Mathematics Sample Paper

Solved SA-1 Sample Question paper -02

JSUNIL TUTORIAL
Chase Excellence

SECTION – A

1. Find the decimal form of $\frac{3}{8}$.

2. Two lines are given to be parallel. The equation of one of the line is $4x + 3y = 14$. What is the equation of the second line.

3. If $\Delta ABC \sim \Delta DEF$, BC = 4 cm, EF = 5 cm and area of $\Delta ABC = 80 \text{ cm}^2$, then find area of ΔDEF.

4. Consider the following frequency distribution :

Monthly Income (in ₹)	Number of families
More than or equal to 10000	100
More than or equal to 13000	85
More than or equal to 16000	69
More than or equal to 19000	50
More than or equal to 22000	33
More than or equal to 25000	15

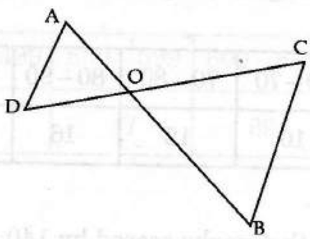
Find the number of families having income range from ₹ 16000 to ₹ 19000.

SECTION – B

5. Check whether the polynomial $g(x) = x^2 + 3x + 1$ is a factor of the polynomial $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$.

6. For what value of k, the pair of equations $kx + 3y = k - 3$, $12x + ky = k$ has unique solution.

7. In the given figure, OA × OB = OC × OD, show that ∠A = ∠C and ∠B = ∠D.



8. If $\cos(A - B) = \frac{\sqrt{3}}{2}$ and $\sin(A + B) = \frac{\sqrt{3}}{2}$, find A and B, where (A + B) and (A – B) are acute angles.

9. Prove that : $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \text{cosec } A - \cot A$

10. Find the mean of first five odd multiples of 5.

SECTION – C

11. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and is to contain cartons of the same drink, what would be the greatest number of cartons each stack would have ?

12. Prove that $\sqrt{3}$ is irrational.

13. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and co-efficients of polynomial.

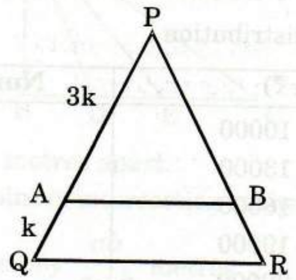
14. The taxi charges in a city comprise of a fixed charge together with the charges for the distance covered. For a journey of 10 km the charge paid is ₹ 75 and for a journey of 15 km the charge paid is ₹ 110.

(i) What will a person have to pay for travelling a distance of 25 km ?

(ii) Which mathematical concept is used in this question ?

(iii) What is its value ?

15. In the given figure $\frac{PA}{AQ} = \frac{PB}{BR} = 3$. If the area of ΔPQR is 32 cm^2, then find the area of the quadrilateral AQRB.



16. D, E and F are the mid points of the sides BC, CA and AB respectively of ΔABC. Determine the ratio of the areas of ΔDEF and ΔABC.

17. Prove that : $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$.

18. Evaluate : $\frac{\sec 41^\circ.\sin 49^\circ + \cos 29^\circ.\text{cosec } 61^\circ - \frac{2}{\sqrt{3}}(\tan 20^\circ.\tan 60^\circ.\tan 70^\circ)}{3(\sin^2 31^\circ + \sin^2 59^\circ)}$.

19. The following tables shows the weights (in gms) of a sample of 100 apples, taken from a large consignment :

Weight (in gms)	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110	110 – 120	120 – 130
No. of Apples	8	10	12	16	18	14	12	10

Find the median weight of apples.

20. The following distribution shows the marks scored by 140 students in an examination. Calculate the mode of the distribution :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	20	24	40	36	20

SECTION – D

21. Prove that $n^2 - n$ is divisible by 2 for every positive integer n.

22. If α and β are the zeroes of polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.

23. Solve the following pair of linear equations grahically :

$$2x + 3y = 12,\ 2y - 1 = x.$$

Determine the co-ordinates of the vertices of the triangle formed by the lines represented by these equations with the x-axis.

24. If a bag containing red and white balls, half the number of white balls is equal to one-third the number of red balls. Thrice the total number of balls exceeds seven times the number of white balls by 6. How many balls of each colour does the bag contain ?

25. If two sides and a median bisecting one of these sides of a triangle are rspectively proportional to the two sides and the corresponding median of another triangle, then prove that the two triangles are similar.

26. A vertical tree 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40 m long on the ground.

(i) Determine the height of the tower.

(ii) Which mathematical concept is used in this problem ?

(iii) What is the value stressed upon in this problem ?

27. Prove that : $\dfrac{\cos^2\theta}{1-\tan\theta}+\dfrac{\sin^3\theta}{\sin\theta-\cos\theta}=1+\sin\theta\cos\theta.$

28. Prove that : $\dfrac{\sec\theta+\tan\theta-1}{\tan\theta-\sec\theta+1}=\dfrac{\cos\theta}{1-\sin\theta}.$

29. Prove that : $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A.$

30. Find the values of x and y, if the median for the following data is 31.

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	**Total**
Frequency	5	x	6	y	6	5	**40**

31. The following table gives the life time of 200 bulbs. Calculate the mean life time of a bulb by step deviation method :

Life time (in hours)	400 – 499	500 – 599	600 – 699	700 – 799	800 – 899	900 – 999
Number of bulbs	24	47	39	42	34	14

10th Mathematics Solution Sample paper -02

SECTION – A

1. $\frac{3}{8} = \frac{3}{2^3} = \frac{3\times 5^3}{2^3\times 5^3} = \frac{375}{10^3} = 0{\cdot}375$ **1**

2. The equation of one line $4x + 3y = 14.$

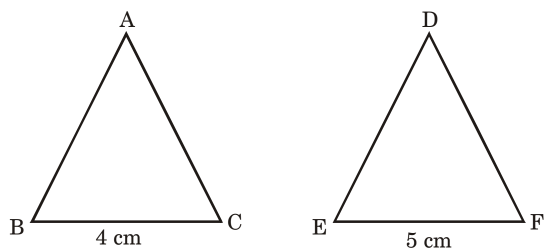
We know that if two lines $a_1x + b_1y + c = 0$ and $a_2x + b_2y + c_2 = 0$ are parallel, then

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$; so there can be infinite such lines. **1**

One of the examples of such a parallel line is given below.

Second parallel line is $-12x = 9y$. where C = 0.

3. For similar triangles, we know that



$$\frac{\text{area of }\Delta ABC}{\text{area of }\Delta DEF} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \quad \frac{80}{\text{area of }\Delta DEF} = \frac{(4)^2}{(5)^2} = \frac{16}{25}$$

$$\Rightarrow \quad \text{area of }\Delta DEF = \frac{80\times 25}{16} = 125 \text{ cm}^2.$$ **1**

4. The number of families having income range from ₹ 16000 to ₹ 19000 = 19. **1**

(The class 16000 – 19000 has frequency 19)

SECTION – B

5. $g(x) = x^2 + 3x + 1, f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$ **½**

$$\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 4 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & (-)\ (-)\ (-) \\
\hline
 & -4x^3 - 10x^2 + 2x \\
 & -4x^3 - 12x^2 - 4x \\
 & (+)\ (+)\ (+) \\
\hline
 & 2x^2 + 6x + 4 \\
 & 2x^2 + 6x + 2 \\
 & (-)\ (-)\ (-) \\
\hline
 & 2 \\
\hline
\end{array}$$

$\therefore$ Remainder, $r(x) = 2$ **1½**

$\because\ r(x) \neq 0, g(x)$ is not a factor of $p(x)$. **½**

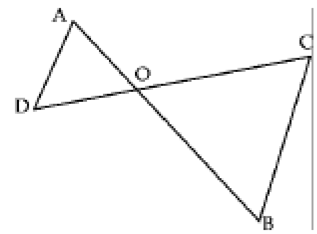
6. Condition for unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow \quad \frac{k}{12} \neq \frac{3}{k}$ **1**

$\Rightarrow \quad k^2 \neq 36$

$\Rightarrow \quad k \neq \pm 6.$ **1**



7. Since, $OA \times OB = OC \times OD$

In ΔAOD and ΔCOB

$\therefore \quad \frac{OA}{OC} = \frac{OD}{OB}$ **1**

$\angle AOD = \angle COB$ (Vertically opposite angles)

$\therefore \quad \Delta AOD \sim \Delta COB$ (SAS similarity)

$\therefore \quad \angle A = \angle C$ and $\angle B = \angle D$. **1**

8. $\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ \Rightarrow A - B = 30^\circ$ ...(i) **½**

$\sin(A + B) = \frac{\sqrt{3}}{2} = \sin 60^\circ \Rightarrow A + B = 60^\circ$ ...(ii) **½**

Adding equations (i) and (ii), $2A = 90^\circ \Rightarrow A = 45^\circ$ **½**

From (ii), $B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$ **½**

9. $\text{LHS} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}}$ **1**

$= \sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}} = \sqrt{\frac{(1-\cos A)^2}{\sin^2 A}}$

$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \text{cosec } A - \cot A = \text{RHS}.$ **Proved. 1**

10. The multiples of 5, according to the problem are :

5, 15, 25, 35, 45 **½**

$\text{Mean} = \frac{5+15+25+35+45}{5}$ **1**

$= \frac{125}{5} = 25.$ **½**

SECTION – C

11. The greatest number of cartons in each stack is the HCF of 144 and 90 **1**

$144 = 2^4 \times 3^2$

$90 = 2 \times 3^2 \times 5$ **1**

$\text{HCF} = 2 \times 3^2 = 18$

$\therefore$ The greatest number of cartons = 18. **1**

12. Let $\sqrt{3}$ be a rational number

$\sqrt{3} = \frac{a}{b}.$ (a and b are integers and co-primes and $b \neq 0$)

On squaring both the sides, $3 = \frac{a^2}{b^2}$ **1**

$\Rightarrow 3b^2 = a^2 \Rightarrow a^2$ is divisible by 3

$\Rightarrow a$ is divisible by 3 ...(i)

We can write $a = 3c$ for some integer c

$\Rightarrow a^2 = 9c^2$

$\Rightarrow 3b^2 = 9c^2$

$\Rightarrow b^2 = 3c^2$ **1**

$\Rightarrow b^2$ is divisible by 3

$\Rightarrow b$ is divisible by 3 ...(ii)

From (i) and (ii), we get 3 as a factor of 'a' and 'b' which is contradicting the fact that a and b are co-primes. Hence our assumption that $\sqrt{3}$ is an rational number is false. So $\sqrt{3}$ is irrational number.

1

13. Let $f(x) = 4x^2 + 4x - 3$; since $\frac{1}{2}$ and $\left(\frac{-3}{2}\right)$ are zeroes of $f(x)$

We must have $f\left(\frac{1}{2}\right) = 0$; $f\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$

$= 1 + 2 - 3 = 0 \Rightarrow f\left(\frac{1}{2}\right) = 0$ **1**

Also, $f\left(-\frac{3}{2}\right) = 4\left(\frac{9}{4}\right) + 4\left(-\frac{3}{2}\right) - 3$

$= 9 - 6 - 3 = 0 \Rightarrow f\left(\frac{-3}{2}\right) = 0$ **1**

$\therefore \frac{1}{2}, -\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$

Now Sum of zeroes $= \frac{1}{2} - \frac{3}{2} = -1 = \frac{-4}{4} = -\frac{\text{coeff. of } x}{\text{coeff. of } x^2}$ **½**

Product of zeroes $= \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4} = \frac{\text{constant term}}{\text{coeff. of } x^2}$

$\therefore$ Relation between zeroes and coeff. of polynomial is verfied. **½**

14. Let the fixed charge of taxi be Rs. x per km and the running charge be ₹ y per km.
According to the question,

$x + 10y = 75$ ...(i)

$x + 15y = 110$ ...(ii) **½**

Subtracting equation (ii) from equation (i), we get

$-5y = -35$

$\Rightarrow y = 7$

Putting $y = 7$ in equation (i), we get $x = 5$ **½**

$\therefore$ Total charges for travelling a distance of 25 km

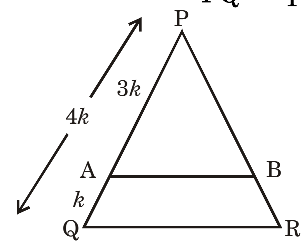
$= x + 25y$

$=$ ₹ $(5 + 25 \times 7)$

$=$ ₹ $(5 + 175)$

$=$ ₹ 180 **1**

15. We have, $\Delta PQR \sim \Delta PAB$ $\left(\because \angle P \text{ is common and } \frac{PA}{PQ} = \frac{PB}{PR}\right)$ **1**



$$\Rightarrow \quad \frac{\text{area } \Delta PQR}{\text{area } \Delta PAB} = \left(\frac{PQ}{PA}\right)^2$$

$$\frac{32}{\text{area } \Delta PAB} = \left(\frac{4k}{3k}\right)^2 \quad \textbf{1}$$

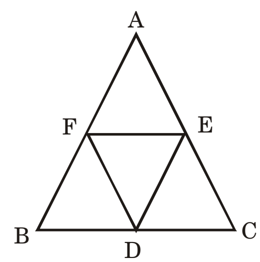
$$\Rightarrow \quad \text{area } \Delta PAB = 18 \text{ cm}^2$$

$\therefore$ area of AQRB = area of ΔPQR – area of ΔPAB = 32 – 18 = 14 cm^2 **1**

16. D, E and F are mid-points of BC, CA and AB respectively. (Given)

$\therefore$ BDEF and DCEF are parallelograms.

($\because$ line joining mid point of two sides of a Δ is parallel to the third side and is one half of it)



In triangles ABC and DEF, $\angle B = \angle E$ and $\angle C = \angle F$ (Opp. angles of a parallelogram)

$\therefore$ $\Delta ABC \sim \Delta DEF$ (AA Similarty) **1**

$$\therefore \quad \frac{\text{ar } \Delta DEF}{\text{ar } \Delta ABC} = \frac{DE^2}{AB^2} = \frac{DE^2}{(2DE)^2} \qquad \left(DE = FB, FB = \frac{1}{2} AB\right) \textbf{1}$$

$$\therefore \quad \frac{\text{ar } \Delta DEF}{\text{ar } \Delta ABC} = \frac{DE^2}{4DE^2} = \frac{1}{4}. \quad \textbf{1}$$

17.

$$\text{LHS} = \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

$$= \frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) - 2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta}{\sin^2\theta - (1 - \sin^2\theta)} \quad \textbf{1}$$

$$= \frac{1+1}{\sin^2\theta - 1 + \sin^2\theta} \quad \textbf{1}$$

$$= \frac{2}{2\sin^2\theta - 1} = \text{RHS.}$$

Proved. 1

18.

$$\frac{\sec 41^\circ . \sin 49^\circ + \cos 29^\circ . \text{cosec } 61^\circ - \frac{2}{\sqrt{3}}(\tan 20^\circ . \tan 60^\circ . \tan 70^\circ)}{3(\sin^2 31^\circ + \sin^2 59^\circ)}$$

$$= \frac{\text{cosec } (90^\circ - 41^\circ)\sin 49^\circ + \cos 29^\circ . \sec\ (90^\circ - 61^\circ) - \frac{2}{\sqrt{3}}[\tan 20^\circ . \sqrt{3} \cot(90^\circ - 70^\circ)]}{3[\sin^2 31^\circ + \cos^2(90^\circ - 59^\circ)]} \quad \textbf{1}$$

[$\because$ cosec (90 – θ) = sec θ, sin (90 – θ) = cos θ]

$$= \frac{\text{cosec } 49^\circ . \sin 49^\circ + \cos 29^\circ . \sec 29^\circ - \frac{2}{\sqrt{3}}[\tan 20^\circ \sqrt{3} . \cot 20^\circ]}{3(\sin^2 31^\circ + \cos^2 31^\circ)} \quad \textbf{1}$$

$$= \frac{1 + 1 - 2}{3} = \frac{2-2}{3} = 0 \quad \textbf{1}$$

19.

C.I.	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100	100 – 110	110 – 120	120 – 130
f	8	10	12	16	18	14	12	10
c. f.	8	18	30	46	64	78	90	100

1

Here, N = 100 ⇒ $\frac{N}{2}$ = 50. So, median class is 90 – 100.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) h$$ **½**

$$= 90 + \left(\frac{50 - 46}{18}\right) \times 10$$ **1**

$$= 90 + \frac{40}{18} = 92{\cdot}2$$

∴ Median weight = 92·2 gm. **½**

20. Modal class : 20 – 30

Here $l = 20, f_1 = 40, f_0 = 24, f_2 = 36, h = 10$

$$\text{Mode} = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$ **1**

$$= 20 + \frac{(40 - 24)}{80 - 24 - 36} \times 10$$ **1**

$$= 20 + \frac{16 \times 10}{20} = 28$$ **1**

SECTION – D

21. Any positive integer is of the form $2q$ or $2q + 1$, for some integer q.

∴ When $n = 2q$

$$n^2 - n = 2q(2q - 1)$$ **1**

$$= 2m, \text{ when } m = q(2q - 1)$$

which is divisible by 2.

When $n = 2q + 1$ **1**

$$n^2 - n = (2q + 1)(2q + 1 - 1)$$

$$= 2q(2q + 1)$$

$$= 2m, \text{ when } m = q(2q + 1)$$ **1**

which is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n. **1**

22. Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\alpha + \beta = -\frac{2}{3}$

and $\alpha\beta = \frac{1}{3}$ **1**

Now for the new polynomial,

$$\text{Sum of the zeroes} = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta} = \frac{(1 - \alpha + \beta - \alpha\beta) + (1 + \alpha - \beta - \alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$\text{Sum of zeroes} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$ **1**

$$\text{Product of zeroes} = \left(\frac{1-\alpha}{1+\alpha}\right)\left(\frac{1-\beta}{1+\beta}\right) = \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$\text{Product of zeroes} = \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{2}{3}} = 3$$ **1**

Hence, Required polynomial $= x^2 - (\text{Sum of zeroes})x + \text{Product of zeroes}$

$= x^2 - 2x + 3.$ **1**

23. $2x + 3y = 12 \Rightarrow y = \frac{12-2x}{3}$

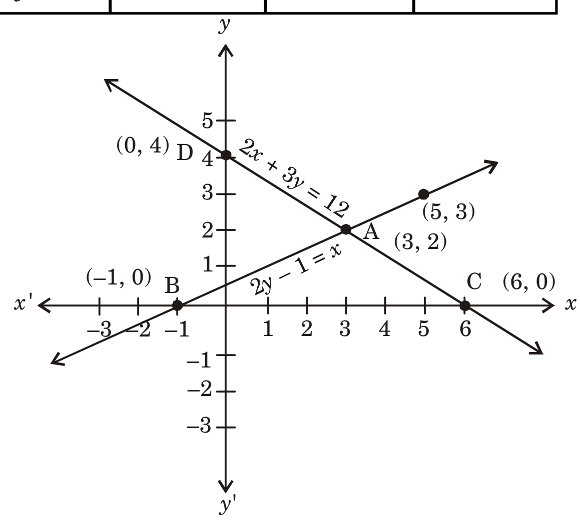
x	0	6	3
y	4	0	2

½

$2y - 1 = x \Rightarrow y = \frac{x+1}{2}$

x	– 1	3	5
y	0	2	3

½



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Plotting the above points we get the graph of the equations $2x + 3y = 12$ and $2y - 1 = x$. Clearly, the two lines intersect at the point A (3, 2). Again the required coordinates of vertices of the triangle ABC are A(3, 2), B (–1, 0) and C(6, 0). **1**

24. Let the number of red balls be x and white balls be y.

According to the question, $\frac{1}{2}y = \frac{1}{3}x$ or $2x - 3y = 0$ ...(i) **1**

and $3(x + y) - 7y = 6$

or, $3x - 4y = 6$ ...(ii) **1**

Multiplying eqn. (i) by 3 and eqn. (ii) by 2 and then subtracting, we get

$$6x - 9y = 0$$

$$6x - 8y = 12$$

$\Rightarrow$ $-y = -12$

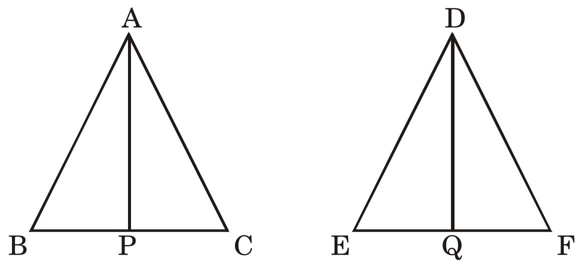
Subtracting from (i), $y = 12$ **1**

$\therefore$ $2x - 36 = 0 \Rightarrow x = 18$

$x = 18, \quad y = 12$ **1**

Hence, number of red balls = 18 and number of white balls = 12.

25. Given : In ΔABC and ΔDEF, AP and DQ are medians, such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AP}{DQ}$ ...(i)



To prove : $\Delta ABC \sim \Delta DEF$ **1**

Proof : From (1), $\frac{AB}{DE} = \frac{\frac{1}{2}BC}{\frac{1}{2}EF} = \frac{AP}{DQ}$

$\Rightarrow$ $\frac{AB}{DE} = \frac{BP}{EQ} = \frac{AP}{DQ}$ **1**

$\Rightarrow$ $\Delta ABP \sim \Delta DEQ$ [SSS similarity] **1**

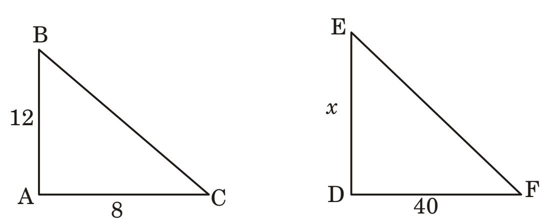
$\Rightarrow$ $\angle B = \angle E$

In ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{EF}$

and $\angle B = \angle E,$ (By SAS criterion)

$\Delta ABC \sim \Delta DEF.$ **Proved. 1**

26. **(i)** Let AB be the vertical tree and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Join BC and EF. Let DE = x.



We have AB = 12 m ½

AC = 8 m ½

and DF = 40 m ½

In ΔABC and ΔDEF, we have $\angle A = \angle D = 90^\circ$

and $\angle C = \angle F$ ½

Therefore by AA criterion of similarity, we have

$$\Delta ABC \sim \Delta DEF$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{AC}{DF}$$

$$\Rightarrow \quad \frac{12}{x} = \frac{8}{40}$$

$$\Rightarrow \quad x = \frac{12 \times 40}{8}$$

$$\Rightarrow \quad x = 60 \text{ m.}$$ 1

(ii) Similarity of Δs and heights and distances. 1

(iii) Growing more and more trees will help to save and protect our enviornment. Trees give us so many things including shade. 1

27. LHS $= \dfrac{\cos^2\theta}{1-\tan\theta} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta}$

$= \dfrac{\cos^2\theta}{1-\dfrac{\sin\theta}{\cos\theta}} + \dfrac{\sin^3\theta}{\sin\theta - \cos\theta}$ 1

$= \dfrac{\cos^3\theta}{\cos\theta - \sin\theta} - \dfrac{\sin^3\theta}{\cos\theta - \sin\theta}$ 1

$= \dfrac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$ 1

$= \dfrac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$ $(\because \sin^2\theta + \cos^2\theta = 1)$

$= 1 + \sin\theta\cos\theta$ = RHS. **Proved. 1**

28. LHS $= \dfrac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\sec\theta + \tan\theta) - (\sec^2\theta - \tan^2\theta)}{\tan\theta - \sec\theta + 1}$

$= \dfrac{(\sec\theta + \tan\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{(\tan\theta - \sec\theta + 1)}$ 1

$= \dfrac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{(\tan\theta - \sec\theta + 1)}$

$= \sec\theta + \tan\theta = \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$ 1

$= \dfrac{1 + \sin\theta}{\cos\theta} = \dfrac{1 + \sin\theta}{\cos\theta} \times \dfrac{1 - \sin\theta}{1 - \sin\theta}$ 1

$$= \frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)} = \frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$= \frac{\cos\theta}{1-\sin\theta} = \text{RHS.}$$ **Proved. 1**

29. We have $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \tan^2 A$ ...(i) **1½**

Again $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$ **1**

$$= \left(\frac{1-\tan A}{\tan A - 1}\times\tan A\right)^2 = (-\tan A)^2$$ **1**

$$= \tan^2 A$$ ...(ii)

From (i) and (ii), we have

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$$ **Proved. ½**

30.

C.I.	f	$c.f.$
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	6	$11 + x$
30 – 40	y	$11 + x + y$
40 – 50	6	$17 + x + y$
50 – 60	5	$22 + x + y$

Here from table, $N = 22 + x + y = 40$ **1**

$\Rightarrow$ $x + y = 18$ ...(i)

Since, median = 31, $\therefore$ Median class is 30 – 40.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right)h$$

$$31 = 30 + \left(\frac{20-(11+x)}{y}\right)\times 10$$ **1**

$$\Rightarrow \quad 1 = \frac{(9-x)\times 10}{y}$$

$$y = 90 - 10x$$

From (i), $10x + y = 90$ ...(ii)**1**

$x + y = 18$

$-\quad -\quad -$

On subtraction, $9x = 72$

$\Rightarrow \qquad x = \frac{72}{9} = 8$

From (i), $\qquad y = 18 - 8 = 10.$ **1**

31. Let assumed mean, $a = 649.5$ and $h = 100$

Life time (in hrs)	x_i	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
400 – 499	449·5	– 2	24	– 48
500 – 599	549·5	– 1	47	– 47
600 – 699	649·5	0	39	0
700 – 799	749·5	1	42	42
800 – 899	849·5	2	34	68
900 – 999	949·5	3	14	42
Total			$\Sigma f_i = 200$	$\Sigma f_i u_i = 57$

2

$\therefore \qquad$ Mean, $\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \times h\right)$ **1**

$= 649{\cdot}5 + \frac{57}{200} \times 100$

$= 649{\cdot}5 + 28{\cdot}5$

$= 678.$ **1**

Average life time is 678 hours.

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