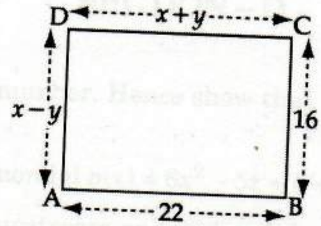
SECTION – A

1. Find the HCF of the smallest composite number and the smallest prime number.
2. If $x = a, y = b$ is the solution of the pair of equations $x - y = 2$ and $x + y = 4$, then find the respective values of a and b.
3. If the ratio of the perimeter of two similar triangles is 4 : 25, then find the ratio of the areas of the similar triangles.
4. Find the modal class in the following frequency distribution :

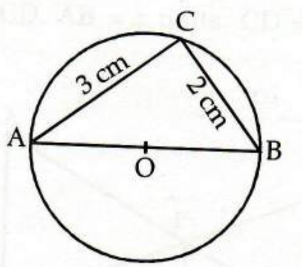
Class	Frequency
0 – 10	3
10 – 20	9
20 – 30	15
30 – 40	30
40 – 50	18
50 – 60	5

SECTION – B

5. Form a quadratic polynomial $p(x)$ with 3 and $-\frac{2}{5}$ as sum and product of its zeroes respectively.
6. In the figure given below, ABCD is a rectangle. Find the values of x and y.



7. In an equilateral triangle ABC, AD is drawn perpendicular to BC meeting BC in D. Prove that $AD^2 = 3BD^2$.
8. In the given figure, AOB is a diameter of circle with centre O. Find tan A. tan B.



9. Evaluate : $\frac{\cos 45°}{\sec 30°} + \frac{1}{\sec 60°}$.

10. Convert the following cumulative distribution to a frequency distribution :

Height (in cm)	less than 140	less than 145	less than 150	less than 155	less than 160	less than 165
Number of students	4	11	29	40	46	51

SECTION – C

11. Prove that $\sqrt{2}$ is irrational.

12. Find the HCF by Euclid's division algorithm of the numbers 1305, 1365, 1530.

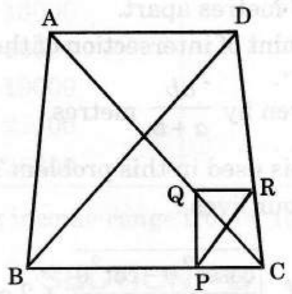
13. If one zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

14. Solve the following pair of equations for x and y :

$$4x + \frac{6}{y} = 15, 6x - \frac{8}{y} = 14$$

and also find the value of p such that $y = px - 2$.

15. In the given figure two triangles ABC and DBC lie on same side of BC such that PQ ∥ BA and PR ∥ BD. Prove that QR ∥ AD.



16. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that DB = 3CD. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.

17. If $\sec\theta = x + \frac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$.

18. Evaluate : $\frac{\cos^2(45° + \theta) + \cos^2(45° - \theta)}{\tan(60° + \theta)\tan(30° - \theta)} + \operatorname{cosec}(75° + \theta) - \sec(15° - \theta)$.

19. The mean of the following distribution is 53. Find the missing frequency p :

Classes	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Frequency	12	15	32	p	13

20. The frequency distribution of agricultural holdings in a village is given below :

Area of land (in hectares)	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11	11 – 13
Number of families	20	45	80	55	40	12

Find the modal agricultural holdings of the village.

SECTION – D

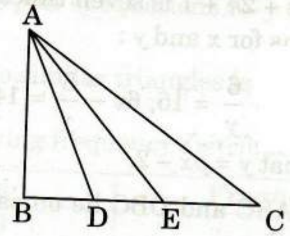
21. Use Euclid's Division Lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

22. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

23. Amit bought two pencils and three chocolates for ₹ 11 and Sumeet bought one pencil and two chocolates for ₹ 7. Represent this situation in the form of a pair of linear equations. Find the price of one pencil and that of one chocolate graphically.

24. Out of a distance of 360 km if 240 km is covered by bus and rest by train, it takes 8 hours to complete the journey. However if 120 km is travelled by the bus and rest by train, it takes one hour less. What is the speed of the bus and the train.

25. In the given figure, D and E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$.



26. Two trees of heights a and b are p metres apart.

(i) Prove that the height of the point of intersection of the lines joining the top of each tree to the foot of the opposite tree is given by $\frac{ab}{a+b}$ metres.

(ii) Which mathematical concept is used in this problem ?

(iii) What is the value of trees in our lives ?

27. If $4 \sin\theta = 3$, find the value of x, if $\sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$.

28. In an acute angled triangle ABC, if $\sin(A + B - C) = \frac{1}{2}$ and $\cos(B + C - A) = \frac{1}{\sqrt{2}}$, find $\angle A$, $\angle B$ and $\angle C$.

29. Evaluate : $\frac{\text{cosec}^2(90° - \theta) - \tan^2\theta}{4(\cos^2 40° + \cos^2 50°)} - \frac{2\tan^2 30°.\sec^2 52°.\sin^2 38°}{3(\text{cosec}^2 70° - \tan^2 20°)}$.

30. Calculate the average daily income (in ₹) of the following data about men working in a company :

Daily Income (in ₹)	< 100	< 200	< 300	< 400	< 500
Number of men	12	28	34	41	50

31. If the mean of the following frequency distribution is 91 and the total frequencies is 150. find the missing frequencies x and y :

Classes	0 – 30	30 – 60	60 – 90	90 – 120	120 – 150	150 – 180
Frequency	12	21	x	52	y	11

10th Mathematics Solution Sample paper -01

SECTION – A

1. The smallest prime number = 2 and smallest composite number is 2^2.
Required HCF (4, 2) = 2. **1**

2.
$$x - y = 2 \quad \text{...(i)}$$
and
$$x + y = 4 \quad \text{...(ii)}$$
Adding both the equations, $2x = 6 \Rightarrow x = 3$
Put the value of x in the eqn. (ii), we get
$$3 + y = 4 \Rightarrow y = 1$$
Hence, $a = 3, b = 1$. **1**

3. For similar triangles,
$$\frac{\text{area of triangle 1}}{\text{area of triangle 2}} = \left(\frac{\text{perimeter of triangle 1}}{\text{perimeter of triangle 2}}\right)^2$$
$$= \left(\frac{4}{25}\right)^2 = \frac{16}{625}.$$ **1**

4. From the table it is clear that the frequency is maximum for the class 30 – 40, so modal class is 30 –40. **1**

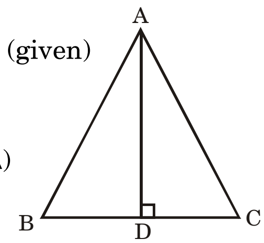
SECTION – B

5. According to question, Sum of zeroes = 3
$$\text{Product of zeroes} = -\frac{2}{5}$$
The required quadratic polynomial $= x^2 - x$ (sum of zeroes) + product of zeroes
$$= x^2 - x(3) - \frac{2}{5}$$ **1**
$$= x^2 - 3x - \frac{2}{5}$$
$$= \frac{1}{5}(5x^2 - 15x - 2)$$
$\therefore$ The quadratic polynomial is $\frac{1}{5}(5x^2 - 15x - 2)$. **1**

6. From fig.,
$$x + y = 22 \quad \text{...(i)}$$
$$x - y = 16 \quad \text{...(ii)}$$ ½
Adding (i) and (ii), we get $2x = 38$
or $x = 19$
Put the value of x in equation (i), we get ½
$$19 + y = 22$$
or $y = 22 - 19 = 3$ ½
Hence, $x = 19$ and $y = 3$. ½

7. In ΔABD, from Pythagoras theorem,

$\Rightarrow$ $AB^2 = AD^2 + BD^2$ **1**



$\Rightarrow$ $BC^2 = AD^2 + BD^2$, as AB = BC = CA (given)

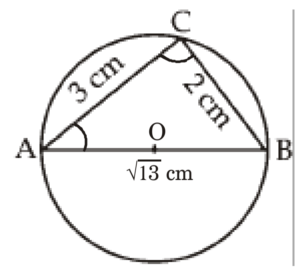
$\Rightarrow$ $(2BD)^2 = AD^2 + BD^2$,

($\perp$ is the median in an equi. Δ)

$\Rightarrow$ $3BD^2 = AD^2$. **1**

8. $\angle C = 90^\circ$ (Angle in a semi-circle)

$\therefore$ $AB = \sqrt{(3)^2 + (2)^2} = \sqrt{9+4} = \sqrt{13}$ (By pythagoras theorem) **½**



$\tan A = \frac{BC}{AC} = \frac{2}{3}$ **½**

$\tan B = \frac{AC}{BC} = \frac{3}{2}$ **½**

$\therefore$ $\tan A \tan B = \frac{2}{3} \cdot \frac{3}{2} = 1.$ **½**

9. $\frac{\cos 45^\circ}{\sec 30^\circ} + \frac{1}{\sec 60^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \frac{1}{2}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{1}{2}$ **1**

$= \frac{\sqrt{6}+2}{4}.$ **1**

10.

Class	Frequency	Cumulative Frequency
135 – 140	4	4
140 – 145	7	11
145 – 150	18	29
150 – 155	11	40
155 – 160	6	46
160 – 165	5	51

2

SECTION – C

11. Let $\sqrt{2}$ is rational,

$\therefore$ $\sqrt{2} = \frac{p}{q}$, where p and q are co-prime integers and $q \neq 0$

$\Rightarrow$ $2 = \frac{p^2}{q^2}$ **½**

$p^2 = 2q^2$

$\Rightarrow p^2$ is divisible by 2

$\therefore$ p is divisible by 2 ...(i) ½

Let $p = 2r$ for some positive integer r

$\Rightarrow$ $p^2 = 4r^2$

$\therefore$ $2q^2 = 4r^2$

$\Rightarrow$ $q^2 = 2r^2$

$\therefore$ q^2 is divisible by 2 **1**

$\Rightarrow$ q is divisible by 2 ...(ii)

From (i) and (ii), p and q are divisible by 2.

Which contradicts the fact that p and q are co-primes.

Hence, our assumption is false.

$\therefore$ $\sqrt{2}$ is irrational. **1**

12. By Euclid's algorithm $1530 = 1365 \times 1 + 165$

$1365 = 165 \times 8 + 45$ **1**

$165 = 45 \times 3 + 30$

$45 = 30 \times 1 + 15$

$30 = 15 \times 2 + 0$ $\therefore$ HCF of 1530 and 1365 is 15. **1**

Now, $1305 = 15 \times 87 + 0$

HCF (1530, 1365, 1305) = 15. **1**

13. Let α and β are the zeroes of the quadratic polynomial, then as per question

$$\beta = 7\alpha$$

$\therefore$ $\alpha + 7\alpha = 8\alpha = -\left(-\frac{8}{3}\right)$ ½

$\Rightarrow$ $\alpha = \frac{1}{3}$ ½

and $\alpha \times 7\alpha = \frac{2k+1}{3}$

$\Rightarrow$ $7\alpha^2 = \frac{2k+1}{3}$

$\Rightarrow$ $7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3}$ **1**

$\Rightarrow$ $7 \times \frac{1}{9} = \frac{2k+1}{3}$

$\Rightarrow$ $\frac{7}{3} - 1 = 2k$

$\Rightarrow$ $\frac{2}{3} = k.$ **1**

14. Let $\frac{1}{y} = a$, the given equations become

$4x + 6a = 15$ ...(i)

$6x - 8a = 14$ ...(ii) ½

Multiply eqn. (i) by 4 and eqn. (ii) by 3 and adding

$16x + 24a = 60$

$18x - 24a = 42$

On adding, $34x = 102$

$\Rightarrow$ $x = \frac{102}{34} = 3$ ½

Put the value of x in eqn. (1), we get

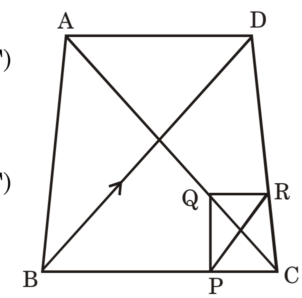
$$4(3) + 6a = 15$$
$$6a = 15 - 12 = 3$$
$$a = \frac{3}{6} = \frac{1}{2}$$

$\because$ $a = \frac{1}{y} = \frac{1}{2} \Rightarrow y = 2$ **1**

Hence $x = 3$ and $y = 2$

Again $y = px - 2 \Rightarrow 2 = p\,(3) - 2 \Rightarrow 3p = 4 \Rightarrow p = \frac{4}{3}$. **1**

15. In ΔABC, $PQ \parallel AB$



$\therefore$ $\frac{BP}{PC} = \frac{AQ}{QC}$ (By BPT) ...(i) **1**

Again in ΔBCD, $PR \parallel BD$

$\Rightarrow$ $\frac{BP}{PC} = \frac{DR}{RC}$ (By BPT) ...(ii) **1**

From (i) and (ii), $\frac{AQ}{QC} = \frac{DR}{RC}$

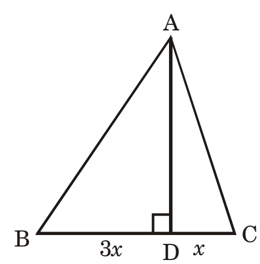
$\Rightarrow$ $QR \parallel AD$ (By converse of BPT) **1**

16. In ΔADB, $AB^2 = AD^2 + BD^2$ (Pythagoras Theorem) ...(i)

(given $AD \perp BC$)

In ΔADC, $AC^2 = AD^2 + CD^2$ (Pythagoras Theorem) ...(ii)

Subtracting eqn. (ii) from eqn. (i), we get



$$AB^2 - AC^2 = BD^2 - CD^2$$ **1**

$$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2 = \frac{BC^2}{2}$$ **1**

$\therefore$ $2(AB^2 - AC^2) = BC^2$

$\therefore$ $2(AB)^2 = 2AC^2 + BC^2$. **1**

17. Let $\sec\theta + \tan\theta = \lambda$ ...(i)

We know that $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow$ $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$ **1**

$\Rightarrow$ $\lambda(\sec\theta - \tan\theta) = 1$

$\Rightarrow$ $\sec\theta - \tan\theta = \frac{1}{\lambda}$ ...(ii)

Adding eqns. (i) and (ii), we get

$$2\sec\theta = \lambda + \frac{1}{\lambda}$$

$\Rightarrow$ $2\left(x + \frac{1}{4x}\right) = \lambda + \frac{1}{\lambda}$

$\Rightarrow$ $2x + \frac{1}{2x} = \lambda + \frac{1}{\lambda}$ **1**

Comparing both sides, we get

$$\lambda = 2x \text{ or } \lambda = \frac{1}{2x}$$

$\Rightarrow$ $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$. **1**

18. $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$ + cosec (75° + θ) – sec (15° – θ)

$= \dfrac{\cos^2(45^\circ+\theta)+\sin^2(90^\circ-45^\circ+\theta)}{\tan(60^\circ+\theta)\cot(90^\circ-30^\circ+\theta)}$ + cosec (75° + θ) – cosec (90° – 15° + θ) **1½**

$= \dfrac{\cos^2(45^\circ+\theta)+\sin^2(45^\circ+\theta)}{\tan(60^\circ+\theta).\cot(60^\circ+\theta)}$ + cosec (75° + θ) – cosec (75° + θ) **½**

$= \dfrac{1}{1} = 1.$ **1**

19.

Class	x_i (Class marks)	f_i	f_ix_i
0 – 20	10	12	120
20 – 40	30	15	450
40 – 60	50	32	1600
60 – 80	70	p	$70p$
80 – 100	90	13	1170
	Total	$\Sigma f_i = 72 + p$	$\Sigma f_ix_i = 3340 + 70p$

1

We know that Mean, $\bar{x} = \dfrac{\Sigma f_ix_i}{\Sigma f_i}$ **½**

$\Rightarrow \quad 53 = \dfrac{3340+70p}{72+p}$ **½**

$\Rightarrow \quad 3340 + 70p = 53(72+p)$

$\Rightarrow \quad 3340 + 70p = 3816 + 53p$

$\Rightarrow \quad 70p - 53p = 3816 - 3340$

$\Rightarrow \quad 17p = 476$

$p = \dfrac{476}{17} = 28.$ **1**

20. Modal class : 5 – 7

Here $l = 5, f_1 = 80, f_0 = 45, h = 2, f_2 = 55$ **1**

$\text{Mode} = l + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$ **1**

$= 5 + \dfrac{80-45}{160-45-55} \times 2 = 5 + \dfrac{35\times 2}{60}$ **1**

$= 6{\cdot}17.$ **1**

SECTION – D

21. Let x be any positive integer, then it is of the form $3q$ or $3q + 1$ or $3q + 2$. where q is a natural number.

Squaring, we get

$(3q)^2 = 9q^2 = 3 \times 3q^2 = 3m, m = 2q^2$

$$(3q + 1)^2 = 9q^2 + 6q + 1 \quad \mathbf{1}$$
$$= 3(3q^2 + 2q) + 1$$
$$= 3m + 1, m = 3q^2 + 2q$$
$$(3q + 2)^2 = 9q^2 + 12q + 4 \quad \mathbf{1}$$
$$= 9q^2 + 12q + 3 + 1$$
$$= 3(3q^2 + 4q + 1) + 1$$
$$= 3m + 1, m = 3q^2 + 4q + 1 \quad \mathbf{1}$$

$\Rightarrow$ Square of any positive integer is of the form $3m$ or $3m + 1$ for some integer m. **1**

22. $$p(x) = 2x^2 + 5x + k$$

$$\text{Sum of zeroes} = -\frac{\text{coeff. of } x}{\text{coeff. of } x^2} \Rightarrow \alpha + \beta = \frac{-5}{2} \quad \mathbf{1}$$

$$\text{Product of zeroes} = \frac{\text{constant}}{\text{coeff. of } x^2} \Rightarrow \alpha\beta = \frac{k}{2}$$

According to question, $$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4} \quad \mathbf{1}$$

$$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta + \alpha\beta = \frac{21}{4} \qquad [\because (\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\,\alpha\beta]$$

$$\Rightarrow \left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \frac{25}{4} - \frac{21}{4} = \frac{k}{2} \quad 1$$

$$\therefore 1 = \frac{k}{2} \Rightarrow k = 2$$

Hence, $$k = 2 \quad \mathbf{1}$$

23. Let the cost of one pencil be ₹ x and the cost of one chocolate be ₹ y.

According to question,

$$2x + 3y = 11 \quad \text{...(i)}$$
$$x + 2y = 7 \quad \text{...(ii)} \ \mathbf{1}$$

Now, $$2x + 3y = 11 \Rightarrow x = \frac{11-3y}{2}$$

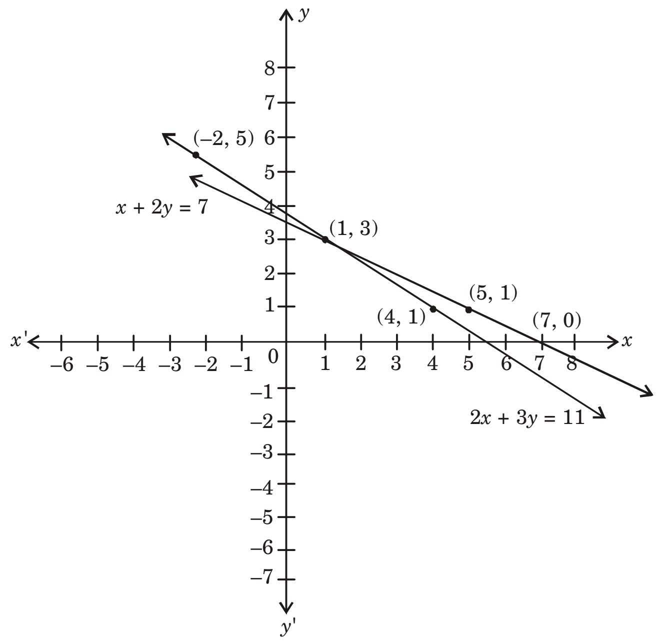
y	1	3	5
x	4	1	–2

½

and $x + 2y = 7$ Þ $x = 7 - 2y$

y	0	1	3
x	7	5	1

½



1

Plotting the above points, we get the graph of the above equations. Clearly the, two lines intersect at point (1, 3).

$\therefore$ Solution of eqns. (i) and (ii) is $x = 1$ and $y = 3$

$\therefore$ Cost of one pencil = ₹ 1 and cost of one chocolate = ₹ 3. **1**

24. Let the speed of bus be x km/hr and the speed of the train be y km/hr.

According to question, $\dfrac{240}{x} + \dfrac{120}{y} = 8$

and $\dfrac{120}{x} + \dfrac{240}{y} = 7$

Let $\dfrac{1}{x} = a$, $\dfrac{1}{y} = b$, then **1**

$$240a + 120b = 8 \quad \text{...(i)}$$

$$120a + 240b = 7 \quad \text{...(ii)}$$

Apply [(i) × 2 – (ii)], we get

$$480a + 240b = 16$$

$$120a + 240b = 7 \quad \text{...(iii)}$$

$$- \quad - \quad -$$

On subtracting, $360a = 9$

$\Rightarrow$ $a = \dfrac{9}{360} = \dfrac{1}{40}$ **1**

Putting this value of a in eqn.(i), we get

$$b = \frac{1}{60}$$ **1**

$$b = \frac{1}{60} = \frac{1}{y} \Rightarrow y = 60$$

$$a = \frac{1}{40} = \frac{1}{x} \Rightarrow x = 40$$

Hence, speed of bus = 40 km/hr and speed of train = 60 km/hr. **1**

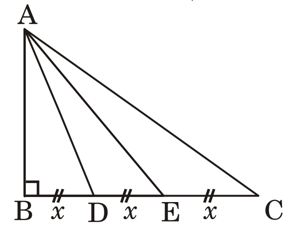
25. **Given :** In figure, D and E trisect BC.

To Prove : $8AE^2 = 3AC^2 + 5AD^2$

Proof : Let $BD = DE = EC = x$ (given D and E trisect + BC)

and $BE = 2x$

$BC = 3x$



$AE^2 = AB^2 + BE^2 = AB^2 + 4x^2$ ...(i) **1**

$AC^2 = AB^2 + BC^2 = AB^2 + 9x^2$

$AD^2 = AB^2 + BD^2 = AB^2 + x^2$ **1**

Now, $8AE^2 = 8AB^2 + 32x^2$ [Multiply eqn. (1) by 8] ...(ii)

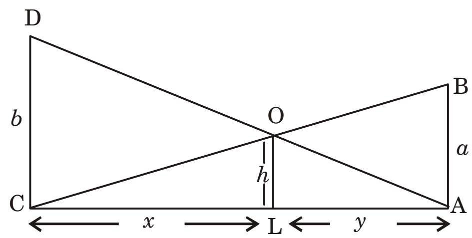
and $3AC^2 + 5AD^2 = 3(AB^2 + 9x^2) + 5(AB^2 + x^2)$ **1**

$= 3AB^2 + 27x^2 + 5AB^2 + 5x^2$

$= 8AB^2 + 32x^2$ ...(iii)

$\therefore$ $3AC^2 + 5AD^2 = 8AE^2$. [From eqn. (ii) & (iii)] **Proved. 1**

26. **(i)** Let AB and CD be the two trees of heights a and b metres such that the trees are p metres apart *i.e.* $AC = p$. Let the lines AD and BC meet at O such that $OL = h$ m.



½

Let $CL = x$ and $LA = y$, then $x + y = p$

In ΔABC and ΔLOC, we have

$\angle CAB = \angle CLO$

$\therefore$ $\angle C \approx \angle C$

$\therefore$ $\Delta CAB \sim \Delta CLO$

$\Rightarrow$ $\dfrac{CA}{CL} = \dfrac{AB}{LO}$

$\Rightarrow$ $\dfrac{p}{x} = \dfrac{a}{h}$

$\Rightarrow$ $x = \dfrac{ph}{a}$ ...(i) ½

In ΔALO and ΔACD, $\angle ALO = \angle ACD$

$\angle A = \angle A$

$\Rightarrow$ $\Delta ALO \sim \Delta ACD$

$\Rightarrow$ $\dfrac{AL}{AC} = \dfrac{OL}{DC}$

$\Rightarrow$ $\dfrac{y}{p} = \dfrac{h}{b}$

$\Rightarrow$ $y = \dfrac{ph}{b}$ ...(ii) ½

Adding eqns. (i) and (ii), we get $x + y = \dfrac{ph}{a} + \dfrac{ph}{b}$ **1**

$\Rightarrow$ $p = ph\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

$$\Rightarrow \qquad \frac{1}{h} = \frac{1}{a} + \frac{1}{b}$$

$$\therefore \qquad h = \frac{ab}{a+b} \text{ m.}$$ ½

(ii) Similarily of triangles. **1**

(iii) Trees are our life line. They should be saved at any cost. **1**

27. $\sin\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{\sqrt{7}}{4}$ and $\tan\theta = \frac{3}{\sqrt{7}}$ **½+½**

$$\therefore \qquad \sqrt{\frac{\text{cosec}^2\theta - \cot^2\theta}{\sec^2\theta - 1}} + 2\cot\theta = \frac{\sqrt{7}}{x} + \cos\theta$$

$$\Rightarrow \qquad \sqrt{\frac{1}{\tan^2\theta}} + 2\times\frac{\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$ **1**

$$\Rightarrow \qquad \frac{1}{\tan\theta} + \frac{2\sqrt{7}}{3} = \frac{\sqrt{7}}{x} + \frac{\sqrt{7}}{4}$$

$$\Rightarrow \qquad \frac{\sqrt{7}}{3} + \frac{2\sqrt{7}}{3} - \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$ **1**

$$\Rightarrow \qquad \frac{4\sqrt{7} - \sqrt{7}}{4} = \frac{\sqrt{7}}{x}$$

$$\Rightarrow \qquad \frac{3\sqrt{7}}{4} = \frac{\sqrt{7}}{x} \Rightarrow x = \frac{4}{3}.$$ **1**

28. We have $\sin(A + B - C) = \frac{1}{2} = \sin 30^\circ$

$\therefore \quad A + B - C = 30^\circ$ ...(i) **1**

and $\cos(B + C - A) = \frac{1}{\sqrt{2}} = \cos 45^\circ$

$\therefore \quad B + C - A = 45^\circ$ ...(ii) **1**

Adding eqns. (i) and (ii), we get

$$2B = 75^\circ$$

$$B = 37{\cdot}5^\circ$$

Now subtracting eqn. (ii) from eqn. (i), we get

$2(A - C) = -15^\circ \Rightarrow A - C = -7{\cdot}5^\circ$ ...(iii)

We know that, $A + B + C = 180^\circ$ **1**

$A + C = 142{\cdot}5^\circ$ ...(iv)

Adding eqns. (iii) and (iv), we get $2A = 135^\circ$

$$A = 67{\cdot}5^\circ$$

$$\Rightarrow \qquad C = 75^\circ$$

Hence, $\angle A = 67{\cdot}5^\circ, \angle B = 37{\cdot}5^\circ\ \angle C = 75^\circ.$ **1**

29. $\text{cosec}^2(90^\circ - \theta) = \sec^2\theta$

$\sec^2\theta - \tan^2\theta = 1$ **1**

$\cos^2 40^\circ + \cos^2 50^\circ = \cos^2(90^\circ - 50^\circ) + \cos^2 50^\circ$

$\sin^2 50^\circ + \cos^2 50^\circ = 1$

$$\tan^2 30^\circ = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3} \qquad \mathbf{1}$$

$$\sec^2 52^\circ \sin^2 38^\circ = \sec^2 52^\circ.\sin^2 (90^\circ - 52^\circ) = \sec^2 52^\circ.\cos^2 52^\circ = 1$$

$$\text{cosec}^2\, 70^\circ - \tan^2 20^\circ = \text{cosec}^2\, (90^\circ - 20^\circ) - \tan^2 20^\circ = \sec^2 20^\circ - \tan^2 20^\circ = 1 \qquad \mathbf{1}$$

$\therefore$ Given expression $= \dfrac{1}{4} - \dfrac{2\times\frac{1}{3}\times 1}{3(1)} = \dfrac{1}{4} - \dfrac{2}{9} = \dfrac{9-8}{36} = \dfrac{1}{36}$. **1**

30.

Classses	x_i (Class marks)	f_i	$f_i x_i$
0 – 100	50	12	600
100 – 200	150	16	2400
200 – 300	250	6	1500
300 – 400	350	7	2450
400 – 500	450	9	4050
Total		$\Sigma f_i = 50$	$\Sigma f_i x_i = 11000$

2

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{11000}{50} \qquad \mathbf{1}$$

$= 220{\cdot}00$

$\therefore$ Average daily income = ₹ 220·00. **1**

31.

Classes	x_i (Class marks)	f_i	$f_i x_i$
0 – 30	15	12	180
30 – 60	45	21	945
60 – 90	75	x	$75x$
90 – 120	105	52	5460
120 – 150	135	y	$135y$
150 – 180	165	11	1815
	Total	$\Sigma f_i = 150$	$\Sigma f_i x_i = 8400 + 75x + 135y$

1

$\therefore \quad 96 + x + y = 150$ ...(i)

$x + y = 54$

$\therefore \quad \bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$$91 = \frac{8400 + 75x + 135y}{150}$$

$13650 = 8400 + 75x + 135y$

$75x + 135y = 5250 \Rightarrow 5x + 9y = 350$ ...(ii) **1**

Solving eqns. (i) and (ii), we get $x = 34$ and $y = 20$. **1**

❑❑