SAMPLE PAPER –4 (SAII) MR AMIT. KV NANGALBHUR

Mathematics

CLASS : X

Time: 3hrs Max. Marks: 90

General Instruction:-

1. All questions are Compulsory.

- The question paper consists of 34 questions divided into 4 sections, A,B,C and D. Section – A comprises of 8 questions of 1 mark each. Section-B comprises of 6 questions of 2 marks each and Section- D comprises of 10 questions of 4 marks each.
- Question numbers 1 to 8 in Section –A multiple choice questions where you are to select one correct option out of the given four.
- There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculator is not permitted.

Section-A

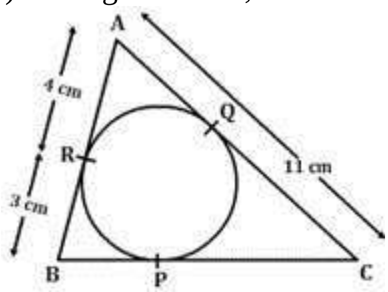
Question number 1 to 10 carry 1 mark each.

1) The discriminant of the quadratic equation .$3\sqrt{3}\, x^2 + 10x + \sqrt{3} = 0$ is
A) 30 B) 36 C) 64 D) 100

2) If $\frac{4}{5}$, a, 2 are three consecutive terms of an A.P., then the value of a.

A) 3/5 B) 5/3 C) 4/5 D) 7/5

3) In figure below, ∆ABC is circumscribing a circle. he length of BC is

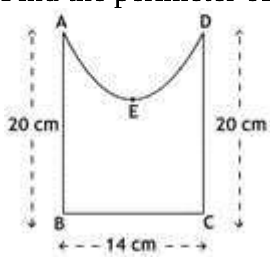


A) 10cm B)7cm C) 4cm D) 3cm

4) HCF and LCM of two numbers are 12 and 36 respectively, then product of these two numbers is:

A) 36 B)432 C) 342 D) 12

5) Find the perimeter of the given figure, where AED is a semi-circle and ABCD is a rectangle.



A) 76cm B)68cm c) 22cm D)54cm

6) To draw a pair of tangents to a circle which are inclined to each other at an angle of 100^0 . it is required to draw tangents at end points of those two radii of the circle, the angle between which should be

A) 100^0 B) 200^0 C) 50^0 D) 80^0

7) Two Tangents making an angle of 120^0 are drawn to a circle of radius 6cm , then the length of each tangent is equal to

A) $2\sqrt{3}$ cm B) $\sqrt{2}$ cm C) $6\sqrt{3}$ cm D) $\sqrt{3}$cm

8) A pole 6m high casts a shadow $2\sqrt{3}$ m long on the ground, then the sun's elevation is

A) 60^0 B) 90^0 C) 30^0 D) 45^0

. Section-B

Question number 9 to 14 carry 2 marks each

9) Solve by factorisation : $\sqrt{7}\, y^2 – 6y - 13\sqrt{7} = 0$.

10) Find a point on x-axis which is equidistant from the points (-2,5) and (2, -).3

11) Determine the ratio in which the point P(m, 6) divides the join of A(-4, 3) and B (2,8).

12) Spherical ball of diameter 21 cm is melted and recasted into cubes , each of side 1 cm . find the number of cubes thus formed.

13) A die is thrown once , find the probability of getting

(i) a prime number (ii) a number divisible by 2

14) All cards of ace, jack and queen are removed from the deck of playing cards. One card is drawn at random from the remaining cards. Find the probability that the card drawn is :(a) a face card (b) not a face card

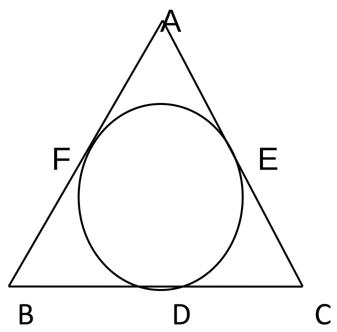
Section-C

Question number 15 to 24 carry 3 marks each

15) Determine the AP whose 3^{rd} term is 16 and when fifth term is subtracted from 7^{th} term, we get 12.

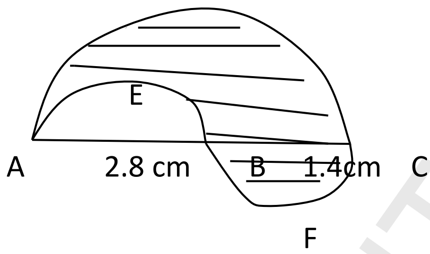
16) Construct a triangle similar to a given Δ ABC in which AB =4cm , BC =6cm and $\angle$ABC = 60^0 , such that each side of the new triangle is ¾ of the corresponding sides of given ΔABC.(Steps of Constructions not required)

17)



In the figure above , the incircle of ΔABC touches the sides BC, CA and AB at D, E and F respectively. If AB = AC , prove that BD = CD.

18)



In the fig. , find the perimeter of the shaded region where ADC, AEB and BFC are semi-circles on the diameter AC, AB, and BC respectively.

19) Largest sphere is carved out of the cube of side 7cm . find volume of the sphere?

20)The curved surface area of the cone is 12320 cm^2. If the radius of the base is 56cm, find its height.

21) The volume of the right circular cylinder of height 7cm is 567π cm^3. Find its curved surface area.

22)An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from his eye is 45^0. What is height of the chimney?

23)Find the value of 'p' for which the points (-5, 1) , (1,p) and (4, -2) are collinear.

OR

If the coordinates of the mid-points of triangle are (1,2) , (0,-1) and (2,-1). Find the coordinates of its vertices.

24) Find the roots of quadratic equation : $a^2b^2x^2 + b^2x - a^2x - 1 = 0$

Section-D

Question number 25 to 34 carry 4 marks each

25) The product of Tanay's age five years ago and his age after 10 years is 16. Find his present age.

26) Find the sum of the first 31 terms of an AP. Whose n^{th} term is given by 3 + 2n/3.

27) OABC is a rhombus whose three vertices A,B and C lie on a circle with centre O. If the radius of the circle is 10cm, find the area of the rhombus.

28) Prove that ' the lengths of the tangents drawn from an external point to a circle are equal. Use this theorem for the following:

If all sides of the parallelogram touches a circle, show that it is a rhombus.

29) From top of a hill the angles of depression of two consecutive kilometre stones due east are found to be 30^0 and 60^0. Find the height of the hill.

30) A school has 5 houses A, B , C , D and E. In a class having 23 students, 4 are from house A , 8 from house B , 5 from house C , 2 from house D and rest are from E . 1 student is selected at random to be a class monitor.

A) Find the probability that the selected student is not from B, C and E.

B) find the probability that the student is from house E.

C) what moral values the student of this class must share?

31) Find the ratio in which the line 2x + y = 4 divides the join of A(2, -2) and B(3,7). Also find the coordinates of the point of their intersection.

32) A bucket made up of metal sheet is in the form of a frustum of a cone of the height 16cm with radii of its lower and upper ends as 8cm and 20cm respectively. Find the cost of the bucket, if the cost of the metal sheet is Rs. 15 per 100 cm^2.

33) A well with 10m inside diameter is dug 14m deep. Earth taken out of it is spread all around to a width of 5m to form an embarkment. Find the height of the embankment.

34) Sum of the areas of the two squares is 468 m^2. If the difference of their perimeters is 24m, find the sides of the two squares.

SOLUTION SAMPLE PAPER 4

SECTION –A

1) C) 64

Explanation: - Here a =3√3, b = 10, c = √3

D = b^2 – 4ac

= 10^2 – 4 X 3√3 X√3

= 100 – 36

= 64

2) D) 7/5

Explanation:- as 4/5, a, 2 are in AP therefore the common difference is same

i.e a- 4/5 = 2 – a

2a = 2 + 4/5

2a = 14/5

a = 7/5

3) A) 10cm

Explanation :- AQ = 4cm (Tangents from same external point to same circle)

⇒ AC = AQ + QC

or 11 = 4 + QC

QC = 7cm

PC = QC = 7cm (Tangents from same external point to same circle)

BP = 3cm (Tangents from same external point to same circle)

BC = PC + BP

= 7 + 3

= 10cm

4) B) 432

Explanation :- Product of two numbers = HCF X LCM

= 12 x 36

= 432

5) A) 76cm

Explanation :- Required perimeter = AB + BC + CD + πr [here r = ½ X BC]

= 20 + 14 + 20 + 22/7 X 7

= 54 +22

=76 cm

6) D) 80^0

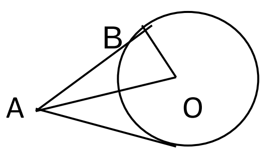
Explanation : As angles between tangents and at centre are supplementary

Required angle = 180^0 -100^0

= 80^0

7) A) 2√3 cm

Explanation :-



$\angle OAB = 120/2$ [the line joining the external point to the centre bisect the angle between the tangents at external point]

Therefore $\angle OAB = 60^0$

Now in ΔAOB

$\tan 60^0 = OB / AB$

$\sqrt{3} = 6 / AB$ (As $\tan 60^0 = \sqrt{3}$)

$AB = 6/\sqrt{3}$

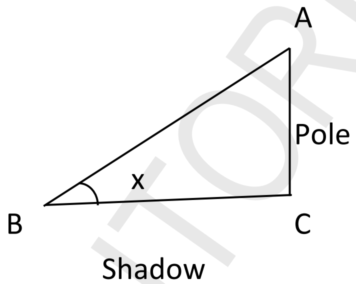
$= 6\sqrt{3}/(\sqrt{3} X \sqrt{3})$

$= 6\sqrt{3}/ 3$

$= 2\sqrt{3}$ cm

8) A) 60^0

Explanation:-



Let the inclination = x

Therefore $\tan x = AC/BC$

$\tan x = 6/2\sqrt{3}$

$= 6\sqrt{3}/(2\sqrt{3} X \sqrt{3})$

$= 6\sqrt{3}/(2X3)$

$= \sqrt{3}$

$= \tan 60^0$

Section –B

9) $\sqrt{7}y^2 - 6y - 13\sqrt{7} = 0$

Or $\sqrt{7}y^2 - 13y + 7y - 13\sqrt{7} = 0$

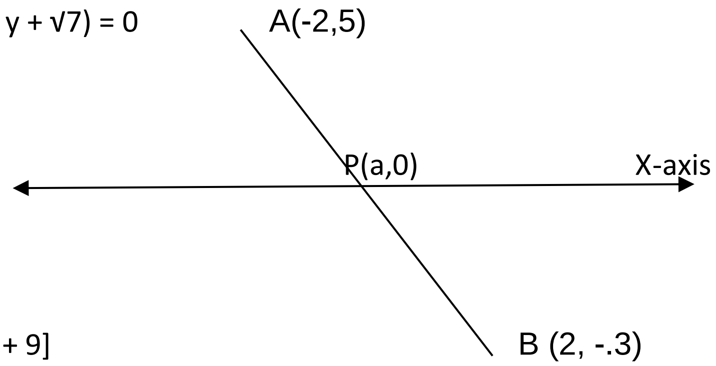
Or $y(\sqrt{7}y - 13) + \sqrt{7}(\sqrt{7}y - 13) = 0$

Or $(\sqrt{7}y - 13)(y + \sqrt{7}) = 0$

Or $(\sqrt{7}y - 13) = 0$ or $(y + \sqrt{7}) = 0$

$Y = 13/\sqrt{7}$ or $y = -\sqrt{7}$

10) Let the point on X axis be P(a,0)



According to question

AP = PB

$\sqrt{[(a+2)^2 + (-5)^2]} = \sqrt{[(a-2)^2 + 3^2]}$

$\sqrt{[a^2 + 4 + 4a + 25]} = \sqrt{[a^2 + 4 - 4a + 9]}$

$a^2 + 4a - a^2 + 4a = 13 - 29$

$8a = -16$

a = -2 (Ans)

11)

A(-4,3) K P(m,6) 1 B(2,8)

Let required ratio = k : 1

Using section formula (on y coordinate) $\{[mx_1 + nx_2]/[m+n] , [my_1 + ny_2]/[m+n]\}$

$6 = (8k +3)/(k+1)$

Or $6(k+1) = 8k + 3$

Or $6k + 6 = 8k +3$

Or $6k – 8k = 3 – 6$

$-2k = -3$

$K = 3/2$

Therefore reqired ratio = 3:2

12) Let the number of cubes formed = N

N X vol of cube = vol of sphere

$N X 1^3 = 4/3 X \pi r^3$

N = 4/3 X 22/7 X 21/2 X21/2 X21/2

N = 38808/8

N = 4851

13) Total outcomes = 6[1,2,3,4,5,6]

i) Total prime numbers =3[2,3,5]

P(E) = 3/6

= ½

ii) Total numbers divisible by 2 =3[2,4,6]

P(E) = 3/6

= ½

14) Number of cards removed = 12 [4A +4J +4Q]

Remaining cards = 52 – 12 =40

a) Number of face cards = 4 [kings]

P(E) = 4/40 = 1/10

b) Probability[not a face card] = 1 – P(E)

= 1 – 1/10

= 9 /10

SECTION – C

15) Given $a_3 = 16$ i.e $a + 2d = 16$-------------------(1)

Also $a_7 – a_5 = 12$

$a + 6d – (a + 4d) =12$

$a +6d – a – 4d = 12$

$2d = 12$

$d = 6$

Put d = 6 in (1)

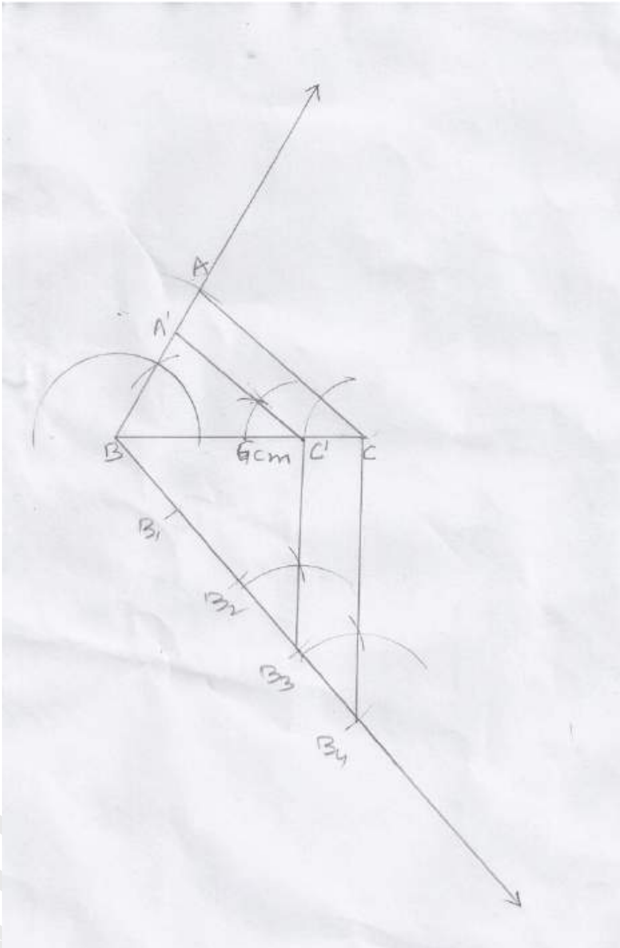
a + 2X6 = 16

a + 12 = 16

a = 4

therefore required AP : 4 , 10 , 16 , 22 ……………..

Q16



16) Given AB = AC

Therefore AF + FB = AE + EC { from fig.)

AF – AE + FB = EC

AF – AF + FB = EC { AF =AE , lengths of two tangents from same external point }

FB = EC { lengths of two tangents from same external point }

BD = DC { lengths of two tangents from same external point }

17) Required perimeter = perimeter(semi-circle ADC) + perimeter(semi-circle AEB) + perimeter(semi-circle BFC)

$= \pi$ X 4.2/2 + π X 2.8/2 + π X 1.4/2

=6.6 + 4.4 + 2.2

= 13.2 cm

18) Diameter of the largest sphere = side of the cube

Therefore diameter = 7 cm

Volume = 4/3 X πr^3

= 4/3 X 22/7 X7/2 X 7/2 X 7/2

=539 / 3 cm^3

19) Given curved surface area = 12320 cm^3

π X rl = 12320

π X 56X l = 12320

Or l = 12320 / (π X 56)

= 12320 X7 /(22X56)

l = 70 cm

Now $h^2 = r^2 + l^2$

Or h = $\sqrt{[l^2 - r^2]}$

= $\sqrt{[70^2 - 56^2]}$

= $\sqrt{[4900 - 3136]}$

= $\sqrt{1764}$

= 42 cm

20) Volume = 567π cm^3 [given]

Therefore

$\pi r^2 h$ = 567π

h = 567π / $r^2\pi$

h = 567/ 7 X 7 [radius = 7cm]

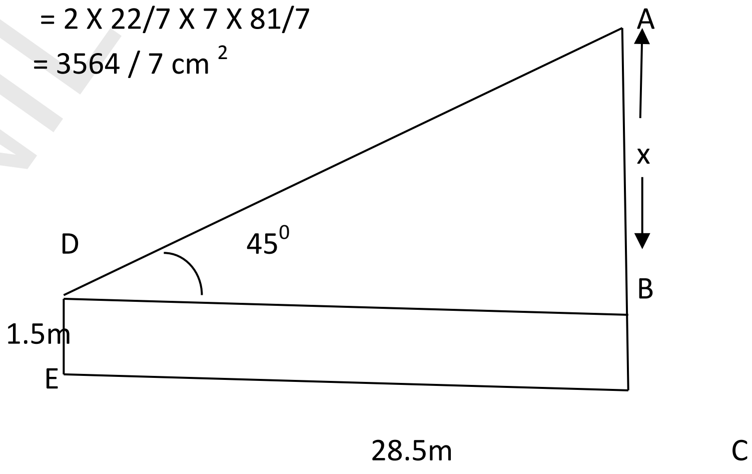
h = 81/7 cm

curved surface area = 2πrh

= 2 X 22/7 X 7 X 81/7

= 3564 / 7 cm^2

21)



Let the height of the chimney = x + 1.5

In ΔADB

Tan 45^0 = x / DB

1 = x / 28.5 { EC = DB}

X = 28.5cm

Therefore height of the chimney = 28.5 + 1.5

= 30 m(Ans)

22) Given that points A(5,1) , B (1,p) and C (4,-2) are collinear

Therefore area $\Delta ABC = 0$

i.e $1/2[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] = 0$

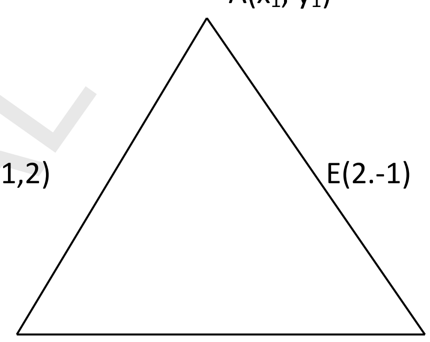
$5(p +2) +1 (-2-1) +4(1-p) =0$

$5p +10 -3+ 4 -4p =0$

$P+11 = 0$

$P = -11$(Ans)

OR



As D(1,2) is the mid point of AB, By mid point formula

$X= (x_1+x_2)/2$ and $y= (y_1+y_2)/2$

Therefore $1 = (x_1+x_2)/2$ and $2 =(y_1+y_2)/2$

$X_1 + x_2 = 2$ ---------------- -------------------------(1)

And $y_1+y_2 = 4$--------------------- -------------------------(2)

Similarly, $x_2 + x_3 =0$--(3)

$Y_2+y_3 = -2$---------------- -----------------------(4)

$x_1+x_3 = 4$--(5)

$y_1+y_3 = -2$--(6)

Add (1), (3) and (5)

$2(X_1 + x_2+ x_3) =6$

$X_1 + x_2+ x_3 =3$---(7)

Put (1) in (7)

$2 + x_3 = 3$

$X_3 =1$

Similarly Putting (3) in (7)

$X_1 +0 = 3$

$X_1 =3$

Similarly Putting (5) in (7)

$X_2 +4 = 3$

$X_2 =-1$

Similarly $y_1 = 2$, $y_2 = 2$ and $y_3 = -4$

23) $a^2b^2x^2 + b^2x – a^2x – 1. = 0$

$b^2x(a^2x +1) -1 (a^2x +1) =0$

$a^2x +1=0$ or $b^2x -1 =0$

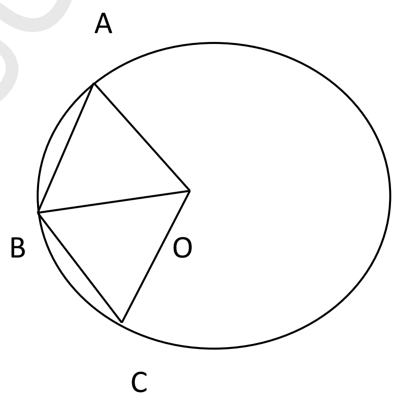
$x = -1/a^2$ or $x = 1/b^2$

section -D

24) let Tanay's present age = x years
therefore Tanay's age 5 year's ago = (x-5) years
and Tanay's age after 10 years = (x + 10) years
According to question
(X – 5) (X +10) =16
$X^2 + 10x -5x -50 =16$
$X^2 + 5x – 66 =0$
$X^2 + 11x -6x – 66 =0$
X(x +11) – 6 (x + 11) =0
x-6 =0 or x +11 = 0
x =6 or x = -11
x = -11 rejected (as age cannot be negative)
therefore Tanay's present age = 6 years

25) Given $a_n = 3 + 2n/3$
Put n= 1 , 2 , 3 ……
a =11/3 , a_2 =13/3, a_3 = 15/3 and so on
therefore a =11/3 and d = 13/3 – 11/3 = 2/3
S_n = n/2[2a + (n-1)d]
= 31/2 [2 X11/3 + 30X 2/3]
= 31/2[22/3 + 60/3]
= 31/2 X 82/3
=1271/3 (Ans)

26)



As its given that OABC is a rhombus

Therefore OA = AB = BC = CO = Radius =10cm[As the sides of rhombus are equal]

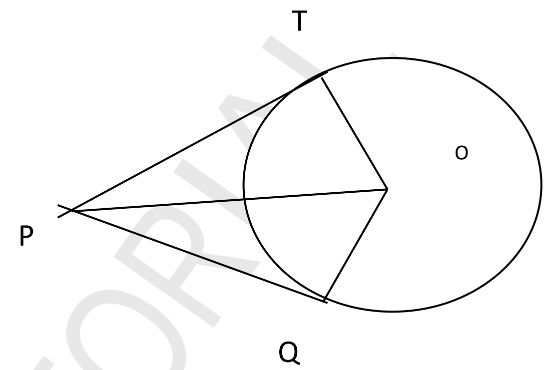
More over OB =10cm~

Area OABC = Area ΔOAB + area ΔOBC

$= \sqrt{3}/4 \times 10^2 + \sqrt{3}/4 \times 10^2$ { area of an equilateral triangle = $\sqrt{3}/4 \times Side^2$ }

$= 50\sqrt{3}\ cm^2$

27) Given:- A circle with center O. Two tangents PT and PQ are drawn from external point P.
To Prove :- PT = PQ
Construction :- Join TO, OP and OQ



Proof:- In ΔPOT and ΔPOQ

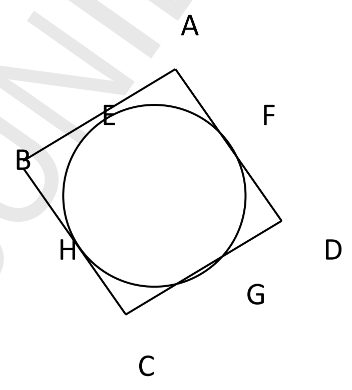
∟OTP = ∟OQP [Each 90^0 as radius and tangents are perpendicular to each other]

OP = OP [Common hypotenuse]

OT = OQ [Radii of the same circle]

Therefore ΔPOT ≌ ΔPOQ [by RHS]

⟹ PT = PQ (c.p.c.t)



Given :- A parallelogram ABCD such that its sides touches a circle at E,F,G and H as shown in fig.

Let AF=AE =a, BE = BH = b, CH = CG =c , DG =DF = d

Now AB = CD { opposite sides of the parallelogram}

Therefore AE+EB = DG +GC

a + b = d + c -------------------------(1)

again BC = AD

BH +HC = AF +FD

b +c = a + d ---------------------------------------(2)

adding (1) and (2)

a +2b +c = a +2d +c

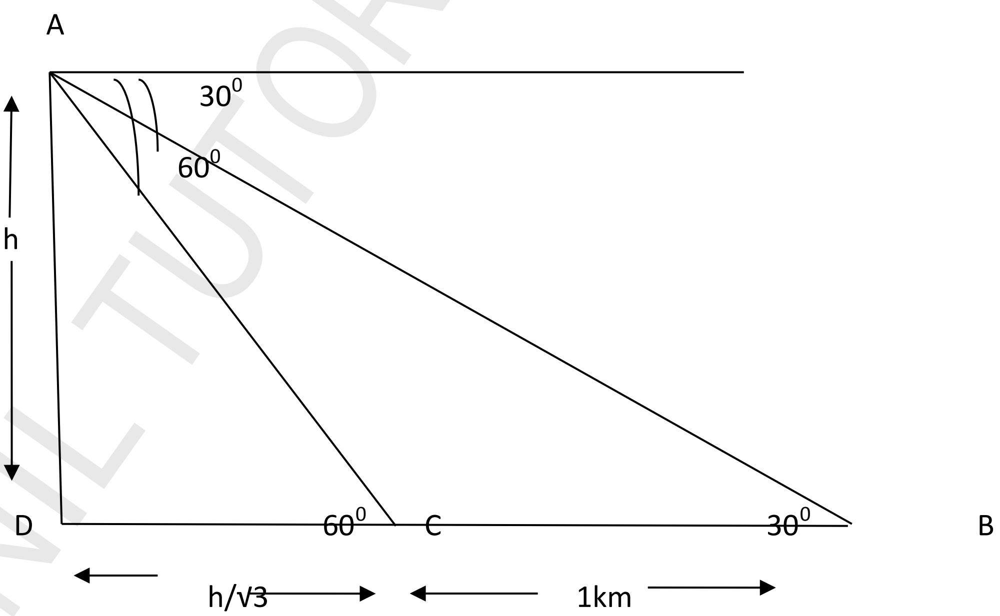
b = d ---------------------------------(3)

But AB= a +c

= a + d (using 3)

=AD Therefore ABCD is a rhombus because if adjacent sides of parallelogram are equal then it's a 3rhombus.

28)



Let AD is hill of height h km and C and B are two kilometre stones

Therefore CB = 1 km

In ∆ADC

Tan 60^0 = h/DC

√3 = h / DC

DC = h /√3--------------------(1)

In ∆ADB

Tan 30^0 = h / (DC + 1)

1/√3 = h / (DC +1)

DC+1 = h√3

1 = h√3 - h /√3 [from 1]

1 = 2h/√3

h = √3/2 km

29)

A) Probability that the student is not from houses B,C and D = No. of students in A and D/ Total No. of students in class

= (4 + 2)/23

= 6/23

B) Number of students in house E = 23 – (4 + 8 + 5 + 2)

= 23 – 19

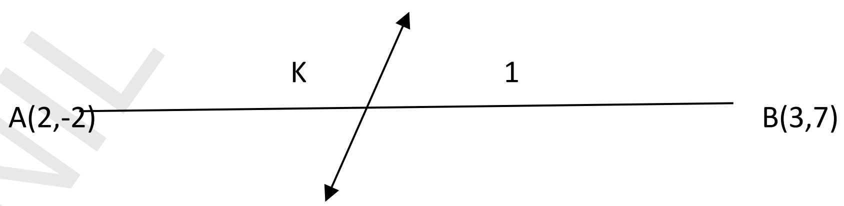
= 4

the probability that the student is from house E= No. of students in E/ Total No. of students in class

= 4 /23

C) Cooperation

31)



Let the required ratio be k:1

Therefore the point of intersection is given by { (3k+2)/(k+1) , (7k- 2)/ (k +1) }

As this point lie on the line 2x + y = 4 therefore it must satisfy the equation

i.e 2(3k+2)/(k+1) + (7k- 2)/ (k +1) =4

or 6k +4 +7k – 2 = 4(k+1) (By taking LCm and cross multiplication)

or 6k + 4 + 7k – 2 = 4k + 4

or 13k – 4k = 4 -2

or 9k = 2

K = 9/2

Therefore the required ratio is 9:2

For point of intersection

[(3 X 9/2 + 2)/ 9/2 +1 , (7X 9/2 – 2)/ 9/2 +1]

(31/11 , 59/11)

D) Here r = 8cm, R=20cm , h = 16cm
Slant height , l = √[h^2 + (R – r)^2]
= √[16^2 + (20 – 8)^2]
= 20 cm
Total surface area = $\pi l(R + r) + \pi r X r$
= 3.14[20(20+8) + 8 X8]
=1956.36 cm^2
Cost of metal used = 1959.36 X 0.15
= Rs. 293.90.

E) Diameter of the well = 10m , height/depth = 14m
Inner radius of the embankment = 5m
Outter radius of the embankment = 10m
As the mud dug out of well is used to make the embankment
Therefore the vol. of well = vol of embankment
$\pi H R^2$ = $\pi h R(outter)^2 - \pi h r(inner)^2$

$\pi 14 X 5 X 5$ = πh(10X10 -5X5)

14X25 = h (100- 25)

14 X25 = h X 75

(14 X25) /75 = h

14/3m = h

F) Let the side of larger square be x m and y m
Therefore the area of larger square = x^2
And the area of smaller square = y^2
Perimeter of larger square = 4x and of smaller square = 4y
According to question
Difference of perimeters = 24
4x – 4y = 24
Or x- y = 6
Or x = 6 + y
According to second condition
Sum of areas = 468
$X^2 + y^2$ = 468
$(6 + y)^2 + y^2$ = 468
36 + y^2 + 12y + y^2 = 468

$2y^2$ + 12y -432 =0

Y^2 + 6y -216 = 0

Y^2+ 18y -12y – 216 =0

Y (y + 18) -12(y + 18) =0

(y +18) (y -12) =0

Y +18 =0 or y – 12 =0

Y = -18 or y = 12

Y = -18 rejected because side cannot be negative

Therefore side of smaller square = 12m

And side of larger square = 18m