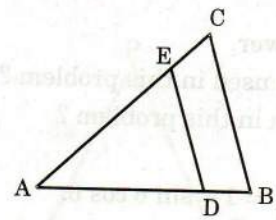
Sample Question Paper — 3

SECTION – A

1. If the HCF of 65 and 117 is expressible in the form of $65m - 117$, then find the value of m.

2. What is the value of k, for equations $2x + ky = 7$, $4x + 8y = 14$ will represent coincident lines.

3. In the figure, DE $\parallel$ BC. If AD = x, DB = $x - 2$, AE = $x + 2$ and EC = $x - 1$, then find the value of x.



4. For the following distribution :

Classes	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	25	30	27	35	21

Find the sum of the lower limit of the median class and the lower limit of the modal class.

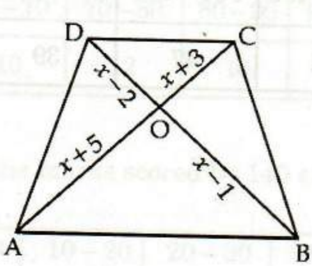
SECTION – B

5. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$. Find the value of p and q.

6. Solve the following system of linear equations by substitution method :

$$2x - y = 2$$
$$x + 3y = 15.$$

7. In the given figure, if AB $\parallel$ DC, find the value of x.



8. Express $\cos 68° + \tan 76°$ in the terms of the angles between $0°$ and $45°$.

9. If $\tan(A + B) = \sqrt{3}$, $\tan(A - B) = \frac{1}{\sqrt{3}}$, $0° < A + B < 90°$ and $A > B$, then find A and B.

10. Convert the following distribution to a more than type cumulative frequency distribution :

Classes	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	12	18	10	15	5

SECTION – C

11. Three bells toll at intervals of 9, 12, 15 minutes respectively. If they start tolling together, after what time will they next toll together.

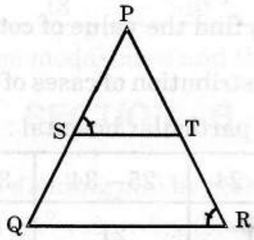
12. Find the HCF and LCM of 510 and 92 and verify that :

HCF × LCM = Product of two given numbers.

13. If α and β are the zeroes of a quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$. Find the quadratic polynomial having α and β as its zeroes. Verify the relationship between the zeroes and coefficients of the polynomial.

14. The well monitored total expenditure per month of a household consists of a fixed rent of the house and mess expenditure depending upon the number of people sharing the house. The total monthly expenditure is ₹ 3900 for 2 people and ₹ 7500 for 5 people. Find the rent of the house and the mess expenditure per head per month.

15. In the given figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isosceles triangle.



16. Prove that the sum of squares on the sides of a rhombus is equal to sum of squares on its diagonals.

17. Simplify : $\dfrac{\sin\theta.\sec(90° - \theta).\tan\theta}{\operatorname{cosec}(90° - \theta).\cos\theta.\cot(90° - \theta)} - \dfrac{\tan(90° - \theta)}{\cot\theta}$.

18. Find the value of sin 60° geometrically.

19. If the mean of the following data is 14·7, find the values of p and q.

Classes	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30	30 – 36	36 – 42	**Total**
Frequency	10	p	4	7	q	4	1	**40**

20. Write the relationship connecting three measures of central tendencies. Hence find the median of the given data if mode is 24·5 and mean is 29·75.

SECTION – D

21. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

22. If the polynomial $f(x) = 3x^4 + 3x^3 - 11x^2 - 5x + 10$ is completely divisible by $3x^2 - 5$, find all its zeroes.

23. Solve the following pair of linear equations graphically :

$$x - y = 1$$

$$2x + y = 8.$$

Also find the co-ordinates of the points where the lines, represented by the above equation, intersect y-axis.

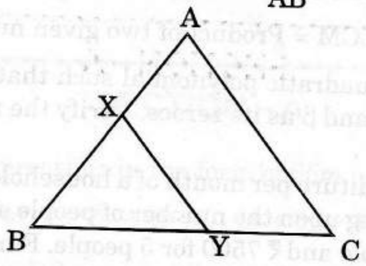
24. Draw the graphs of following equations :

$$2x - y = 1, x + 2y = 13.$$

Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y-axis.

25. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio. Prove it.

26. In the given figure, the line segment XY is parallel to the side AC of ΔABC and it divides the triangle into two parts of equal area. Find the ratio $\frac{AX}{AB}$.



27. If $\cos\theta + \sin\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, prove that $q(p^2 - 1) = 2p$.

28. Prove that : $(\sin A + \sec A)^2 + (\cos A + \operatorname{cosec} A)^2 = (1 + \sec A \operatorname{cosec} A)^2$.

29. If $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$, then find the value of $\cot^2\theta + \tan^2\theta$.

30. The following table shows the age distribution of cases of a certain disease (caused due to spread of a virus) admitted during a year in a particular hospital :

Classes	5 – 14	15 – 24	25 – 34	35 – 44	45 – 54	55 – 64
Frequency	6	11	21	23	14	5

(i) Find the average age for which maximum cases occured ?

(ii) Which mathematical concept is used in this problem.

(iii) What is its value ?

31. Draw a more than ogive for the following data :

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	5	9	10	12	8	7	5	4

10th Mathematics Solution Sample paper -03

SECTION – A

1. $65 = 13 \times 5$

and $117 = 13 \times 9$

Hence, $\text{HCF} = 13$

According to question $65m - 117 = 13$

$\Rightarrow 65m = 13 + 117 = 130$

$\Rightarrow m = \frac{130}{65} = 2.$ **1**

2. For coincident lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

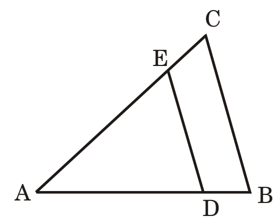
$\Rightarrow \frac{2}{4} = \frac{k}{8} = \frac{7}{14}$

$\Rightarrow k = 4.$ **1**

3. In the Δ ABC,

DE || BC (given)

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By B.PT)



$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$

$\Rightarrow x(x-1) = (x-2)(x+2)$

$\Rightarrow x^2 - x = x^2 - 4$

$\Rightarrow x = 4.$ **1**

4. In the distribution, Median class $= 20 - 25$

Hence, Lower limit of median class $= 20$

and Model class $= 25 - 30$

So, Lower limit of modal class $= 25$

Sum of lower limit of median class and lower limit of modal class $= 25 + 20 = 45.$ **1**

SECTION – B

5. Let, $f(x) = 2x^2 - 5x - 3$

Let the zeroes of polynomial are α and β, then

$$\text{Sum of zeroes } \alpha + \beta = \frac{5}{2}, \text{ product of zeroes } \alpha\beta = -\frac{3}{2}$$

According to question, zeroes of $x^2 + px + q$ are 2α and 2β

$$\text{Sum of zeroes} = -\frac{\text{coeff. of } x}{\text{coeff. of } x^2} = \frac{-p}{1}$$

$= 2\alpha + 2\beta = 2(\alpha + \beta) = 2 \times \frac{5}{2} = 5 \Rightarrow p = -5$ **1**

$$\text{Product of zeroes} = \frac{\text{Constant}}{\text{Coeff. of } x^2} = \frac{q}{1}$$

$$= 2\alpha \times 2\beta = 4\alpha\beta = 4\left(-\frac{3}{2}\right) = -6$$

$\therefore$ $p = -5$ and $q = -6$. **1**

6. $2x - y = 2$

$\Rightarrow$ $y = 2x - 2$ ...(i)

$x + 3y = 15$ ...(ii)

Substituting the value of y from (i) in (ii), we get

$x + 6x - 6 = 15$ **1**

$\Rightarrow$ $7x = 21$

$\Rightarrow$ $x = 3$

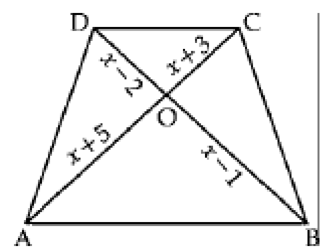
From (i), $y = 2 \times 3 - 2 = 4$

$\therefore$ $x = 3$ and $y = 4$. **1**

7. Since the diagonals of a trapezium divide each other proportionally, we have

$$\frac{OA}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{x+5}{x+3} = \frac{x-1}{x-2}$$ **1**



$\Rightarrow$ $(x + 5)(x - 2) = (x - 1)(x + 3)$

$\Rightarrow$ $x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$

$\Rightarrow$ $3x - 2x = 10 - 3$

$\therefore$ $x = 7$. **1**

8. $\cos 68^\circ + \tan 76^\circ = \cos(90^\circ - 22^\circ) + \tan(90^\circ - 14^\circ)$ **1**

$= \sin 22^\circ + \cot 14^\circ$, $[\because \cos(90^\circ - \theta) = \sin\theta$ and $\tan(90^\circ - \theta) = \cot\theta\,]$ **1**

9. $\tan(A + B) = \sqrt{3} = \tan 60^\circ$

$\Rightarrow$ $A + B = 60^\circ$ ...(i)½

Again, $\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30^\circ$

$\Rightarrow$ $A - B = 30^\circ$ $(\because A > B)$ ...(ii)½

Adding equations (i) and (ii), we get

$$2A = 90^\circ \Rightarrow A = \frac{90^\circ}{2} = 45^\circ$$

Putting this value of A in equation (i), we get

$B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$

Hence, $A = 45^\circ$ and $B = 15^\circ$. **1**

10.

Class	Cumulative frequency
More than 50	60
More than 60	48
More than 70	30
More than 80	20
More than 90	5

2

SECTION – C

11. The bells will next full to gether after time equal to LCM (9, 12, 15) **1**

To find L.C.M. (9, 12, 15).

$$9 = 3 \times 3$$
$$12 = 2 \times 2 \times 3$$
$$15 = 3 \times 5 \qquad \textbf{1}$$
$$\therefore \quad \text{L.C.M.} = 3 \times 3 \times 2 \times 2 \times 5 = 180 \text{ minutes}$$

The bells will toll together after 180 minutes. **1**

12. By Euclid's division algorithm, $510 = 92 \times 5 + 50$

$$92 = 50 \times 1 + 42$$
$$50 = 42 \times 1 + 8$$
$$42 = 8 \times 5 + 2 \qquad \textbf{1}$$
$$8 = 2 \times 4 + 0.$$
$$\text{HCF}(510, 92) = 2$$
$$92 = 2^2 \times 23$$
$$510 = 2 \times 3 \times 5 \times 17$$
$$\text{LCM} = 2^2 \times 23 \times 3 \times 5 \times 17 = 23460$$
$$\text{HCF} \times \text{LCM} = 2 \times 23460 = 46920 \qquad \textbf{1}$$
$$\text{Product of two given numbers} = 510 \times 92$$
$$\Rightarrow \quad \text{HCF} \times \text{LCM} = \text{Product of two given numbers.} \qquad \textbf{1}$$

13. Given,

$$\alpha + \beta = 24 \qquad \text{...(i)}$$
$$\alpha - \beta = 8 \qquad \text{...(ii)}$$

Adding equations (i) and (ii), we get

$$2a = 32$$
$$\Rightarrow \quad a = 16$$

Put the value of α in equation (i)

$$16 + \beta = 24$$
$$\beta = 24 - 16 = 8 \qquad \textbf{1}$$

Hence, the quadratic polynomial is

$$= x^2 - (\text{Sum of zeroes})\, x + (\text{Product of zeroes})$$
$$= x^2 - (\alpha + \beta)\, x + \alpha\beta$$
$$= x^2 - (16 + 8)\, x + (16)\,(8)$$
$$= x^2 - 24x + 128$$

Verification :

$$\alpha + \beta = \frac{-b}{a} = -\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2}$$
$$\Rightarrow \quad 24 = -\left(-\frac{24}{1}\right) \qquad \textbf{1}$$

and

$$\alpha\beta = \frac{c}{a} = \frac{\text{Constant}}{\text{Coeff. of } x^2}$$
$$\Rightarrow \quad 128 = \frac{128}{1}$$

Hence, the relationship is verified . **1**

14. Let the monthly rent of the house be ₹ x and the mess expenditure per head per month be ₹ y.

According to the given condition,

$$x + 2y = 3900 \qquad \text{...(i)}$$

$$x + 5y = 7500 \qquad \text{...(ii)} \quad \mathbf{1}$$

Subtracting equation (ii) from equation (i), we get

$$-3y = -3600$$

$$\Rightarrow \quad y = \frac{3600}{3} = 1200 \qquad \mathbf{1}$$

Putting this value of y in eqn. (i), we get

$$x + 2400 = 3900$$

$$\Rightarrow \quad x = 3900 - 2400$$

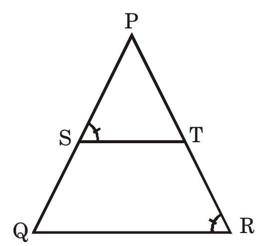
$$= 1500$$

Hence, monthly rent = ₹ 1500 and mess expenditure per head per month = ₹ 1200. **1**

15. Given :

$$\frac{PS}{SQ} = \frac{PT}{TR}$$

$$\angle PST = \angle PRQ$$



To prove : ΔPQR is isosceles triangle.

Proof :

$$\frac{PS}{SQ} = \frac{PT}{TR}$$

By converse of B.P.T., we get $\quad ST \parallel QR$ **1**

$\therefore \quad \angle PST = \angle PQR$ (Corresponding angles) **1**

$\therefore \quad \angle PST = \angle PRQ$ (Given)

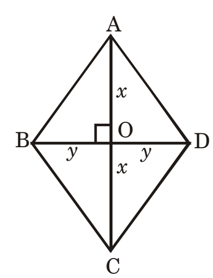
But $\quad \angle PQR = \angle PRQ$

So, ΔPQR is an isosceles triangle. **Proved 1**

16. ABCD is a rhombus, $\therefore$ AO = OC = x (say), BO = OD = y (say) and $\angle AOB = 90°$

$\Rightarrow \quad AB^2 = OA^2 + BO^2 = x^2 + y^2$ (By Pythagoras theorem) **1**

Similarly, $\quad AD^2 = x^2 + y^2 = BC^2 = CD^2$ **1**



$$\therefore \quad AB^2 + BC^2 + CD^2 + DA^2 = 4(x^2 + y^2)$$

$$= (2x)^2 + (2y)^2$$

$$= AC^2 + BD^2$$

Proved. 1

17. We know that,

sec (90° – θ) = cosec θ, tan (90° – θ) = cot θ, cot (90° – θ) = tan θ, cosec (90° – θ) = sec θ **1**

$$\text{Hence, } \frac{\sin\theta.\sec(90° - \theta)\tan\theta}{\text{cosec}\,(90° - \theta)\cos\theta.\cot(90° - \theta)} - \frac{\tan(90° - \theta)}{\cot\theta} = \frac{\sin\theta.\text{cosec}\,\theta.\tan\theta}{\sec\theta.\cos\theta.\tan\theta} - \frac{\cot\theta}{\cot\theta} \qquad \mathbf{1}$$

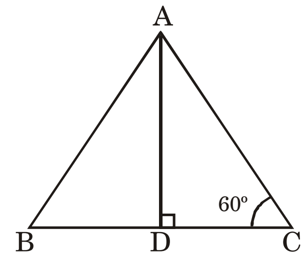
$$= \frac{\sin\theta \times \dfrac{1}{\sin\theta} \times \tan\theta}{\dfrac{1}{\cos\theta} \times \cos\theta.\tan\theta} - 1 = 1 - 1 = 0. \qquad \mathbf{1}$$

18. Let ΔABC is an equilateral triangle, and

$$AB = BC = AC = 2a$$
$$AD \perp BC$$

AD is also median. In equilateral Δ ABC

$$CD = a = BD$$
$$\angle A = \angle B = \angle C = 60^\circ \qquad \mathbf{1}$$



In ΔADC,
$$AC = 2a$$
$$DC = a$$

$\therefore$ $$AD = \sqrt{(2a)^2 - (a)^2} = \sqrt{3a^2} \qquad \mathbf{1}$$
$$= \sqrt{3}\,a$$

$\therefore$ $$\sin 60^\circ = \frac{AD}{AC} = \frac{\sqrt{3}a}{2a}$$

$\therefore$ $$\sin 60^\circ = \frac{\sqrt{3}}{2}. \qquad \mathbf{1}$$

19.

Classes	x_i	f_i	x_if_i
0 – 6	3	10	30
6 – 12	9	p	$9p$
12 – 18	15	4	60
18 – 24	21	7	147
24 – 30	27	q	$27q$
30 – 36	33	4	132
36 – 42	39	1	39
	Total	**Sf_i = 26 + p + q**	**Sx_if_i = 408 + 9p + 27q**

Given, $$\Sigma f_i = 40,$$

$\Rightarrow$ $$26 + p + q = 40$$

$\Rightarrow$ $$p + q = 14 \qquad \text{...(i)} \tfrac{1}{2}$$

$\therefore$ $$\text{Mean, } \bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} \qquad \tfrac{1}{2}$$

$$14{\cdot}7 = \frac{408 + 9p + 27q}{40}$$
$$588 = 408 + 9p + 27q \qquad \mathbf{1}$$
$$180 = 9p + 27q$$
$$p + 3q = 20 \qquad \text{...(ii)}$$

Subtracting eq. (i) from eq. (ii), we get

$$2q = 6$$

$\Rightarrow$ $$q = 3 \qquad \tfrac{1}{2}$$

Putting this value of q in eq. (i), we get

$$p = 14 - q = 14 - 3 = 11 \qquad \tfrac{1}{2}$$

20. According to question $$\text{mode} = 24{\cdot}5$$

and $$\text{mean} = 29{\cdot}75$$

The relationship connecting measures of central tendencies is :

$$3\text{ Median} = \text{Mode} + 2\text{ Mean} \qquad \mathbf{1}$$
$$3\text{ Median} = 24{\cdot}5 + 2 \times 29{\cdot}75$$
$$= 24{\cdot}5 + 59{\cdot}50 \qquad \mathbf{1}$$
$$3\text{ Median} = 84{\cdot}0$$

$\therefore$ $$\text{Median} = \frac{84}{3} = 28. \qquad \mathbf{1}$$

SECTION – D

21. $n^3 - n = n(n^2 - 1) = n(n+1)(n-1) = (n-1)\,n(n+1)$

$=$ product of three consecutive positive integers.

Now, we have to show that the product of three consecutive positive integers is divisible by 6.

We know that any positive integer a is of the form $3q$, $3q + 1$ or $3q + 2$ for some positive integer q.

Let a, $a + 1$, $a + 2$ be any three consecutive integers. **½**

Case I. If $a = 3q$.

$$a(a+1)(a+2) = 3q(3q+1)(3q+2)$$
$$= 3q \text{ (even number, say } 2r) = 6qr,$$

($\because$ Product of two consecutive integers $(3q + 1)$ and $(3q + 2)$ is an even integer)

which is divisible by 6. **1**

Case II. If $a = 3q + 1$.

$$\therefore \quad a(a+1)(a+2) = (3q+1)(3q+2)(3q+3)$$
$$= (\text{even number say } 2r)(3)(q+1)$$
$$= 6r(q+1),$$ **1**

which is divisible by 6.

Case III. If $a = 3q + 2$.

$$\therefore \quad a(a+1)(a+2) = (3q+2)(3q+3)(3q+4)$$
$$= \text{multiple of 6 for every } q$$
$$= 6r \text{ (say)},$$ **1**

which is divisible by 6.

Hence, the product of three consecutive integers is divisible by 6. **½**

22. Since $3x^2 - 5$ divides $f(x)$ completely

$\therefore$ $(3x^2 - 5)$ is a factor of $f(x)$

$3(x^2 - \frac{5}{3})$ is a factor of $f(x)$

$\therefore$ $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$ is a factor of $f(x)$

$\therefore$ $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are zeroes of $f(x)$ **1**

$$
\begin{array}{r|l}
3x^2 - 5 & 3x^4 + 3x^3 - 11x^2 - 5x + 10 \;(\; x^2 + x - 2 \\
 & 3x^4 \qquad\quad - 5x^2 \\
 & - \qquad\qquad + \\
\hline
 & \qquad 3x^3 - 6x^2 - 5x \\
 & \qquad 3x^3 \qquad\quad - 5x \\
 & \qquad - \qquad\qquad + \\
\hline
 & \qquad\qquad - 6x^2 + 10 \\
 & \qquad\qquad - 6x^2 + 10 \\
 & \qquad\qquad + \qquad - \\
\hline
 & \qquad\qquad\quad \times
\end{array}
$$ **1½**

$\therefore$ $(x^2 + x - 2)$ is a factor of $p(x)$

$\therefore$ $(x^2 + 2x - x - 2)$ is a factor of $p(x)$

$(x + 2)(x - 1)$ is a factor of $p(x)$ **1**

$\therefore$ $\quad -2$ and 1 are zeroes of $p(x)$ **1**

$\therefore$ all the zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -2$ and 1. ½

23. $x - y = 1 \Rightarrow y = x - 1$

x	2	3	-1
y	1	2	-2

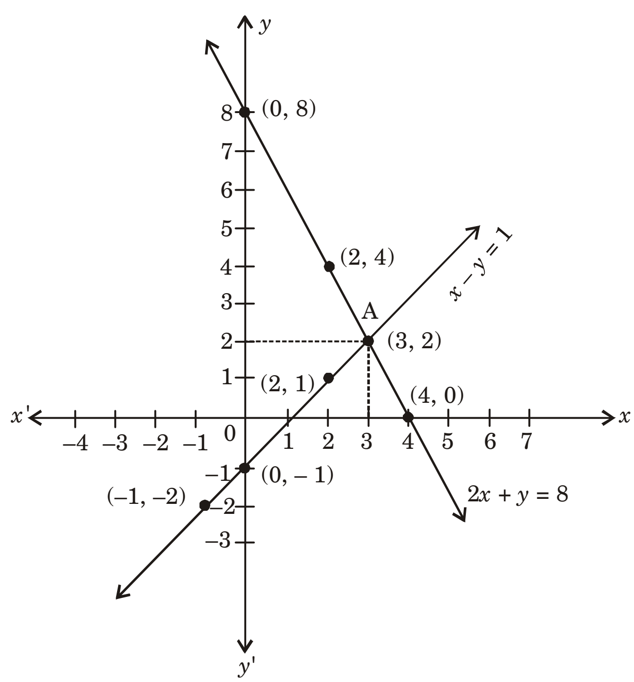
½

$2x + y = 8 \Rightarrow y = 8 - 2x$

x	2	4	0
y	4	0	8

½

Plotting the above points we get the graphs of the equations $x - y = 1$ and $2x + y = 8$.



1

Clearly, the two lines intersect at point A (3, 2). **1**

$\therefore$ Solution of given equations is $x = 3, y = 2$.

Again, $\quad x - y = 1$ intersects y-axis at $(0, -1)$ **1**

and $\quad 2x + y = 8$ intersects y-axis at $(0, 8)$.

24. $2x - y = 1 \quad \Rightarrow \quad y = 2x - 1$

x	0	1	3
y	-1	1	5

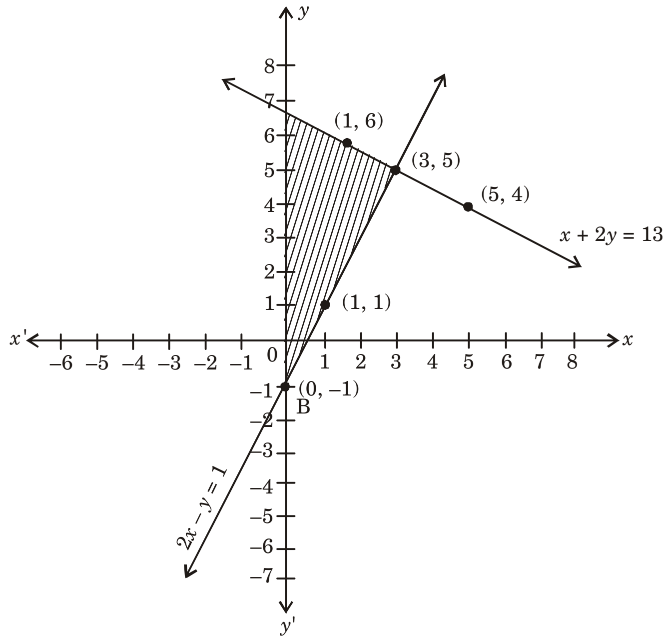
½

and $\quad x + 2y = 13 \quad \Rightarrow \quad y = \dfrac{13 - x}{2}$

x	1	3	5
y	6	5	4

½

Plotting the above points we get the graph of above equations.



1

Clearly, two lines intersect at point A (3, 5).

Hence, $x = 3$ and $y = 5$ is the solution of above equations. **1**

ABC is the triangular shaded region formed by the lines and the y-axis. **1**

25. **Given :** ABC is a triangle in which DE || BC.

To prove : $\frac{AD}{BD} = \frac{AE}{CE}$

Construction : Draw DN ⊥ AE and EM ⊥ AD. Join BE and CD. **1**

Proof : In ΔADE, $\text{area}(\Delta ADE) = \frac{1}{2} \times AE \times DN$ ...(i)

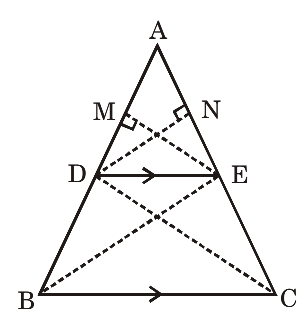
$\text{area}(\Delta DCE) = \frac{1}{2} \times CE \times DN$ ...(ii)

Divide (i) by (ii) $\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DCE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times CE \times DN}$

$\Rightarrow \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEC)} = \frac{AE}{CE}$ ...(iii) **1**

$\text{area}(\Delta ADE) = \frac{1}{2} \times AD \times EM$ ...(iv)

$\text{area}(\Delta DEB) = \frac{1}{2} \times EM \times BD$ ...(v)



Divide (iv) by (v), $\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{\frac{1}{2} \times AD \times EM}{\frac{1}{2} \times BD \times EM}$

$$\Rightarrow \quad \frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AD}{BD} \quad \text{...(iv)} \ \mathbf{1}$$

ΔDEB and ΔDEC lies on the same base DE and between same parallel lines DE and BC.

$$\therefore \quad \text{area}(\Delta DEB) = \text{area}(\Delta DEC)$$

From equation (iii),
$$\frac{\text{area}(\Delta ADE)}{\text{area}(\Delta DEB)} = \frac{AE}{CE} \quad \text{...(vii)}$$

From equations (vi) and (vii), we get

$$\frac{AE}{CE} = \frac{AD}{BD}.$$ **Proved. 1**

26. Given : The line segment XY is parallel to side AC of ΔABC. XY divides ΔABC in to two parts equal in area.

To find : In ΔBAC and ΔBXY,

$$\angle B = \angle B \quad \text{(Common)}$$
$$\angle BAC = \Delta BXY \quad \text{(Corresponding angles)}$$
$$\therefore \quad \Delta BAC \sim \Delta BXY \quad \text{(By AA similarity)} \ \mathbf{1}$$

We have,
$$\frac{\text{ar}(\Delta BAC)}{\text{ar}(\Delta BXY)} = \left(\frac{BA}{BX}\right)^2$$

($\because$ ratio of areas of similar triangles is proportional to the squares on the sides of the triangles)

$$\frac{2 \times \text{ar}(\Delta BXY)}{\text{ar}(\Delta BXY)} = \left(\frac{AB}{BX}\right)^2 \quad \mathbf{1}$$

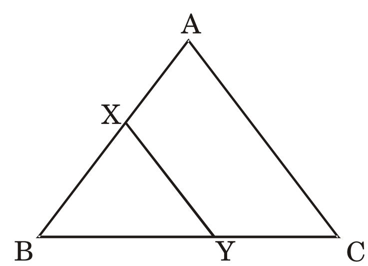
$$\Rightarrow \quad \frac{AB}{BX} = \sqrt{2}$$

$$\Rightarrow \quad \frac{BX}{AB} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad 1 - \frac{BX}{AB} = 1 - \frac{1}{\sqrt{2}} \quad \mathbf{1}$$

$$\Rightarrow \quad \frac{AB - BX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}. \quad \mathbf{1}$$



27. **Given :** $\cos\theta + \sin\theta = p$ and $\sec\theta + \text{cosec}\,\theta = q$

$$\therefore \quad \text{LHS} = q(p^2 - 1) = (\sec\theta + \text{cosec}\,\theta)\,[(\cos\theta + \sin\theta)^2 - 1]$$
$$= (\sec\theta + \text{cosec}\,\theta)\,[1 + 2\sin\theta\cos\theta - 1]$$
$$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta\cos\theta] \quad \mathbf{2}$$
$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta} \times 2\sin\theta\cos\theta$$
$$= 2(\sin\theta + \cos\theta) \quad \mathbf{1}$$
$$= 2p$$
$$= \text{RHS}.$$ **Proved. 1**

28.
$$\text{LHS} = (\sin A + \sec A)^2 + (\cos A + \text{cosec}\,A)^2$$
$$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2 \quad \mathbf{½}$$

$$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + \frac{1}{\sin^2 A} + 2\frac{\cos A}{\sin A}$$ ½

$$= \sin^2 A + \cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + 2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin^2 A \cos^2 A} + 2\left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}\right)$$ **1**

$$= 1 + \frac{1}{\sin^2 A \cos^2 A} + \frac{2}{\sin A \cos A}$$

$$= \left(1 + \frac{1}{\sin A \cos A}\right)^2$$

$$= (1 + \sec A.\text{cosec } A)^2.$$ **Proved.**

29. Let $\cot\theta = x$, $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$ becomes

$$\sqrt{3}x^2 - 4x + \sqrt{3} = 0$$ **1**

or $$(x - \sqrt{3})(\sqrt{3}x - 1) = 0$$

$\therefore$ $$x = \sqrt{3} \text{ or } \frac{1}{\sqrt{3}}$$ **1**

$\Rightarrow$ $$\cot\theta = \sqrt{3} \text{ or } \cot\theta = \frac{1}{\sqrt{3}}$$

$\therefore$ $$\theta = 30^\circ \text{ or } \theta = 60^\circ$$

If $\theta = 30^\circ$, then $$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + \left(\frac{1}{\sqrt{3}}\right)^2 = 3 + \frac{1}{3} = \frac{10}{3}$$ **1**

If $\theta = 60^\circ$, then $$\cot^2 60^\circ + \tan^2 60^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 = \frac{1}{3} + 3 = \frac{10}{3}$$ **1**

30. **(i)** Here class intervals are not in inclusive form. So, we first convert them in inclusive form by subtracting 1/2 from the lower limit and adding 1/2 to the upper limit of each cases. where h is the difference between the lower limit of a class and the upper limit of the preceding class. The given frequency distribution in inclusive form is as follows :

Age (in years)	**No. of cases**
4·5 -14·5	6
14·5 -24·5	11
24·5 -34·5	21
34·5 -44·5	23
44·5 -54·5	14
54·5 -64·5	5

It is clear from the table that the modal class is 34·5 - 44·5. **1**

Here, $l = 34{\cdot}5$, $h = 10$, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$ **1**

Now, $$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$\Rightarrow$ $$\text{Mode} = 34{\cdot}5 + \frac{23-21}{46-21-14} \times 10$$

$$= 34{\cdot}5 + \frac{2}{11} \times 10$$

$$= 34{\cdot}5 + 1{\cdot}81 = 36{\cdot}31$$ **1**

(ii) Mode of grouped data. **1**

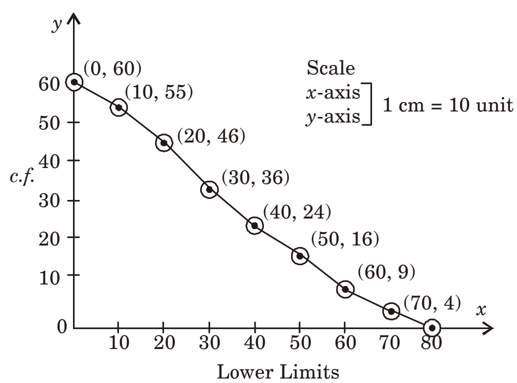
(iii) Habits of neatness and cleanliness help us lead a healty life. **1**

31.

Classes	More than or equal to	*c.f.*
0 – 10	0	60
10 – 20	10	55
20 – 30	20	46
30 – 40	30	36
40 – 50	40	24
50 – 60	50	16
60 – 70	60	9
70 – 80	70	4

2

More than ogive is as under :



2

❑ ❑