SAMPLE QUESTION PAPER 2017-18 MATHEMATICS

Time allowed : 3 hrs **Maximum marks : 80**

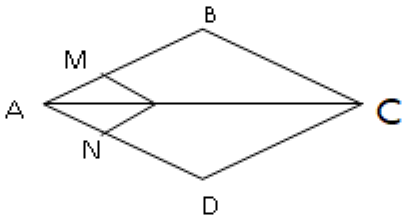
General Instructions :

All questions are compulsory

1. The question paper consists of 30 questions divided into four sections A,B,C,D
2. Section A contains 6 questions of 1 marks each
3. Section B contains 6 questions of 2 marks each
4. Section C contains 10 questions of 3 marks each
5. Section D contains 8 questions of 4 marks each
6. There is no overall choice. However an internal choice has been provided in four questions of 3 marks each. You have to attempt only one of the alternatives in all such questions.
7. Use of calculator is not permitted

SECTION – A

1. Without performing actual division, state whether the rational number $\frac{13}{125}$ will have a terminating decimal expansion or a Non-terminating repeating decimal expansion.
2. Find the 10^{th} term from the last term of the A.P : 3,8,13,…….,253
3. Find the value(s) of k, for which the quadratic equation $kx(x-2)+6=0$ has two equal roots.
4. Find the coordinates of the point on y-axis which is nearest to the point (5,-2).
5. In this given figure LM||CB and LN||CD prove that $\frac{AM}{AB}=\frac{AN}{AD}$



6. Express Sin 67°+Cos 75° in terms of trigonometric ratios of angles between 0° and 45°.
7. If two positive integers p and q are written as p=a^2b^3 and y=a^3b ; a,b are prime numbers, then verify $\mathrm{LCM(p,q)\ X\ HCF(p,q)\ =\ pq}$
8. For which value of 'k' will the following pair of linear equations have no solution?

$$3x+y=1$$
$$(2k-1)x+(k-1)y=2k+1$$

9. Find the sum of first 24 terms of the list of numbers whose n-th term is given by : a_n=3+2n
10. If (1,$\frac{p}{3}$) is the midpoint of the line segment joining the prints (2,0) and (0,$\frac{2}{9}$), then show that the line $5x+3y+2=0$ *passes through the point* $(-1,3p)$
11. An Integer is chosen between 0 and 100. What is the probability that it's not divisible by 7.

12. A bag contains 5 red balls and some blue balls. The probability of drawing a blue ball from the bag is thrice that of a red ball, find the number of blue balls in the bag.

SECTION – C

13. Find all the zeroes of $2x^4-3x^3-3x^2+6x-2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$

14. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rays more. Find the number of students in the class.

15. In what ratio does the x-axis divides the line segment joining the points (-4,-6) and (-1,7) ? Find the coordinates of the point of division?

OR ,

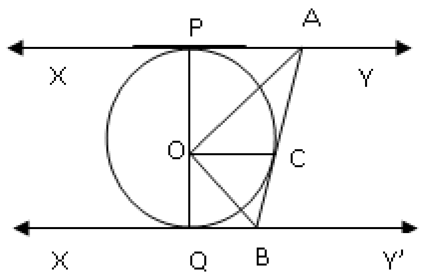
The points A(4,-2), B(7,2), C(0,9) and D(-3,5) form a parallelogram. Find the length of the altitude of the parallelogram on the base AB.

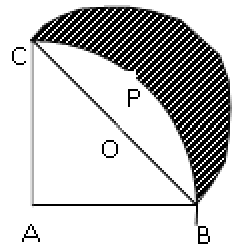
16. Show that exactly one of the numbers n, n+2, n+4 is divisible by 3.

17. $\triangle$ABC is right angled at C. If BC = a, CA = b and AB = c and P is the length of perpendicular drawn from C AB, then prove that: (I) PC = ab (II) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

OR, In figure, the line segment XY is parallel to side AC of $\triangle$ABC and it divides the triangle into two parts of equal areas. Find the ratio $\frac{AX}{AB}$

18. In the given figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X'Y' at B. Prove that <AOB = 90^0.





19. Evaluate: $\frac{cos^2 5° + cos^2 55°}{cosec^2 15° - tan^2 75°} + \sqrt{3}(tan13° tan23° tan30° tan67° tan77°)$

20. In the given figure, ABPC is a quadrant of a circle of radius 14cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

21. In a rain water harvesting system, the rainwater from a roof of 22m x 20m drains into a cylindrical tank having diameter of base 2m and height 3.5m. If the tank is full, find the rainfall in cm. Write your views on the water conservation. OR,

A cubical block of side 10cm is surmounted by a hemisphere. What is the largest diameter that the hemisphere can have? Find the cost of painting the total surface area of the solid so formed, at the rate of Rs 5 per 100 sq.cm

22. Find the mode of the following distribution of marks obtained by the students in an examination:

Marks obtained	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of students	15	18	21	29	17

Given the mean of the distribution is 53, using empirical relationship estimate the value of its median.

SECTION – D

23. A plane left 40 minutes late due to bad weather and in order to reach its destination 1600km away in time, it had to increase its speed by 400km/h from its usual speed. Find the usual speed of the plane. OR,

Check whether the equation $5x^2 - 6x - 2 = 0$ has real roots and if it as, find them by the method of completing the square. Also, verify that roots obtained satisfy the given equation.

24. If the ratio of the sum of the first n-terms of two A.P. is 7n+1 : 4n+27, the find the ratio of their 9^{th} terms.

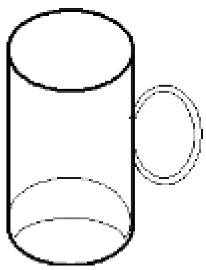
25. State and prove Pythagoras theorem. OR , State and prove the Basic Proportionality Theorem.

26. Draw a $\triangle$ABC with side BC = 7cm, <B = 75°, < A = 105°. Then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of $\triangle$ABC. Write the steps of construction and justify your answer.

27. Prove that $\frac{Cos\ A - Sin\ A + 1}{Cos\ A + Sin\ A - 1}$ = Cosec A + Cot A using the identity $Cosec^2$ A = 1 + Cot^2 A.

28. As observed from the top of a 75 m high light house from the sea – level, the angles of depression of two ships are 30° and 45° . If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships.

29. Two dairy owners A and B sell flavored milk filled to capacity in mugs of negligible thickness, which are cylindrical in shape with a raised hemispherical bottom.



Or,

The mugs are 14 cm high and have a diameter of 7 cm as shown in the given figure. Both A and B sell flavored milk at the rate of Rs 80 per litre. The dairy owner A uses the formula $\pi r^2 h$ to find the volume of milk in the mug and charges Rs 43.12 for it. The dairy owner B is of the view that the price of the actual quantity of milk should be charged. What according to him should be the price one mug of milk ? Which value is exhibited by the dairy owner B ?

30. In the retail market fruit vendors were selling apples kept in packing boxes. These boxes contained varying number of apples. If the median number of apples in a box was 42 and the total number of boxes was 550, then find the missing frequencies f_1 and f_2 in the following distribution table.

Number of apples in a box	25 - 30	30 - 35	35 - 40	40 – 45	45 - 50	50 – 55	55 - 60
Number of boxes	20	67	f_1	f_2	125	35	25

OR, Draw the move than type,give for the following data. Also, find the median from the graph.

Class Interval	0-10	10-20	20-30	30-40	40-50
Frequency	10	18	40	20	12

Marking Scheme (class – X)

Sub : Mathematics Session : 2017-18

SECTION A

The number given = $\frac{13}{125}$.

LCM of 125 = 5^3

$\therefore \frac{13}{5^3 \times 2^0}$ It is terminating. ($\because$ Any denominator no. that can be expressed in the form of 2^m x 5^n is terminating)

AP: 3, 8, 13, …, 253

a = 3, d = 5, l = 253

$\therefore$ From the last term:

a' = 253, d'= -5, l' = 253

$\therefore$ a'_{10} (from last) = a' + (n-1)d'

= 253 + 9(-5)

=253 -45

= 208

f(x) = kx(x-2) + 6 = 0

= kx^2 – 21x +6 = 0

f(x) have equal and real roots

$\therefore$ D = 0

$b^2 - 4ac = 0$

$(-2k)^2 - 4.k.6 = 0$

$4k^2 = 24k$

k = 6

The point is (5, -2)

The point on y-axis : (0 , y)

$\therefore$ The point (0,-2) is nearest to te point (5, -2) on y-axis.

Given, LM||CB , LN||CD, ABCD is a quadrilateral.

To prove: $\frac{AM}{AB} = \frac{AN}{AD}$

Proof: In $\triangle$ABC , LM||BC

$\because \frac{AL}{LC} = \frac{AM}{MB}$ ($\because$ BPT) ……….(i)

In $\triangle$ADC,

$\frac{AL}{MD} = \frac{AN}{ND}$($\because$ BPT) ……….(ii)

Sin 67^0 + cos 75^o

Sin $(90 – 23)^o$ + cos $(90 – 15k)^o$

Cos 23^o + sin 15^o

p = a^2b^3, q=a^3b

L.C.M => a^3b^3

HCF => a^2b

LCM X HCF = pq

a^3b^3 x a^2b = a^2b^3 x a^3b

$a^3b = a^3b$

1=1 (proved)

When no solution:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

3x+y-1=0

(2k-1)x+(k-1)y-(2k+1)=0

$a_1=3$; $b_1=1$; $c_1=-1$

$a_2=(2k-1)$; $b_2=(k-1)$; $c_2=-(2k+1)$

$\frac{3}{2k-1} = \frac{1}{k-1} / \frac{1}{2k+1}$

Case 1

3k-3 = 2k-1

k=2

9) a_n=3+2n ; a_2=3+4 =>7

a_1= 3+2

a_1=5

a=5 ; d=2

a_2=13

$S_{2n} = \frac{24}{2}(10 + 23.12)$

= 12(10+46)

= 672

10)

1: (2,0) ; $\frac{p}{3} = \frac{\frac{2}{9}+0}{2}$ (1, $\frac{p}{3}$) (0, $\frac{2}{9}$)

$$\frac{p}{3} = \frac{2}{9\,X\,2}$$

$$P = \frac{1}{3}$$

0(-1,3p) => (-1,1)

so, x = -1 ; y=1

5x+3y+2= 0

-5+3+2= 0

-5+5 = 0 (Hence Proved)

11) Numbers between 0 to 100 which are divisible by 7 is 14. So, numbers which are not divisible by 7 are 100-14=86

So, P = $\frac{86}{100}$ = $\frac{43}{50}$

12) A bag Contains 5 red balls and some blue balls.

Let the number of blue balls be 'x'

So, Total no of balls = x+5

A T Q : $3(\frac{5}{5+x}) = \frac{x}{x+5}$

$\frac{15}{5+x} = \frac{x}{x+5}$

So, x =15

So, number of blue balls are 15.

13. a=bq+r , $a \leq r < b$

b=3 => r=0,1,2

r=0, => n=3q, 3q+2, 3q+4

r=1, => n=3q+1, 3q+3, 3q+5

r=2, => n=3q+2, 3q+4, 3q+6

Exactly one in the above 3 sets is divisible by 3.

14. The roots given $\sqrt{2}$ and - $\sqrt{2}$

$\therefore$(x- $\sqrt{2}$) and (x + $\sqrt{2}$)

$\therefore$(x- $\sqrt{2}$) (x+ $\sqrt{2}$)

= $(x)^2 - (\sqrt{2})^2$

= $x^2 - 2$

$\therefore$ The other zeros are

$x^2 - 2$) $2x^4 - 3x^3 - 3x^2 + 6x - 2$ ($2x^2 - 3x + 1$

$\cancel{2x^4}$ $- 4x^2$

- +

$\cancel{-3x^3} + x^2 + \cancel{6x} - 2$

$\cancel{-3x^3}$ $+ \cancel{6x}$

+ -

$x^2 - 2$

$x^2 - 2$

x__

$\therefore$ $2x^2 - 3x + 1$

$= 2x^2 - 2x - x + 1$

$= 2x(x-1) - 1(x-1)$

$= (2x-1)(x-1)$

$\therefore$ The total zeros are

$\frac{1}{2}, 1, \sqrt{2}, -\sqrt{2}$

15. Let the total rows be x

Let the total columns be y.

ATQ

$\therefore$ xy= (x+3) (y-1)

$xy = \cancel{xy} - \cancel{y} + 3y - 3$

$3 = -x + 3y$ ..... (I)

$\therefore$ xy =(x – 3) (y +2)

$xy = \cancel{xy} + \cancel{2x} - 3y - 6$

$6 = 2x - 3y$ .... (II)

$-x + 3\cancel{y} = 3$

$2x - 3y = \cancel{6}$

x=9

$\therefore$ -9 + 3y = 3

3y = 12

y=4

The total rows is 9 The total columns is 4.

16.

$0 = \frac{7k-6}{2}$

6 FTk

$k = \frac{6}{7}$ ...... (II)

$\therefore$ Ratio is 6:7

$\therefore$ $x = \frac{-6-28}{13}$

$x = \frac{-34}{13}$

$= \frac{-34}{13}$

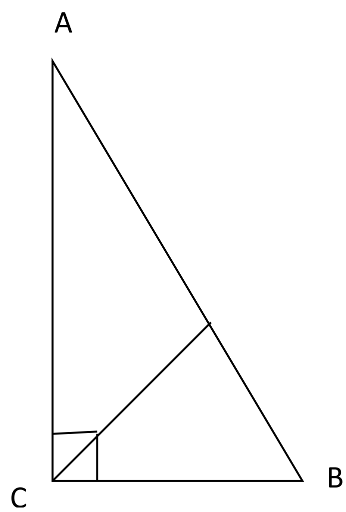
17. Area ofABC= $\frac{1}{2}$ cp =1/2ab

Or, pc = ab

Or, 1/p = c/ab

Or, $1/p^2 = c^2/a^2b^2 = a^2 + b^2/a^2b^2$

$= 1/a^2 + 1/b^2$



18.Given : XY ∥ X′Y′

AB are tangent.

To prove : ∠ADB = 90°

Proof :

In △ POA and △ OAC

∠OPA = ∠OCA = 90°

OR

18. $\angle PAC + \angle QBA = 180°$ $[Cointerior\ angles]$
ΔPOA ≅ ΔCOA [By]

19.XY | | AC
∠BXY = $\angle A \& \angle BYX = \angle C$
△ABC ~ △XBY [By AA]

$$\frac{AR.\ \Delta ABC}{AR.\ \Delta XBY} = \frac{AB^2}{XB^2}$$
$$AR.\Delta ABC = 2.AR.\Delta XBY$$

$\frac{AR.\ \Delta ABC}{AR.\Delta XBY} = \frac{2}{1}$
$(\frac{AB}{XB})^2 = \frac{2}{1}$

$$\frac{AB}{XB} = \sqrt{2}$$
$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$

$$\frac{AB - XB}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$
$$\frac{AX}{AB} = \frac{\sqrt{2} - 1}{\sqrt{2}}$$
$$\frac{AX}{AB} = \frac{2 - \sqrt{2}}{2}$$

OR

19. $\frac{\cos^2 35° + \cos^2 55°}{cosec^2 15° - \tan^2 75°} + \sqrt{3}(\tan 13° \tan 23° \tan 30° \tan 67° \tan 77°)$

$$= \frac{\cos^2 35° + \sin^2 35°}{\sec^2 75° - \tan^2 75°} + \sqrt{3}(\tan 13° \tan 77° \tan 23° \tan 67° \tan 30°)$$

$= \frac{1}{1} + \sqrt{3}(\cot 77° \tan 77° \cot 67° \tan 67° \tan 30°)$
$= 1 + \sqrt{3}\left(\frac{1}{\tan 77°} \times \tan 77° \times \frac{1}{\tan 67°} \times \tan 30°\right)$
$= 1 + \sqrt{3} \times \frac{1}{\sqrt{3}}$
= 1 + 1
= 2

20. $BC^2 = AC^2 + AB^2$
$= 14^2 + 14^2$

$$BC^2 = 312$$

BC = $14\sqrt{2}$
$r_2 = 7\sqrt{2}$
Given , ABPC is a quadrant,
AC = AB= 14 cm = r_1
$\theta = 90°$
Area of shaded region = $AR.\Delta ABC + AR\ of\ semicircle - AR.quadrant\ ABPC$

$$= \frac{1}{2} \times AB \times AC + \frac{\pi r_2^2}{2} - \frac{\theta}{360}\pi r_1^2$$

$$= \frac{1}{2} \times 14 \times 14 + \frac{\frac{22}{7} \times 7\sqrt{2} \times 7\sqrt{2}}{2} - \frac{90}{360} \times \frac{22}{7} \times 14 \times 14$$

$= 98 + 154 - 154$
=252 -154
= 98 cm^2
21. Given,

length of the roof(l) = 22m
breadth of the roof(b) = 20m
height of the roof (h_1) = ?
base diameter of tank = 2m
base radius of tank (r) = 1m
h_2= 3.5m
Therefore,
Vol. of tank = vol. of roof
Or, $\pi r^2 h_2 = l.b.h_1$
Or, $\frac{22}{7} \times 1 \times 1 \times 3.5 = 22 \times 20 \times h_1$
Or, h_1 = 2.2cm
Views:
a. Water is very necessary to carry out daily activities.
b. We should not waste water as our lives depend on it.
OR
21. TSA of the solid = CSA of the hemisphere + TSA of Cube – Area of the Top
= $2\pi r^2 + 6a^2 - a^2$
= $2\pi r^2 + 5a^2$
= 2 x 3.14 x 5 x 5 + 5 x 10 x 10
= 657cm^2
Therefore, cost @ Rs5/100cm^2 = $\frac{5}{100} \times 657$
= Rs 32.85
22. Here modal class: 60 – 80
l = 60, f_1= 29, f_2= 17, f_0 = 21, h = 20
$\therefore$ Mode $= l + (\frac{f_1 - f_0}{2f_1 - f_0 - f_2})n$
$= 60 + (\frac{29-21}{58-21-17})n$
$= 60 + \frac{8}{20} \times 20$
$=68$
Now,
3Median = Mode + 2Mean
= 68 + 2(53)
= 68 + 106
=174
Median $= \frac{174}{3} = 58$
$\therefore$Median=58
23. For real roots, $D \geq 0$
$\therefore$ D = $(-6)^2$ - 4x5x(-2)
= 76 $\geq$ 0
So, the Eqn. has real roots.
OR
23. Let, The usual speed be 'x' km/hr.
$\frac{1600}{x} - \frac{1600}{x+400} = \frac{2}{3}$
Or, 800 $(\frac{x+400-x}{x(x+400)}) = \frac{1}{3}$
Or, x= $\frac{-400 \pm \sqrt{(1600+3840000)}}{2}$
= $\frac{-400 \pm \sqrt{(200)}}{2}$
= 800 km/hr.

$5x^2 - 6x - 2 = 0$

$$x^2 - \frac{6}{5}x - \frac{2}{5} = 0$$
$$\left(x - \frac{6}{10}\right)^2 - \left(\frac{6}{10}\right)^2 - \frac{2}{5} = 0$$
$$x - \frac{6}{10} = \pm\sqrt{\frac{19}{5}}$$

∴ $x = \frac{3 \pm \sqrt{19}}{5}$

24. Given ratio of the n^{th} term of 2 A.P.s is 7n+1:4n+27

So, $\frac{\frac{n}{2}(2a_1 + (n-1)d_1)}{\frac{n}{2}(2a_2 + (n-1)d_2)} = \frac{7n+1}{4n+27}$

$$\frac{2(a_1 + (\frac{n-1}{2})d)}{\left(2(a_2 + \left(\frac{n-1}{2}\right)d\right)} = \frac{7n+1}{4n+27}$$
$$\frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{\left(a_2 + \left(\frac{n-1}{2}\right)d_2\right)} = \frac{7n+1}{4n+27}$$

Now, ratio of 9^{th} term $= \frac{a_1 + (9-1)d_1}{a_{2+(9-1)d_2}}$

$= \frac{a_1 + 8d_1}{a_{2+8d_2}}$

Comparing both equation,

$$\frac{n-1}{2} = 8$$

n-1 = 16 ; n=17

So, Ratio of 9^{th} term $= \frac{7(17)+1}{4(17)+27} = \frac{120}{95} = \frac{24}{19}$

26 . Correct Statement $\frac{1}{2}$

Figure Given $\frac{1}{2}$

To prove $\frac{1}{2}$

Construction $\frac{1}{2}$

Proof $1\frac{1}{2}$

Or

Same.

26. Correct △ ABC

Correct a △ A´B´C´

27. L.H.S $= \frac{\cos A + cosec\, A - 1}{\cot A - cosec\, A + 1}$ (dividing the numerator & denominator by sin A)

$= \frac{\cos A + cosec\, A - (cosec\, A + \cot A)(cosec\, A - \cot A)}{\cot A - cosec\, A + 1}$

= cosec A + cot A

28. $\frac{x}{75} = \cot 45°$

Or x= 75

$$\frac{x+y}{75} = cot 30°$$

Or, x+y = $75\sqrt{3}$

Or, y=$75(\sqrt{3} - 1)$

29. Height of the mug = 14cm =1.4dm

Diameter = 7cm = 0.7dm

Radius = 3.5cm = 0.35dm

Actual volume of the milk in the mug = volume of the cylinder – volume of the raised hemispherical bottom

Now,

Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times \frac{0.35}{100} \times \frac{0.35}{100} \times \frac{1.4}{10}$

= 0.5390 dm^3

Volume of the hemispherical bottom = $\frac{2}{3}\ \pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{0.35}{100} \times \frac{0.35}{100} \times \frac{0.35}{100}$

=0.0898333dm^3

So, Actual volume of the milk in the mug = 0.5390 – 0.08983

=0.44917dm^3

Volume of the milk = 0.44917

The price of one litre = Rs. 80

The price of 0.44917 litres = 80 × 0.44917

= Rs. 27.93 ≅ 28

According to the dairy B, the price of the milk should be Rs. 28 approximately.

The dairy owner B is a humane person with morals. He believes in serving people at fair price and does not deceive people unlike dairy owner A.

30.

Number of Apples	Class mark	Frequency	Cumulative frequency
25-30	27.5	20	20
30-35	32.5	67	87
35-40	37.5	f1	87+f1
40-45	42.5	f2	87+f1+f2
45-50	47.5	125	212+f1+f2
50-55	52.5	35	247+f1+f2
55-60	57.5	25	272 + f1 + f2 550

Median = 42

Median of the data = l + $\left(\frac{\frac{n}{2}+cf}{f}\right) \times h$

Accordingly,

$42 = 55 + \left(\frac{275-(247+f1+f2)}{25}\right) \times 5 \quad = 55 + \left(\frac{275-275-247-f1-f2}{5}\right) \times 5 \quad = 55 + \left(\frac{28-f1-f2}{5}\right)$

$42 = \frac{275+28-f1-f2}{5}$

210 = 275 +28-f1-f2

f1+f2 = 303 – 210 = <u>93</u> (1)

Now, The total number of boxes = 550

So, 272+f1+f2 = 550

30.

Class Interval	F	Cf	Type
0-10	10	100	More than 0
10-20	18	90	More than 10
20-30	40	72	More than 20
30-40	20	32	More than 30
40-50	12	12	More than 40