Gurukul The School

SAMPLE PAPER-1
HALF YEARLY EXAMINATION, 2018-19

MATHEMATICS

Time Allowed : 3hrs **CLASS – X** **Maximum Marks : 80**

Name ________________________________ **Sign of Invigilator ___________**

General Instructions :

1. *The question paper comprises of* ***thirty questions divided into four Sections- A, B, C and D****.*
2. ***Section A*** *comprises of* ***six questions Q1 to Q6*** *of* ***one mark*** *each.*
3. ***Section B*** *comprises of* ***six questions Q7 to Q12*** *of* ***two marks*** *each.*
4. ***Section C*** *comprises of* ***ten questions Q13 to Q22*** *of* ***three marks*** *each.*
5. ***Section D*** *comprises of* ***eight questions Q23 to Q30*** *of* ***four marks*** *each.*
6. ***All*** *questions are* ***compulsory.***
7. *Use of* ***calculators*** *is* ***not permitted.***

SECTION – A

1 The decimal expansion of the rational number $\frac{43}{2^4 \times 5^3}$ will terminate after how many places of decimals? **1**

2 If $a+b$ and $a-b$ are the two zeroes of a polynomial, then find that polynomial. **1**

3 Find the values of k, such that the following pair of linear equations is inconsistent $3x+y=1$ and $(2k-1)x+(k-1)y=2k+1$. **1**

4 The n^{th} term of an A.P. is 6n+2. Find the common difference. **1**

5 If $sec\theta - cosec2\theta = 0$ then find the value of $sin^2\theta + tan^2\theta$. **1**

6 Find the probability of 53 Sundays in the year 2018. **1**

SECTION – B

7 105 goats and 175 cows have to be taken across the river. There is only one boat which will have to make up many trips to do so. If boatman want to carry all the animals in least number of trips and each trip must have equal number of animals, then find the number of animals in each trip he can carry on his boat. **2**

8 If -5 is a root of the quadratic equation $2x^2+px-15=0$ and the quadratic equation $p(x^2+x)+k=0$ has equal roots, find the value of k. **2**

9 If A$(-2,-1), B\ (a,0), C(4,b)\ and\ D(1,2)$ are are the vertices of a parallelogram ABCD, find the values of $a\ and\ b$. **2**

10. Evaluate $\frac{cos^2 20° + cos^2 70°}{sec^2 50° - cot^2 40°} + 2cosec^2 58° - 2cot58° tan32° - 4tan13° tan37° tan45° tan53° tan77°$ **2**

11 Find the mean of the following distribution **2**

x_i	4	6	9	10	15
f_i	5	10	10	7	8

12 Cards numbered from 31 to 83 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn card is **2**

(i) An even number
(ii) A perfect cube number.

SECTION – C

13 Prove that the square of any positive integer is of the form $5q, 5q+1$ *and* $5q+4$, for some integer q. **3**

14 If α and β are the zeroes of a quadratic polynomial $P(x) = kx^2 + 4x + 4$ .Then find the value of k if it is given that $\alpha^2 + \beta^2 = 24$. **3**

15 Solve the pair of linear equations graphically $x - y = 1$ and $2x + y = 8$.Shade the area bounded by the graphs of the given equations and the Y-axis. Also determine its area. **3**

16 Solve the following quadratic equation $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0.$ **3**

17 Find the middle term of the A.P. 7, 10, 13 ,….,187. **3**

18 Find the coordinates of the points which divides the line segment joining the points A $(2,-2)$ and B (10,8) into four equal parts. **3**

19 Prove that $(sinA + cosecA)^2 + (cosA + secA)^2 = 7 + tan^2A + cot^2A.$ **3**

20 The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant, the angles of elevation of a balloon from these windows are observed to be 60 and 30 respectively. Find the height of the balloon above the ground. **3**

21 If the median of the following frequency distribution is 32.5, find the missing frequencies. **3**

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
frequency	f_1	5	9	12	f_2	3	2	40

22 If all the aces are removed from a deck of playing cards and then from the remaining cards if one card is picked at random. Then find the probability of getting: **3**

(i) A face card.
(ii) Either a king or a red card.
(iii) Neither a queen nor a black card.

SECTION – D

23 Obtain all the zeroes of a polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$. **4**

24 A boat goes 30 km upstream and 40 km downstream in 8 hours. It can go 36 km upstream and 32 km downstream in the same time. Find the speed of the boat in still water and the speed of the stream. **4**

25 A swimming pool is filled with three pipes with uniform flow. The first two pipes operating simultaneously fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately. **4**

26 The sum of the three numbers in A.P. is 12 and the sum of their cubes is 288. Find the numbers. **4**

27 If the coordinates of the mid points of the sides of a triangle are $(1,1), (2,-3))$ and $(3,4)$.Find the coordinate of all the vertices of the triangle and hence find the centroid of the triangle. **4**

28 Prove that $\left[\frac{1+sinA-cosA}{1+sinA+cosA}\right]^2 = \frac{1-cosA}{1+cosA}$. **4**

29 The angle of elevation of an aeroplane from a point on the ground is 45°. After a flight of 15 seconds, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 meters, find the speed of the aeroplane. **4**

30 The frequency distribution of score obtained by 230 candidates in a medical entrance test is as follows: **4**

Scores	400-450	450-500	500-550	550-600	600-650	650-700	700-750	750-800
f_i	20	35	40	32	24	27	18	34

Construct a less than type as well as a more than type cumulative frequency curves, and hence obtain the median rainfall.

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MARKING SCHEME - SAMPLE PAPER-1
HALF YEARLY EXAMINATION, 2018-19
MATHEMATICS
CLASS – X

	SECTION – A	
1	After 4 places of decimal. (0.0043)	**1**
2	P(x)=k$\{x^2 + 2ax + (a^2 - b^2)\}$	**1**
3	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (condition for inconsistency) $\Rightarrow \frac{3}{(2k-1)} = \frac{1}{(k-1)} \neq \frac{1}{2k+1}$; $\Rightarrow k = 2$	**1**
4	$a_n = 6n + 2$ $\Rightarrow a_1 = 6 + 2 = 8$ $\Rightarrow a_2 = 6 \times 2 + 2 = 14$ $\therefore\ d = a_2 - a_1 = 14 - 8 = 6$	**1**
5	If $sec\theta - cosec2\theta = 0$ $\Rightarrow sec\theta = \text{cosec}\, 2\theta$ $\Rightarrow sec\theta = \sec(90° - 2\theta)$ $\Rightarrow \theta = 90° - 2\theta$ $\Rightarrow 3\theta = 90°$ $\Rightarrow \theta = 30°$ $\Rightarrow \therefore\ \ sin^2 30° +\ tan^2 30° = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$	**1**

6	2018 is a non-leap year =365 days=52 weeks+1 day (the last day of the year); 52 weeks means every day of the week is coming 52 times, but whichever day of the week will fall on the last day, will become the only day which will be coming 53 times in the year. That last day of the year has seven possibilities (outcomes) = {S, M, T, W, Th., F, Sat} $\therefore$ Probability(53 Sundays in the year) = $\frac{1}{7}$;	**1**
	SECTION – B	
7	No. of animals in each trip =HCF(105,175) =35	**2**
8	Putting x = -5 in the equation $2x^2 + px - 15 = 0$, $we\ get\ p = 7$ $\therefore\ p(x^2 + x) + k = 0$ becomes $7x^2 + 7x + k = 0$ For equal roots, $b^2 - 4ac = 0$ $\therefore k = \frac{7}{4}$	**2**
9	As diagonals AC and BD of the parallelogram bisect each other at O. So, O the midpoint of both AC and BD $\left(\frac{-2+4}{2}, \frac{-1+b}{2}\right) = \left(\frac{a+1}{2}, \frac{0+2}{2}\right)$ $\therefore a = 1\ and\ b = 3$	**2**
10	$\frac{cos^2 20° + cos^2 70°}{sec^2 50° - cot^2 40°} + 2cosec^2 58° - 2cot58° tan32° - 4tan13° tan37° tan45° tan53° tan77°$ $= \frac{sin^2 70° + cos^2 70°}{sec^2 50° - tan^2 50°} + 2(cosec^2 58° - cot^2 58°) - 4tan13° tan37° tan45° cot53° cot13°$ $= \frac{1}{1} + 2(1) - 4 \times 1 \times 1 \times 1$ $=1+2-4=-1$	**2**
11	Find the mean of the following distribution	**2**

x_i	4	6	9	10	15	Total
f_i	5	10	10	7	8	40
$x_i f_i$	20	60	90	70	120	360

$mean = \frac{\sum f_i x_i}{\sum f_i} = \frac{360}{40} = 9$

12 No. of cards = 83 – 31 +1 =53 **2**

(i) P(getting an even No. card)= $\frac{26}{53}$

(ii) P(getting a perfect cube No. card)= $\frac{1}{53}$

SECTION – C

13 Let a be any positive no, and b = 5. **3**

So, by Euclid's division algorithm,

$a = 5q + r, \quad where\ 0 \le r < 5$

$when\ r = 0,\ a = 5q$

$\Rightarrow a^2 = (5q)^2 = 25q^2 = 5(5q^2) = 5m,\ where m = 5q^2$

when r =1; $a = 5q + 1 \Longrightarrow a^2 = (5q+1)^2 = 25q^2 + 10q + 1$

$a = 5(5q^2 + 2q) + 1 = 5m + 1; where\ m = 5q^2 + 2q,$

when r =2; $a = 5q + 2 \Longrightarrow a^2 = (5q+2)^2 = 25q^2 + 20q + 4$

$a = 5(5q^2 + 4q) + 4 = 5m + 4; where\ m = 5q^2 + 4q$

similarly for other values for r , we find that the square of any positive integer is of the form 5m or 5m+1 or 5m+4 .[we can also use q in place of m , which is simply representing a positive integer.]

14 If α and β are the zeroes of a quadratic polynomial $P(x) = kx^2 + 4x + 4$ .Then find the value of k if it is given that $\alpha^2 + \beta^2 = 24$. **3**

= $\alpha^2 + \beta^2 = 24$

= $(\alpha + \beta)^2 - 2\alpha\beta = 24$

= $(\frac{-4}{k})^2 - 2\frac{4}{k} = 24$

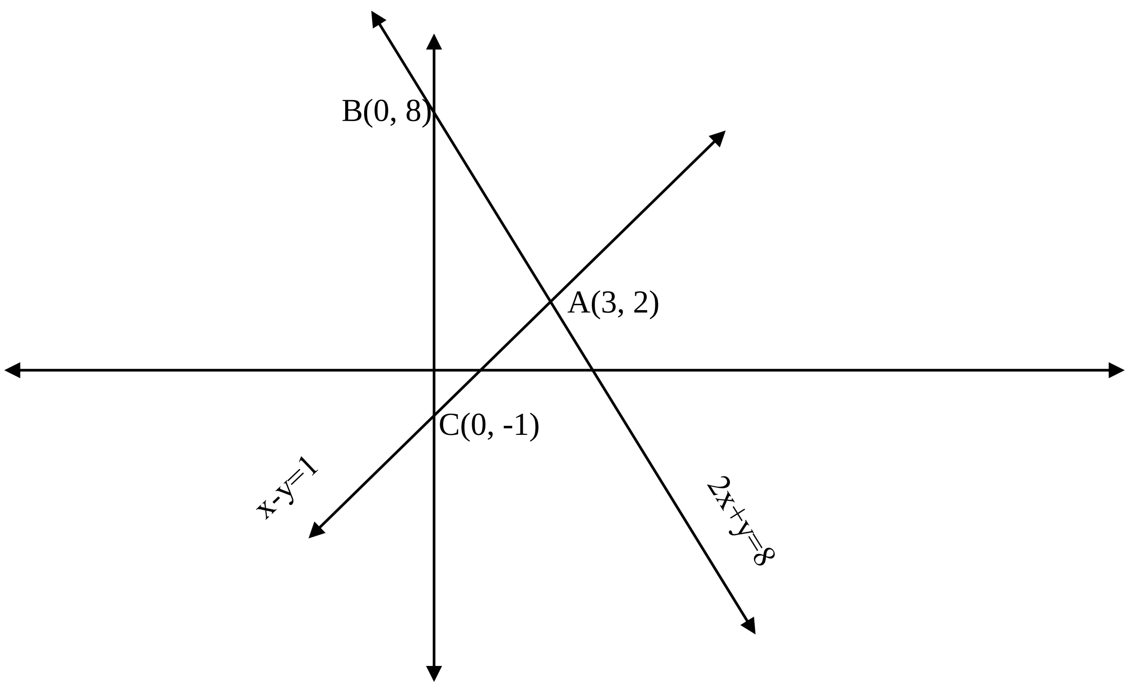
= $(\frac{-4}{k})^2 - 2\frac{4}{k} = 24$

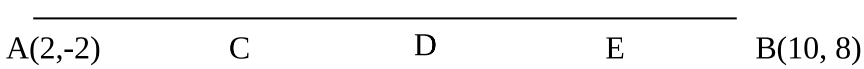
= $\frac{16}{k^2} - \frac{8}{k} = 24$

= $\frac{2}{k^2} - \frac{1}{k} = 3$

= $3k^2 + k - 2 = 0$

= $(k+1)(2k-2) = 0$

	either $(k+1)=0\ or\ (3k-2)=0$ either $k=-1\ or\ k=\frac{2}{3}$ Type equation here.	
15	 Solution is $x=3\ and\ y=2$. Area of the triangle ABC=$\frac{1}{2}\times BC\times AD=\frac{1}{2}\times 9\times 3=13.5\ sq.unit$	**3**
16	Solve the following quadratic equation $9x^2-9(a+b)x+(2a^2+5ab+2b^2)=0$. $$using\ quadratic\ formula: x=\frac{-B\pm\sqrt{B^2-4AC}}{2A}$$ $$\Rightarrow x=\frac{-\{-9(a+b))\}\pm\sqrt{\{-9(a+b))\}^2-4\times 9\times(2a^2+5ab+2b^2)}}{2\times 9}$$ $$\Rightarrow x=\frac{9a+9b\pm\sqrt{81a^2+81b^2+162ab-72a^2-180ab-72b^2)}}{18}$$ $$\Rightarrow x=\frac{9a+9b\pm\sqrt{(9a^2+9b^2-18ab)}}{18}$$ $$\Rightarrow x=\frac{9a+9b\pm\sqrt{(3a-3b)^2}}{18}$$	**3**

	$$\Rightarrow x = \frac{(9a+9b) \pm (3a-3b)}{18}$$ $either\ \ x = \frac{(2a+b)}{3}$ or $x = \frac{(a+2b)}{3}$	
17	Find the middle term of the A.P. 7, 10, 13 ,….,187. $a = 7, d = 10 - 7 = 3, let\ a_n = 187$ $a + (n-1)d = 187$ $7 + (n-1)3 = 187$ $3n + 4 = 187$ $n = 61$ $middle\ term = \frac{n+1}{2} = \frac{61+1}{2} = \frac{62}{2} = 31^{st}\ term$ middle term is $a_{31} = a + (31-1)d = 7 + 30 \times 3 = 97$.	**3**
18	Find the coordinates of the points which divides the line segment joining the points A $(2, -2)$ and B $(10,8)$ into four equal parts.  Using mid point formula for AB ; coordinates of D $\left(\frac{2+10}{2}, \frac{-2+8}{2}\right) = D(6,3)$ Using mid point formula for AD; coordinates of C $\left(\frac{2+6}{2}, \frac{-2+3}{2}\right) = C(4, \frac{1}{2})$ Using mid point formula for DB ; coordinates of E $\left(\frac{6+10}{2}, \frac{3+8}{2}\right) = D(8, \frac{11}{2})$	**3**

19 Prove that $(sinA + cosecA)^2 + (cosA + secA)^2 = 7 + tan^2A + cot^2A.$ **3**

$$L.H.S. = (sinA + cosecA)^2 + (cosA + secA)^2$$

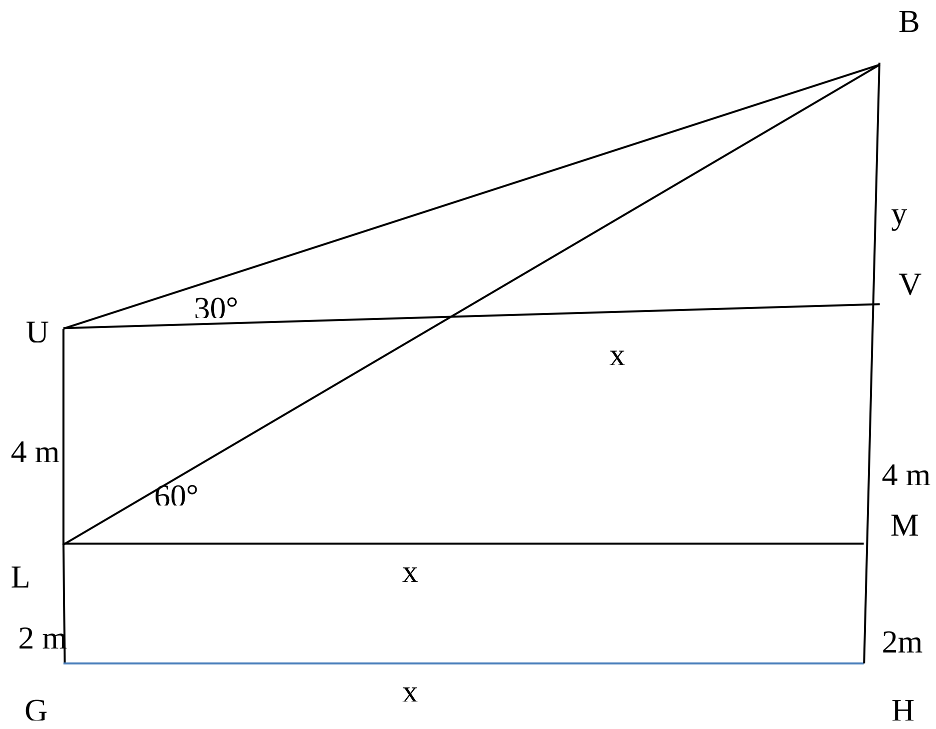
$$= sin^2A + cosec^2A + 2sinA.cosecA + cos^2A + sec^2A + 2\,cosA.secA$$

$$= (sin^2A + cos^2A) + (1 + cot^2A) + 2sinA.\frac{1}{sinA} + (1 + tan^2A) + 2\,cosA.\frac{1}{cosA}$$

$$= 1 + (1 + cot^2A) + 2 + (1 + tan^2A) + 2$$

$$= 7 + tan^2A + cot^2A$$

20 The lower window of a house is at a height of 2 m above the ground and its upper window is 4 m vertically above the lower window. At certain instant, the angles of elevation of a balloon from these windows are observed to be 60 and 30 respectively. Find the height of the balloon above the ground. **3**



$$in\ the\ \perp \Delta BVU; \quad tan30° = \frac{BV}{UV}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{x}$$

$$x = \sqrt{3}y \ldots\ldots\ldots\ldots\ldots\ldots..(eq.1)$$

$$in\ the\ \perp \Delta BML; \quad tan60° = \frac{BM}{LM}$$

$$\frac{\sqrt{3}}{1} = \frac{y+4}{x}$$

	$$x = \frac{y+4}{\sqrt{3}} \dots\dots\dots\dots\dots\dots..(eq.2)$$ $$from\ eq.1\ and\ eq.2;\ \sqrt{3}y = \frac{y+4}{\sqrt{3}}$$ $$3y = y + 4$$ $$y = 2;$$ $$height\ of\ the\ balloon\ is\ BH = y + 4 + 2 = 2 + 6 = 8\ m$$	
21	Median = 32.5 So, median class = 30 – 40	**3**

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
frequency	f_1	5	9	12	f_2	3	2	40
cf	f_1	5+ f_1	14+ f_1	26+ f_1	26+ $f_1 + f_2$	29+ $f_1 + f_2$	31+ f_1+ f_2	31+ f_1+ f_2=40

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h \qquad l = 30; \quad \frac{n}{2} = 20; \quad cf = 14 + f_1\ ; \quad h = 10 \quad and \quad f = 12$$

$$32.5 = 30\left(\frac{20 - (14 + f_1)}{12}\right) \times 10$$

$$2.5 = \frac{6 - f_1}{6} \times 5$$

$$f_1 = 3$$

$$Now,\ f_1 + f_2 = 9$$

$$\therefore\ f_2 = 6$$

22	Total cards left =52 – 4 =48 (i) Face cards = 3x4 =12 Required Probability = $\frac{12}{48} = \frac{1}{4}$ (ii) Kings = 4 (including 2 of reds); Red cards = 12+12=24(excluding 2 red aces) So, cards which are either king or a red card = 4+24-2=26 Required Probability = $\frac{26}{48} = \frac{13}{24}$ (iii) Queens = 4; Black cards = 12+12=24(excluding 2 black aces) So no. of cards which are neither a queen nor a black card= 48 – 4 – 24 = 20 Required Probability = $\frac{26}{48} = \frac{13}{24}$	**3**
	SECTION – D	
23	$\sqrt{\frac{5}{3}}$ *and* $-\sqrt{\frac{5}{3}}$ *are the zeroes* $\therefore \left(x^2 - \frac{5}{3}\right)$ *will be a factor of* $3x^4 + 6x^3 - 2x^2 - 10x - 5$ *on dividing* $3x^4 + 6x^3 - 2x^2 - 10x - 5$ *by* $\left(x^2 - \frac{5}{3}\right)$ *we get quotient* $= 3x^2 + 6x + 3$ $3x^2 + 6x + 3 = (3x+3)\,(x+1)$ $\therefore$ *other two zeroes are* -1 *and* -1	**4**
24	Let speed of boat in still water is x km/h and speed of stream is y km/hr speed of boat upstream = (x-y) km/h speed of boat downstream = (x+y) km/h $\therefore \frac{30}{x-y} + \frac{40}{x+y} = 8\ ;\ \ \frac{36}{x-y} + \frac{32}{x+y} = 8$ *taking* $\frac{1}{x-y} = a$ *and* $\frac{1}{x+y} = b$ We get, *30a+40b=8……….(i)* *and, 36a+32b=8………(ii)* Solving we get:	**4**

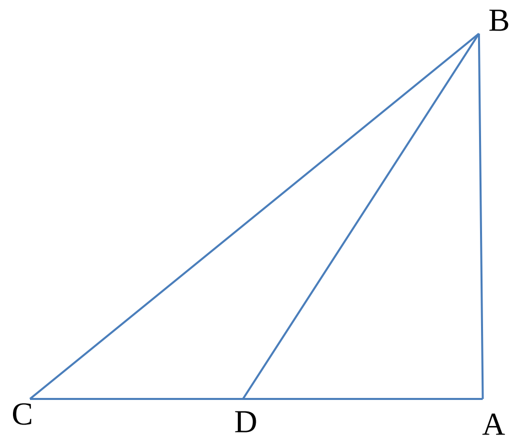
	$a = \frac{2}{15}$ and $b = \frac{1}{10}$ $\therefore \frac{1}{x-y} = \frac{2}{15}$ and $\frac{1}{x+y} = \frac{1}{10}$ $\therefore x - y = \frac{15}{2}$ and $x + y = 10$ solving we get: $x = \frac{35}{4}$ km/h and $y = \frac{5}{4}$ km/h	
25	Let volume of pool = V Let time taken by second pipe to fill the pool = x hrs. time taken by first pipe to fill the pool = $x+5$ hrs. time taken by third pipe to fill the pool = $x-4$ hrs. In one hour first pipe can fill = $\frac{V}{x+5}$ part of the pool In one hour second pipe can fill = $\frac{V}{x}$ part of the pool In one hour third pipe can fill = $\frac{V}{x-4}$ part of the pool According to question, $\frac{V}{x+5} + \frac{V}{x} = \frac{V}{x-4}$ $(2x+5)(x-4) = (x+5)x$ $x^2 - 8x - 20 = 0$ $x = 10$ or $x = -2$(neglected) So, time taken by first pipe = 15 hrs. time taken by second pipe = 10 hrs. and time taken by third pipe = 6 hrs.	**4**
26	Let the numbers are :	**4**

	$a-d, a, a+d$ $a-d+a+a+d=12$ $a=4$ $Now, (4-d)^3+4^3+(4+d)^3=288$ $d=2$ $\therefore$ *the numbers are*: 2, 4 *and* 6	
27	Let P(1,1), Q(2, - 3) and R(3,4) are the mid points of AB, BC and CA respectively. *let* $A=(x_1, y_1),\ B(x_2, y_2)$ *and* $C(x_3, y_3)$ *taking P as the mid point of BC* $P(1,1)=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $\Rightarrow x_1+x_2=1$ *and* $y_1+y_2=2$ *similarly, we get*: $x_2+x_3=4 \qquad y_2+y_3=-6$ $x_1+x_3=6 \qquad y_1+y_3=8$ $Centroid=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)=\left(2, \frac{2}{3}\right)$	**4**
28	$LHS=\left(\frac{1+\sin A-\cos A}{1+\sin A+\cos A}\times\frac{1+\sin A-\cos A}{1+\sin A-\cos A}\right)^2$ $=\left(\frac{1+\sin^2 A+\cos^2 A+2\sin A-2\sin A\cos A-2\cos A}{1-\cos^2 A+\sin^2 A+2\sin A}\right)^2$ $=\left(\frac{2+2\sin A-2\sin A\cos A-2\cos A}{\sin^2 A+\sin^2 A+2\sin A}\right)^2=\left[\frac{(1+\sin A)(2-2\cos A)}{2\sin A(\sin A+1)}\right]^2=\left[\left(\frac{1-\cos A}{\sin A}\right)\right]^2$ $=\frac{(1-\cos A)(1-\cos A)}{(1-\cos A)(1+\cos A)}=\frac{(1-\cos A)}{(1+\cos A)}=RHS$	**4**
29	AB = 3000 m (height of the aeroplane from the ground)	**4**

$\angle$ BDA = 45^0

$\angle$ BCA = 30^0

Distance travelled in 15 seconds = DC



$In \Delta BAD,$

$\tan 45^0 = \frac{AB}{AD} = \frac{3000}{AD}$

$AD = 3000m$

$In \Delta BAC,$

$\tan 30^0 = \frac{AB}{AC} = \frac{3000}{AC}$

$AC = 3000\sqrt{3}m$

$\therefore CD = AC - AD = 3000(\sqrt{3} - 1)m$

$\therefore speed = \frac{3000(\sqrt{3}-1)m}{15} = 146 \ m/\sec$

30

C.I.	f		Less than cf		More than cf
400 - 450	20	Less than 450	20	More than or equal to 400	230
450-500	35	Less than 500	55	More than or equal to 450	210
500-550	40	Less than 550	95	More than or equal to 500	175
550-600	32	Less than 600	127	More than or equal to 550	135
600-650	24	Less than 650	151	More than or equal to 600	103
650-700	27	Less than 700	178	More than or equal to 650	79
700-750	18	Less than 750	196	More than or equal to 700	52
750-800	34	Less than 800	230	More than or equal to 750	34
Total	230				

After plotting both the ogives on a graph, the point where the meet will be the median.

Median = 581(approx.)

4

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