

PRINCE PUBLIC SCHOOL
HALF YEARLY EXAMINATION (2019-20)
SAMPLE PAPER-1
MATHEMATICS
X

TIME ALLOWED: 3 HOURS

MAXIMUM MARKS: 80

General Instructions

1. This question paper consists of 40 questions. All questions are compulsory.
2. Questions 1- 20 in Section- A are very short type questions carrying 1 mark each.
3. Questions 21- 26 in Section-B are short answer type questions carrying 2 marks each.
4. Question 27-34 in Section C are short answer type-II questions carrying 3 marks each.
5. Question 35-40 in Section D are long answer type questions carrying 4 marks each.
6. There is no overall choice.
7. Use of calculator is not allowed.

SECTION-A

- Q1. If $\sin A + \sin^2 A = 1$, then what is the value of $\cos^2 A + \cos^4 A$?
- Q2. If α and β are the zeroes of the polynomial $p(x) = x^2 + x + 1$, then the sum of their reciprocals is _____.
- Q3. Write the set of values of a and b for which the following system of equations has infinitely many solutions. $2x + 3y = 7$ and $2ax + (a + b)y = 28$ _____.
- Q4. If a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5, then calculate the least prime factor of $a + b$.
- Q5. If one of the zeroes of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is -3, then what is the value of k ?
- Q6. Find the value of k , for which one root of the quadratic equation $kx^2 - 14x + 8 = 0$ is six times the other.
- Q7. The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.
- Q8. Given that H.C.F (325,500) =25, find L.C.M (325,500)= _____
- Q9. The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 15 cm. Find the length of longer side. _____
- Q10. If the sum of the zeroes of the polynomial $p(x) = 2x^3 - 3ax^2 + 4x - 5$ is 6, then value of a is _____.
- Q11. If the mean of 4, 5, a , 6, b , 9 and 11 is 10, then find the value of $(a + b)$. _____
- Q12. If one root of the equation $px^2 - 14x + 8 = 0$ is six times the other, then p is equal to _____
- Q13. Find the whether the following pair of equations has no solution, unique solution or infinitely many solution, $5x - 8y + 1 = 0$; $3x - \frac{24}{5}y + \frac{3}{5} = 0$. _____
- Q14. Find the probability of getting a sum 9 when 2 dice are thrown. _____
- Q15. If $\sum fi = 11$, $\sum fixi = 2p + 52$ and the mean of any distribution is 6, find the value of p .
- Q16. Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$. _____
- Q17. If the H.C.F of 85 and 153 is expressed in the form $85m - 153$, then value of m is _____.
- Q18. If mean is 30 and mode is 34, then find the median. _____
- Q19. There are 25 tickets bearing numbers from 1 to 25. One ticket is drawn at random. The probability that the number on it is a multiple of 5 or 6 is _____.
- Q20. If the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio 3 : 2, then m is equal to _____

SECTION-B

- Q21. Use Euclid's division lemma to show the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Q22. If $5\tan\theta = 4$, find the value of $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta}$.

Q23. If α, β and γ are zeroes of the polynomial $f(x) = px^3 + qx^2 + rx + s$, then find the value of $\alpha^2 + \beta^2 + \gamma^2$.

Q24. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k .

Q25. Prove that: $\frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = 2\operatorname{cosec}\theta$.

Q26. The ratio of incomes of two persons is $9 : 7$ and the ratio of their expenditure is $4 : 3$. If each of them manages to save ₹ 2000 per month, find their monthly incomes.

SECTION-C

Q27. Prove that one of every three consecutive integers is divisible by 3.

Q28. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Q29. In fig.1, $\triangle FEC \cong \triangle GBD$ and $\angle 1 = \angle 2$. Prove that $\triangle ADE \sim \triangle ABC$.

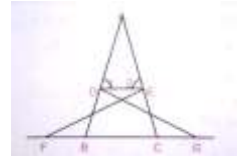


fig. 1

Q30. Solve for x , $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$, $x \neq -1, -2, 4$.

OR

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Q31. Cards marked with numbers 3, 4, 5, ..., 50 are placed in a bag and mixed thoroughly. One card is drawn at random from the bag. Find the probability that number on the card drawn is .

- a) divisible by 7 b) a perfect square c) a multiple of 6.

Q32. The arithmetic mean of the following frequency distribution is 50. Find the value of p .

Class	Frequency
0-20	17
20-40	p
40-60	32
60-80	24
80-100	19

OR

Varun decided to teach math to weak students in a slum school. He recorded the marks obtained by the students in their last term. The following frequency distribution gives the marks obtained by the students.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of students	5	3	4	3	3	4	7	9	7	8

Find the median of the data.

Q33. Prove by using suitable identities, $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \cdot \operatorname{cosec}\theta$

OR

Prove by using suitable identities, $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$.

Q34. A test consists of 'TRUE' or 'FALSE' questions. One mark is awarded for every correct answer while $\frac{1}{2}$ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by cheating. He answered 120 questions and got 90 marks. If answer to all questions he attempted by cheating were wrong, then how many questions did he answer correctly? How the habit of cheating will affect his character building?

SECTION-D

Q35. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

- a) a two-digit number
- b) a perfect square number
- c) a number divisible by 5
- d) a prime number

Q36. Solve the given quadratic equation for x : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Q37. Find the missing frequencies in the following frequency distribution table, if $N = 100$ and median is 32.

Marks obtained	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	Total
Number of students	10	?	25	30	?	10	100

Q38. State Basic Proportionality Theorem and prove it.

OR

In fig.2, the perpendiculars from A on side BC of a ΔABC intersects BC at D such that $DB = 3CD$. Prove that $2AB^2 = 2AC^2 + BC^2$.

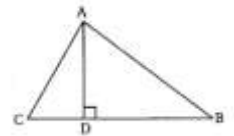


fig. 2

Q39. The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Q40. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a .