

CHAPTER - 4

Quadratic Equations

Level - 1 (01 Marks Question)

Q.1 Check whether the following are quadratic equation or not

i) $(x-3)(2x+1) = x(x+5)$

ii) $(x+2)^3 = 2x(x^2-1)$

Q.2 Solve by factorisation method

$$x^2 - 7x + 12 = 0$$

Q.3 Find the discriminant

$$x^2 - 3x - 10 = 0$$

Q.4 Find the nature of root

$$2x^2 + 3x - 4 = 0$$

Q.5 Find the value k so that quadratic equation $3x^2 - kx + 27 = 0$ has equal root

Q.6 Determine whether given value of x is a solution or not

(1) $x^2 - 3x - 1 = 0$: $x = 1$

Level 2 (02 Marks)

Example 1 Solve the quadratic equation $9x^2 - 3x - 20 = 0$ by factorisation method

and find the roots of quadratic equation.

Solution $9x^2 - 3x - 20 = 0$

$$9x^2 - 15x + 12x - 20 = 0$$

$$3x(3x-5) + 4(3x-5) = 0$$

$$(3x-5)(3x+4) = 0$$

either $3x - 5 = 0$ or $3x + 4 = 0$

$$3x = 5$$

$$3x = -4$$

$$x = \frac{5}{3}$$

$$x = \frac{-4}{3}$$

- Q.1 Solve by quadratic equation $16x^2 - 24x - 1 = 0$ by using quadratic formula.
- Q.2 Determine the values of k for which the quadratic equation $2x^2 + 3x + K = 0$ have both roots real.
- Q.3 Find the roots of equation $2x^2 + x - 6 = 0$
- Q.4 Find the roots of equation $x - \frac{1}{x} = 3$ $x \neq 0$

Level 3 (03 Marks)

- Q.1 The sum of the squares of two consecutive positive integers is 265. Find the integers.
- Q.2 Divide 39 into two parts such that their product is 324.
- Q.3 The sum of number and its reciprocal is $\frac{17}{4}$. Find the number.
- Q.4 The length of rectangle is 5cm more than its breadth if its area is 150 Sq cm. Find the length and breadth.
- Q.5 The altitude of a right triangle is 7cm less than its base. If the hypotnuse is 13 cm. Find the other two sides.

SOLUTION

CHAPTER - 4

Quadratic Equations

Level - 1 (01 Marks Questions)

Q.1 i) $(x-3)(2x+1) = x(x+5)$

$$2(2x+1) - 3(2x+1) = x(x+5)$$

$$2x^2 + 2x - 6x - 3 = x^2 + 5x$$

$$2x^2 + 2x - 6x - 3 - x^2 - 5x = 0$$

$$x^2 - 9x - 3 = 0$$

Yes, this is a quadratic equation because it is in the form of $ax^2+bx+c=0$

ii) $(x+2)^3 = 2x(x^2-1)$

$$(x)^3 + (2)^3 + 3(x)(2)(x+2) = 2x^3 - 2x$$

$$x^3 + 8 + 6x(x+2) = 2x^3 - 2x$$

$$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$$

$$x^3 + 8 + 6x^2 + 12x - 2x^3 + 2x = 0$$

$$-x^3 + 6x^2 + 14x + 8 = 0$$

No, it is not a quadratic equation.

Q.2 $x^2 - 7x + 12 = 0$

$$x^2 - 4x - 3x + 12 = 0$$

$$x(x-4) - 3(x-4) = 0$$

$$(x-3)(x-4) = 0$$

$$x-3 = 0 \quad \text{and} \quad x-4 = 0$$

$$x = 3 \quad \quad \quad x = 4$$

Q.3 $x^2 - 3x - 10 = 0$

Here, $a = 1, b = -3, c = -10$

\therefore Discriminant $= b^2 - 4ac$

$$= (-3)^2 - 4 \times (1) \times (-10)$$

$$= 9 + 40 = 49$$

Q.4 $2x^2 + 3x - 4 = 0$

Here, Discriminant $= b^2 - 4ac = (3)^2 - 4 \times (2) \times (-4)$

$$= 9 + 32$$

$$= 41$$

$D = 41 > 0$ or roots are real and equal

Q.5 $3x^2 - kx + 27 = 0$

Here, it is given that roots are equal

$$D = 0$$

or $b^2 - 4ac = 0$

$$(-K)^2 - 4 \times 3 \times 27 = 0$$

$$K^2 = 324$$

$$K^2 = (18)^2$$

or $K = 18$

Q.6 $x^2 - 3x - 1 = 0$ and $x = 1$

$$p(x) = x^2 - 3x - 1$$

$$p(1) = (1)^2 - 3(1) - 1$$

$$= 1 - 3 - 1$$

$$= 1 - 4 = -3$$

No, $x = 1$ is not a solution of $p(x)$

Level (2) (02 Marks)

Q.1 $16x^2 - 24x - 1 = 0$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-24) \pm \sqrt{(-24)^2 - 4 \times 16 \times (-1)}}{2 \times 16}$$

Here, $a = 16$, $b = -24$, $c = -1$

$$= \frac{24 \pm \sqrt{576 + 64}}{2 \times 16}$$

$$= \frac{24 \pm \sqrt{640}}{32}$$

$$= \frac{24 \pm 8\sqrt{10}}{32}$$

$$= \frac{3 \pm \sqrt{10}}{4}$$

$$= \frac{3 \pm \sqrt{10}}{4}$$

$$x = \frac{3 \pm \sqrt{10}}{4} \text{ or } \frac{3 - \sqrt{10}}{4}$$

Q.2 $2x^2 + 3x + K = 0$

$$\begin{aligned} \text{Here, } D &= b^2 - 4ac = (3)^2 - 4 \times 2 \times K \\ &= 9 - 8K \end{aligned}$$

It will have real roots, if $D=0$ or $D>0$

$$\Rightarrow D \geq 0$$

$$\text{or } 0 \leq D$$

$$\text{or } 0 \leq 9 - 8k$$

$$k \leq \frac{9}{8}$$

$$\text{or } 8k \leq 9$$

Q.3 $2x^2 + x - 6$

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{(1)^2 - 4 \times 2 \times -6}}{2 \times 2} \\ &= \frac{-1 \pm \sqrt{1 + 48}}{4} \\ &= \frac{-1 \pm \sqrt{49}}{4} \\ &= \frac{-1 \pm 7}{4} \end{aligned}$$

$$x = \frac{-1 \pm 7}{4} = \frac{6}{4} = \frac{3}{2}$$

$$\text{or } x = \frac{-1-7}{4} = \frac{-8}{4} = -2$$

Q.4 $x - \frac{1}{x} = 3, x \neq 0$

$$\frac{x^2 - 1}{x} = \frac{3}{1} \text{ or } x^2 - 1 = 3x$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times -1}}{2 \times 1}$$

$$x = \frac{3 + \sqrt{13}}{2}, \quad x = \frac{3 - \sqrt{13}}{2}$$

$$\text{or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Level - 3 (03 Mark Questions)

Q.1 Let the first integer = x

∴ the second integer = x + 1

According to Question

$$(x)^2 + (x+1)^2 = 265$$

$$x^2 + x^2 + 1 + 2x = 265$$

$$2x^2 + 2x + 1 - 265 = 0$$

$$2x^2 + 2x - 264 = 0$$

$$= \frac{-2 \pm \sqrt{4+2112}}{4}$$

$$= \frac{-2 \pm \sqrt{2116}}{4}$$

$$= \frac{-2 \pm 46}{4}$$

$$= \frac{-2 \pm 46}{4} = \frac{44}{4} = 11$$

$$\text{or } x = \frac{-2 - 46}{4} = \frac{-48}{4} = -12$$

If $x = 11$, then $x+1 = 12$

If $x = -12$, then, $x+1 = -11$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 2 \times -264}}{2 \times 2}$$

Q.2 Let one of the number = x

\therefore then the other number will be = $39-x$

According to Q

$$(x) \times (39 - x) = 324$$

$$39x - x^2 = 324$$

$$-x^2 + 39x - 324 = 0$$

$$= \frac{-39 \pm \sqrt{(39)^2 - 4x - 1x - 324}}{2x - 1}$$

$$= \frac{-39 \pm \sqrt{1521 - 1296}}{-2}$$

$$= \frac{-39 \pm \sqrt{225}}{-2}$$

$$= \frac{-39 \pm 15}{-2}$$

$$x = \frac{-39 \pm 15}{-2} = \frac{-24}{-2} = +12$$

$$\text{or } x = \frac{-39 - 15}{-2} = \frac{-54}{-2} = +27$$

If $x = 27$, then $39 - x = 12$

If $x = 12$, then $39 - x = 27$

$$\frac{1}{x}$$

Q.3 Let the number = x

\therefore its Reciprocal =

According to Q

$$x + \frac{1}{x} = \frac{17}{4}$$

$$\frac{x^2 + 1}{x} = \frac{17}{4}$$

$$4(x^2 + 1) = 17x$$

$$4x^2 + 4 - 17x = 0$$

$$4x^2 - 17x + 4 = 0$$

$$x = \frac{+17 \pm \sqrt{(-17)^2 - 4 \times 4 \times 4}}{2 \times 4} = \frac{17 \pm \sqrt{289 - 64}}{8}$$

$$= \frac{17 \pm \sqrt{225}}{8} = \frac{17 \pm 15}{8}$$

$$\text{Let } x = \frac{17+15}{8} = \frac{32}{8} = 4$$

$$\text{or } x = \frac{17-15}{8} = \frac{2}{8} = \frac{1}{4}$$

$$\text{If } x=4, \text{ then } \frac{1}{x} = \frac{1}{4}$$

$$\text{If } x = \frac{1}{4}, \text{ then } \frac{1}{x} = 4$$

Q.4 Let Breadth of the rectangle = x

∴ Length of the rectangle = x + 5

According to Question

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4 \times 1 \times -150}}{2 \times 1}$$

$$(x)(x+5) = 150$$

$$x^2 + 5x - 150 = 0$$

$$= \frac{-5 \pm \sqrt{25+600}}{2}$$

$$= \frac{-5 \pm \sqrt{625}}{2}$$

$$= \frac{-5 \pm 25}{2}$$

$$x = \frac{-5+25}{2} = \frac{20}{2} = 10$$

$$x = \frac{-5+25}{2} = \frac{-30}{2} = -15$$

The Breadth = 10cm

Length = 15 cm

Q.5 Let the Base of $\Delta = x$ cm

the attitude = $x - 7$ cm

the hypoten use = 13cm

By using Pythagoras Theorem

$$13^2 = x^2 + (x-7)^2$$

$$169 = x^2 + x^2 + 49 - 14x$$

$$\therefore x = \frac{+14 \pm \sqrt{(-14)^2 - 4 \times 2 \times -120}}{2 \times 2}$$

$$\text{or } 0 = 2x^2 - 14x - 120$$

$$= \frac{+14 \pm \sqrt{196+960}}{4}$$

$$= \frac{+14 \pm \sqrt{1156}}{4}$$

$$= \frac{+14 \pm 34}{4}$$

$$= \frac{+14+34}{4} = \frac{48}{4} = 12$$

$$x = \frac{+14 - 34}{4} = \frac{-20}{4} = -5$$

⇒ The Base of $\Delta = 12\text{cm}$

$$\text{Attitude} = x - 7 = 12 - 7 = 5\text{cm}$$