Gurukul The School

SAMPLE PAPER 2
HALF YEARLY EXAMINATION, 2018-19

MATHEMATICS

Time Allowed : 3hrs **CLASS – X** **Maximum Marks : 80**

Name _________________________________ **Sign of Invigilator ___________**

General Instructions :

1. *The question paper comprises of* ***thirty questions divided into four Sections- A, B, C and D****.*
2. ***Section A*** *comprises of* ***six questions Q1 to Q6*** *of* ***one mark*** *each.*
3. ***Section B*** *comprises of* ***six questions Q7 to Q12*** *of* ***two marks*** *each.*
4. ***Section C*** *comprises of* ***ten questions Q13 to Q22*** *of* ***three marks*** *each.*
5. ***Section D*** *comprises of* ***eight questions Q23 to Q30*** *of* ***four marks*** *each.*
6. ***All*** *questions are* ***compulsory.***
7. *Use of* ***calculators*** *is* ***not permitted.***

SECTION – A

1 'a' and 'b' are the two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then find the least prime factor of (a + b). **1**

2 If 3 and 5 are the two zeroes of a polynomial, then find that polynomial. **1**

3 Find the values of k, such that the pair of linear equations $4y = kx - 3$ and $6x - 12y = 9$ will have infinitely many solutions. **1**

4 For what value of p are $2p + 1, 13$ and $5p - 3$ forms an A.P. **1**

5 If $cos\theta - sin2\theta = 0$ then find the value of $tan^2\theta + cot^2\theta$. **1**

6 Two friends were born in the year of 2000. What is the probability that their birthday falls on the same day. **1**

SECTION – B

7 Two tankers contain 620 litres and 840 litres of diesel respectively. Find the maximum capacity of a container which can measure the diesel of both the tankers in exact number of times. **2**

8 Solve the equation for x: $\sqrt{2x+9} + x = 13$. **2**

9 Prove that the points $(a, 0), (0, b)$ and $(1, 1)$ are collinear if $\frac{1}{a} + \frac{1}{b} = 1$. **2**

10. Evaluate $\frac{sec37°.cosec53° - tan37°.cot53° + sin^2 55° + sin^2 35°}{tan10°.tan20°.tan60°.tan70°.tan80°}$. **2**

11 If the median of the following distribution is 24, find the value of f. **2**

Class interval	0-10	10-20	20-30	30-40	$40-50$
frequencies	5	25	f	18	7

12 Cards numbered from 11 to 60 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn card is **2**

(i) A prime number
(ii) A perfect square number.

SECTION – C

13 Prove that $\sqrt{3}-\sqrt{2}$ is an irrational number. **3**

14 If α and β are the zeroes of a quadratic polynomial $P(x)=2x^2+5x+k$ .Then find the value of k if it is given that $\alpha^2+\beta^2+\ \alpha\beta=\frac{21}{4}$. **3**

15 Solve the pair of linear equations $2(ax-by)+(a+4b)=0$; and $2(bx+ay)+(b-4a)=0$. **3**

16 If the roots of the equation $(a-b)x^2-(b-c)x+(c-a)=0$ are equal, prove that $2a=b+c$. **3**

17 Find the common difference of an A.P. whose first term is 5 and the sum of its first four terms is half the sum of the next four terms. **3**

18 Find the coordinates of the points of trisection of a line segment joining the points A $(2,-2)$ and B $(-7,4)$. **3**

19 Prove that $(sinA+secA)^2+\ (cosA+cosecA)^2=\ (1+secA.cosecA)^2$. **3**

20 The angle of depression of the top and bottom of a $50\ m$ high building from the top of a tower are $45°\ and\ 60°$ respectively. Find the height of the tower and the horizontal distance between the tower and the building. (use $\sqrt{3}=1.73$) **3**

21 The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. Find the median consumption of electricity. **3**

Monthly consumption (in units)	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

22 If all the face cards are removed from a deck of playing cards and then from the remaining cards if one card is picked at random. Then find the probability of getting: **3**

(i) A card with even number on it
(ii) Either an ace or a red card.
(iii) Neither a club nor an ace.

SECTION – D

23 Obtain all the zeroes of a polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$. **4**

24 Joseph travels 370 km partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car. **4**

25 Two water taps together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than other to fill the tank separately, find the time in which each pipe would fill the tank separately. **4**

26 Find the sum of the following series: **4**

$$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 \ldots. + (-5) + 81 + (-3)$$

27 Prove that, the quadrilateral formed by joining the four points $A(2,-1)$, $B(3,4)$, $C(-2,3)$ and $D(-3,-2)$, is a rhombus but not a square. Hence find the area of the rhombus so formed. **4**

28 If $secA + tanA = p$ then prove that $sinA = \frac{p^2-1}{p^2+1}$. **4**

29 The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°.Find the height of the cloud. **4**

30 The annual rainfall record of a city for 66 days is given in the following table: **4**

Rainfall (in cm)	0-10	10-20	20-30	30-40	40-50	50-60
Number of days	22	10	8	15	5	6

Construct a less than type as well as a more than type cumulative frequency curves, and hence obtain the median rainfall.

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MATHEMATICS

CLASS – X

	SECTION – A	
1	Let a = 3x, b = 5y So, a + b = 3x + 5y Both the terms are odd numbers. When we add two odd numbers, the resulting sum will always be an even number. Hence, the least prime factor of a + b is 2.	**1**
2	$x^2 - 8x + 15$	**1**
3	K=2	**1**
4	$13 - 2p - 11 = 5p - 3 - 13$ $p = 6$	**1**
5	$cos\theta = sin2\theta$ This is possible only when θ =30° Therefore, $tan^2 30 + cot^2 30 = \frac{1}{3} + 3$ $= \frac{10}{3}$	**1**
6	P(having same birthday)= $\frac{1}{366}$	**1**
	SECTION – B	
7	HCF of 620 and 840, Maximum capacity of container = 20 l	**2**
8	$\sqrt{2x+9} = 13 - x$ Squaring both sides, $\Rightarrow 2x + 9 = 169 + x^2 - 26x$ $\Rightarrow x^2 - 28x + 160 = 0$ $\Rightarrow x$ =20 and 8	**2**
9	$\frac{1}{a} + \frac{1}{b} = 1.$ $\Rightarrow a + b = ab$ ---(1) area of triangle formed by given three points,	**2**

area of triangle=1/2 {a (b-1)+0+1 (0-b)}

$$= ab - a - b$$

$$= ab - (a + b)$$

$$= ab - ab\ (from\ (1))$$

$$0$$

Hence, points are collinear.

10. **2**

$$\frac{cosec(90-37)°.cosec53°-\cot(90-37)°.cot53°+sin^2 55°+cos^2(90-35)°}{\cot(90-10)°.\cot(90-20)°.tan60°.tan70°.tan80°}$$

$$= \frac{cosec^2 53° - cot^2 53° + sin^2 55° + cos^2 55°}{cot\,80°.cot70°.tan60°.tan70°.tan80°}$$

$$=\frac{1+1}{1\times1\times\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3}$$

11 **2**

Class interval	Frequencies	cf
0-10	5	5
10-20	25	30
20-30	f	30+f
30-40	18	48+f
40-50	7	55+f

Median = 24

$$\Rightarrow l + \left(\frac{\frac{n}{2} - cf}{f}\right)h = 24$$

$$\Rightarrow 20 + \left(\frac{\frac{55+f}{2} - 30}{f}\right)10 = 24$$

$$\Rightarrow f = 25$$

12 **2**

(i) $P(\ prime\ number) = \frac{13}{50}$

(ii) $P(perfect\ square\ number) = \frac{2}{25}$

SECTION – C

13	Let us consider $\sqrt{3}-\sqrt{2}$ be a rational number. $$\sqrt{3}-\sqrt{2}=\frac{a}{b}, where\ a,b\ are\ co-prime\ integers, q\neq 0.$$ $\Rightarrow \sqrt{3}=\frac{a}{b}-\sqrt{2}$ Squaring both sides, $$\Rightarrow 3 = \frac{a^2+2b^2-2\sqrt{2}ab}{b^2}$$ $\Rightarrow 2\sqrt{2}a = a^2-b^2$ $$\Rightarrow \sqrt{2}=\frac{a^2-b^2}{2a}$$ Since, irrational $\neq$ Rational. This contradiction arises because of our wrong assumption. Hence, $\sqrt{3}-\sqrt{2}$ is irrational.	**3**
14	$\alpha+\beta=\frac{-5}{2}\ \ and\ \ \alpha\beta=\frac{k}{2}$ $(\alpha+\beta)^2=\left(\frac{-5}{2}\right)^2$ $\alpha^2+\beta^2+\alpha\beta+\alpha\beta=\frac{25}{4}$ $\Rightarrow \frac{21}{4}+\frac{k}{2}=\frac{25}{4}$ $\Rightarrow k=2$	**3**
15	$2ax-2by=-(a+4b)$ $$2bx+2ay=-(b-4a)$$ Solving above two equations by any method, $$x=\frac{-1}{2}+\frac{4ab}{b^2-a^2}$$ $$y=\frac{2+a^2}{b^2-a^2}$$	**3**
16	Putting $D = 0$, $$b^2-4ac=0$$ $\{-(b-c)\}^2-4(a-b)(c-a)=0$ It will lead to $2a=b+c$	**3**
17	Let d is common difference of AP now first 4term is 5,5+d,5+2d,5+3d and next 4term 5+4d,5+5d,5+6d,5+7d According to question , 20+6d=(20+22d)/2	**3**

20+6d=10+11d
d=2

18 Coordinates of the points of trisection of a line segment are $(-1,0)\ and\ (-4,2)$. **3**

19 $LHS = (sinA + \frac{1}{\cos A})^2 + \ (cosA + \frac{1}{sinA})^2$. **3**

$$= \frac{(\sin A\, cosA + 1)^2}{cos^2 A} + \frac{(\sin A\, cosA + 1)^2}{sin^2 A}$$

$$= (\sin A\, cosA + 1)^2 \left(\frac{1}{sin^2 A} + \frac{1}{cos^2 A}\right)$$

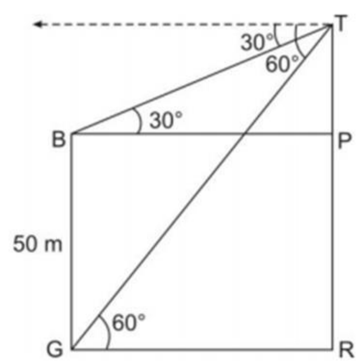
$$= (\sin A\, cosA + 1)^2 \left(\frac{sin^2 A + cos^2 A}{sin^2 A. cos^2 A}\right)$$

$$= \left(\frac{\sin A\, cosA + 1}{sinA. cosA}\right)^2$$

$=(1 + secA. cosecA)^2$.

20 In ΔBTP, **3**

tan 30° = TP/BP
BP = TP√3



In ΔGTR,
tan 60° $=\frac{TR}{GR}$
GR = $\frac{TR}{\sqrt{3}}$
As BP = GR
TP√3 $=\frac{TR}{\sqrt{3}}$
3 TP = TP + PR
2 TP = BG
TP = $\frac{50}{2}$ = 25 m
Now, TR = TP + PR
TR = (25 + 50) m
Height of tower = TR = 75 m
Distance between building and tower = GR = $\frac{TR}{\sqrt{3}}$
GR =25√3 m =43.25 m

21 **3**

Monthly consumption	**Number of consumers**	***cf***
65-85	4	4
85-105	5	9

105-125	13	22
125-145	20	42
145-165	14	56
165-185	8	66
185-205	4	70

$$Median = l + \left(\frac{\frac{n}{2} - c}{f}\right)h$$

$$= 125 + \left(\frac{35 - 22}{20}\right)20$$

$$= 138$$

22

(i) $P(card\ with\ even\ number) = \frac{20}{40}$

(ii) $P(Either\ an\ ace\ or\ a\ red\ card) = \frac{22}{40}$

(iii) $P(Neither\ a\ club\ nor\ an\ ace) = \frac{25}{40}$

3

SECTION – D

23 $(x + 2 + \sqrt{3})(x - 2 + \sqrt{3}) = 0$ **4**

P(x) must be divisible by $x^2 - 4x + 1$

Quotient is $(2x^2 - x - 1)$

The other zeroes are 1 and $\frac{-1}{2}$

24 Let speed of the train be $x\ km/h$ and that of the car be $y\ km/h$. **4**

$$\frac{250}{x} + \frac{120}{y} = 4$$

$$\frac{130}{x} + \frac{240}{y} = \frac{43}{10}$$

Equations can be solved by any method.

$x = 100\ km/h$ and $y = 80\ km/h$

25 Let the time taken by 1st pipe a be $x\ min$ **4**

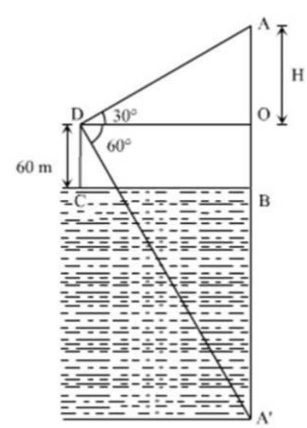
then pipe taken by 2nd pipe b =$(x + 5)min$

time taken by both pipe together 11 1/9 or 100/9

$$\frac{1}{x} + \frac{1}{(x + 5)} = \frac{9}{100}$$

$$x = 20m$$

$$Time\ taken\ by\ 1st\ pipe = 20\ min$$

	$Time\ taken\ by\ 2nd\ pipe = 25\ min$	
26	$5 + (-41) + 9 + (-39) + 13 + (-37) + 17 \ldots. + (-5) + 81 + (-3)$ S = [5 + 9 + 13 + 17 +......81] + [(-41) + (-39) + (-37) +.....+ (-5) + (-3)] A1 = 5 + 9 + 13 + ... +81 So 81 = 5 + (n-1)4 n = 20 Sum, S(A1) = 20/2 [2×5 + (12)4] S (A1) = 580 Similarly For A2 -3 = -41 + (n-1) (2) So n = 20 Thus S(A2) = 10 [-6 + 19×2] = 320 S = 320 + 580 = 900	**4**
27	We can prove that by showing that diagonals are of different length. Area = 24 *sq units*	**4**
28	$RHS = \frac{(secA + tanA)^2 - 1}{(secA + tanA)^2 + 1}$ $= \frac{sec^2A + tan^2A + 2.secA.tanA - 1}{sec^2A + tan^2A + 2.secA.tanA + 1}$ $= \frac{2tan^2A + 2secA.tanA}{2sec^2A + 2secA.tanA}$ $= \frac{2tanA(1 + secA)}{2secA(1 + secA)}$ $= sinA$	**4**
29	Let AO=H CD=OB=60m A'B=AB=(60+H)m In △AOD, tan30°=$\frac{H}{OD}$ OD=$\sqrt{3}$ H Now, in △ A'OD, tan60'= $\frac{OA\prime}{OD}$ H=60m Thus, height of the cloud above the lake = AB+A'B  	**4**

	=(60+60) = 120 m	
30		**4**

Rainfall (in cm)	cf
Less than 10	22
Less than 20	32
Less than 30	40
Less than 40	55
Less than 50	60
Less than 60	66

Correct ogive can be drawn.

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