quad \left[\begin{array}{r} 1 \\ 45\overline{)\,55} \\ \underline{45} \\ \underline{10} \end{array}\right] \quad \left[\begin{array}{r} 4 \\ 10\overline{)\,45} \\ \underline{40} \\ \underline{5} \end{array}\right]$$

We observe that the remainder at this stage is zero.
So, the last divisor *i.e.* 5 is the HCF of 210 and 55.

$\therefore \quad 5 = 210 \times 5 + 55y$

$\Rightarrow \quad 55y = 5 - 210 \times 5 = 5 - 1050$

$\Rightarrow \quad 55y = -1045 \Rightarrow y = \dfrac{-1045}{55} = -19$

EXAMPLE 12 : *An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. The maximum number of columns in which they can march is*

Multiple Choice Question

(a) 7 (b) 8 (c) 9 (d) 10 **[NCERT]**

SOLUTION : 8 columns. the HCF of 616 and 32

EXAMPLE 13 : *The largest number which divides 615 and 963 leaving remainder 6 in each case is*

Multiple Choice Question

(a) 85 (b) 86 $\boxed{\text{(c) 87}}$ (d) 88

SOLUTION : Here, $615 - 6 = 609$ and $963 - 6 = 957$.
Clearly, the required number is the HCF of the numbers 609 and 957.
find the HCF of 609 and 957 by Euclid's alogorithm as given alongside.

$$957 = 609 \times 1 + 348$$
$$609 = 348 \times 1 + 261$$
$$348 = 261 \times 1 + 87$$
$$261 = 87 \times 3 + 0$$

EXAMPLE 14 : *The largest number that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively is*

Multiple Choice Question

$\boxed{\text{(a) 17}}$ (b) 18 (c) 19 (d) 20

SOLUTION : Here, $398 - 7 = 391$, $436 - 11 = 425$ and $542 - 15 = 527$.
HCF of the numbers 391, 425, 527.

$$\boxed{\begin{aligned} 425 &= 391 \times 1 + 34 \\ 391 &= 34 \times 11 + 17 \\ 34 &= 17 \times 2 + 0 \end{aligned}}$$

EXAMPLE 15 : *A merchant has 120 litres of oil of one kind, 180 litres of oil of another kind. He wants to sell the oil by filling the two kinds of oil in tins of equal capacity. The greatest capacity of such a tin is*

Multiple Choice Question

(a) 55 litres $\boxed{\text{(b) 60 litres}}$ (c) 70 litres (d) none

SOLUTION : Clearly, the greatest capacity of tin is the HCF of 120 and 180 in litres.

EXAMPLE 16 : *During a sale, colourpencils were being sold in packs of 24 each, crayons in packs of 32 each and water colour in packs of 48 each. If you want full packs of all three and the same number of pencils, water colour and crayons, how many of each would you need to buy ?*

SOLUTION : In order to arrange the colour items as required, we have to find the largest number that divides 24, 32 and 48 exactly. $\boxed{\text{The HCF of 8 and 48 is 8.}}$

No. of packets of colour pencils $= \dfrac{24}{8} = \mathbf{3}$. No. of packets of crayons $= \dfrac{32}{8} = \mathbf{4}$

$$\boxed{\text{No. of packets of water colours } = \dfrac{48}{8} = \mathbf{6}.}$$

EXAMPLE 17 : *In a seminar, the number of participants in Hindi, English and mathematics are 60, 84 and 108 respectively. If in each room the same number of participants are to be seated and all of them being in the same subject the minimum number of rooms required is*

Multiple Choice Question

(a) 21 (b) 22 (c) 23 (d) 24

the HCF of 60 and 84 is 12. $\therefore$ in each room maximum 12 participants can be seated.

Total number of participants $= 60 + 84 + 108 = 252$ the no. of rooms required $= \frac{252}{12} = 21$

EXAMPLE 18 : *15 pastries and 12 biscuit packets have been donated for a school function. These are to be packed in smaller identical boxes with the same number of pastries and biscuit packets in each. The no. of biscuit packets and pastries are respectively* (a) 4, 5 (b) 5, 6 (c) 6, 7 (d) 7, 8

Multiple Choice Question

SOLUTION : the HCF of 12 and 15 is 3. the no. of pastries in each box $= \frac{15}{3} = 5$

No. of biscuit packets in each box $= \frac{12}{3} = 4$

EXAMPLE 19 : *Find the HCF of 65 and 117 and express it in the form 65m + 117n.*

SOLUTION : using division lemma

$117 = 65 \times 1 + 52$

$65 = 52 \times 1 + 13$

$52 = 13 \times 4 + 0$

So, 13 is the HCF of 65 and 117.

$13 = 65 - 52 \times 1$

$\Rightarrow 13 = 65 - (117 - 65 \times 1)$

$\Rightarrow 13 = 65 - 117 + 65$

$\Rightarrow 13 = 65 \times 2 + 117 \times (-1)$

$\Rightarrow \boxed{13 = 65m + 117n,}$ where $m = 2$ and $n = -1$.

EXAMPLE 20 : *Show that there are infinitely many primes of the form $4n + 3$.*

SOLUTION : If possible, let number of primes of form $4n + 3$ be finite.

Since $4a - 1$ is odd, 2 can not be a factor

Thus, factors could be of type $4n + 1$ or $4n + 3$.

$\therefore$ the number of primes of the form $4n + 3$ is infinite.

EXAMPLE 21: **Multiple Choice Question**

The decimal expansion of the rational number $\dfrac{43}{(2)^4 (5)^3}$ will terminat after how many places of decimal ?

(a) 2 (b) 3 (c) 4 (d) 5

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SOLUTION :

$$\frac{43}{(2)^4 (5)^3} = \frac{43}{(2)^4 (5)^3} \times \frac{5}{5} = \frac{43 \times 5}{(2 \times 5)^4} = \frac{215}{10000} = 0.0215$$

$\therefore$ $\dfrac{43}{(2)^4 (5)^3}$ will terminate after 4 places of decimal.

So, the option **(c)** is correct, which is the required answer, *i.e.* answer (c

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Multiple Choice Question

Mathematics are 60, 84 and 108 respectively. The minimum number of rooms required, if in each room the same number of participants are to be seated and all of them being in the same subject, is

(a) 17 (b) 18 (c) 19 (d) 21

SOLUTION :

The HCF of 60, 84 and 108 is $2^2 \times 3 = 12$.

$\therefore$ in each room, 12 participants can be seated.

$\therefore$ The no. of rooms required $= \dfrac{\text{Total no. of participants}}{12} = \dfrac{60 + 84 + 108}{12} = \dfrac{252}{12} = 21$