Mathematics (Standard)- Theory

Time allowed: 3 hours **Maximum marks : 80**

General instruction

(i) This question paper comprise four sections – A, B, C and D this question paper carries 40 questions . All question are compulsory.

(ii) Section A:Q. No.1 to 20 comprises of 20 questions of one marks each.

(iii) Section B :Q. No.21 to 26 comprises of 6 questions of two marks each.

(iv) Section C :Q. No.27 to 34 comprises of 8 questions of three marks each.

(v) Section D :Q. No.35 to 40 comprises of 6 questions of four marks each.

(vi) There is no overall choice in the question paper . However , an internal choice has been provided in 2 questions of one marks each, 2 questions of two marks each , 3 questions of three marks each and 3 questions of four marks each. You have to attempt only one of the choice in such questions.

(vii) In addition to this , separate instructions are given with each section and question , wherever necessary.

(viii) Use of calculators is not permitted.

SECTION-1

1. The exponent of 2 in the prime factorization of 144, is

(a) +2 (b) 4

(c) 1 (d) 6

Sol. (b)

$144 = 12 \times 12 = 4 \times 3 \times 4 \times 3 = 2^2 \times 3 \times 2^2 \times 3 = 2^4 \times 3^2$

2. The common difference of an AP, whose n^{th} term is $a_n = (3n + 7)$, is

(a) 3 (b) 7

(c) 10 (d) 6

Sol. (a)

$a_n = 3n + 7$

$a_1 = 3 \times 1 + 7 = 10$

$a_2 = 3 \times 2 + 7 = 13$

$d = a_2 - a_1 = 13 - 10 = 3$

3. The HCF of 135 and 225 is

(a) 15 (b) 75

(c) 45 (d) 5

Sol. (c)

$135 = 15 \times 7 = 5 \times 3 \times 3 \times 3$

$225 = 15 \times 15 = 5 \times 3 \times 5 \times 3$

HCF of 135 and 225 = $5 \times 3 \times 3 = 45$

4. If ΔABC ~ DEF such that AB = 1.2 mm and DE = 1.4 cm, the ratio of the areas of ΔABC and Δ DEF is

(a) 49 : 36 (b) 6 : 7

(c) 7 : 6 (d) 36 : 49

Sol. (d)

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{(1.2)^2}{(1.4)^2} = \frac{36}{49}$$

5. The value of λ for which $(x^2 + 4x + \lambda)$ is a perfect square, is

(a) 16 (b) 9

(c) 1 (d) 4

Sol. (d)

$x^2 + 4x + \lambda = 0$

for perfect root, D = 0

$b^2 - 4ac = 0$

$4^2 - 4\lambda = 0$

$\Rightarrow \lambda = 4$

6. The value of k, for which the pair of linear equations $kx + y = k^2$ and $x + ky = 1$ have infinitely many solutions is

(a) ±1 (b) 1

(c) −1 (d) 2

Sol. (b)

For infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{a_1} = \frac{1}{k} = \frac{-k^2}{-1}$$

$\Rightarrow k^2 = 1$

$k = \pm 1$

Also $k^3 = 1 \Rightarrow k = 1$

$\therefore$ value of k satisfying the whole equation is only 1

7. The value of k for which the points A(0, 1), B(2, k) and C(4, −5) are collinear is

(a) 2 (b) −2

(c) 0 (d) 4

Sol. (b)

Area of ΔABC = 0

$$\frac{1}{2}\left[0(k+5)+2(-5-1)+4(1-k)\right]=0$$

$\Rightarrow -12 + 4 - 4k = 0$

$\Rightarrow -4k = 8$

$\Rightarrow k = -2$

8. The value of p for which (2p + 1), 10 and (5p + 5) are three consecutive terms of an AP is

(a) −1 (b) −2

(c) 41 (d) 2

OR

The number of terms of an AP 5, 9, 18, … 185 is

(a) 31 (b) 51

(c) 41 (d) 40

Sol. (d)

$$a_2 - a_1 = a_3 - a_2$$

$$10-(2p+1)=(5p+5)-10$$

$$10-2p-1=5p+5-10$$

$9 - 2p = 5p - 5$

$9 + 5 = 5p + 2p$

$\Rightarrow 14 = 7p$

$\Rightarrow p = 2$

OR

$a_n = a + (n - 1)d$

$185 = 5 + (n - 1) \times 4$

$185 - 5 = 4n - 4$

$180 = 4n - 4$

184 = 4n

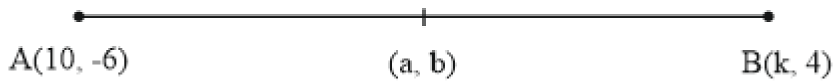
$$\Rightarrow n = \frac{184}{4} = 46$$

9. If (a, b) is the mid-point of the line segment joining the points A(10, –6) and B(k, 4) and a – 2b = 18, the value of k is

(a) 30 (b) 22

(c) 4 (d) 40

Sol. (b)



$$b = \frac{4-6}{2}$$

= -1

a – 2b = 18

a – 2(-1) = 18

a + 2 = 18

a = 16

$$a = \frac{k+10}{2}$$

$$16 = \frac{k+10}{2}$$

$$k = 22$$

10. In fig. 1, the graph of the polynomial p(x) is given. The number of zeroes of the polynomial is

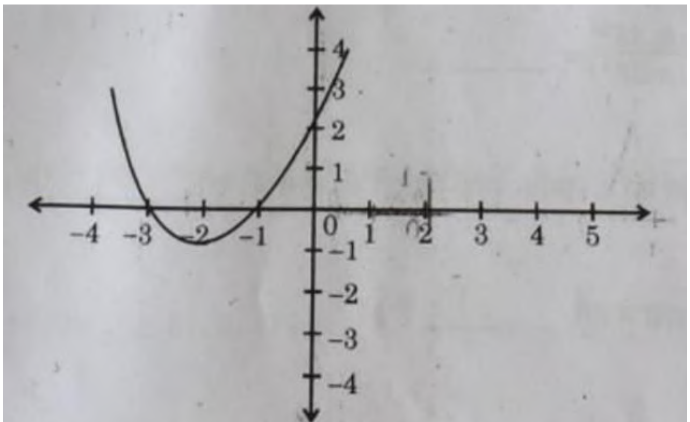


Fig. 1

(a) 1 (b) 2

(c) 3 (d) 0

Sol. Two zeroes

In Q. Nos. 11 to 15, fill in the blanks. Each question is of 1 Mark:

11. In fig. 2, PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then the measure of ∠OAB is______.

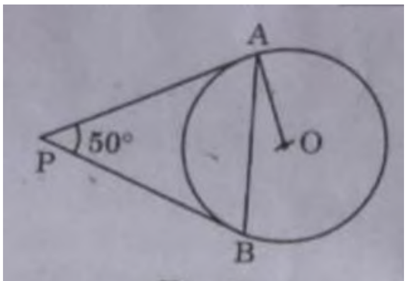


Fig. 2

OR

In Q. Fig. 3, PQ is a chord of a circle and PT is tangent at P such ∠QPT = 60°, then the measure of ∠PRQ is____.

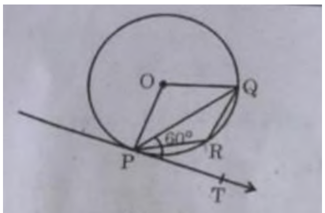


Fig. 3

Sol. ΔAPB is an isosceles triangle

⇒ ∠PAB = ∠PBA

⇒ ∠APB + ∠PAB + ∠PBA = 180°

⇒ 50° + ∠PAB × 2 = 180

⇒ ∠PAB = 65°

⇒ ∠OAB = 90°

Thus, ∠ OAB = ∠ OAP - ∠ PAB = 90° - 65° = 25°

OR

∠POR = 120°

∴ reflex ∠POR = 240

∠PRQ = 1\2 reflex ∠POR = 120

12. $\frac{3\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2}\left(\frac{\cos 35^\circ}{\sin 55^\circ}\right) =$ ______.

Sol. cot 40 = tan 50

cos 35 = sin 55

$\therefore\ 3-\frac{1}{2}=\frac{5}{2}$

13. The distance between two parallel tangents of a circle of radius 4 cm is______.

Sol. Distance = diameter = 2r

r = 4

distance = 8 cm

14. the distance of the point (−3, 4) from Y-axis is_____.

Sol. Distance from y axis = 3 units = positive of x coordinate.

15. Value of $\frac{2\tan^2 60^\circ}{1+\tan^2 30^\circ}$ is_____.

Sol.

$$\frac{2\left(\sqrt{3}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$=\frac{2\times 3}{1+\frac{1}{3}}$$

$$=\frac{9}{2}$$

Q. Nos. 16 to 20 are short answer type questions of 1 mark each.

16. The probability that it will rain tomorrow is 0.85. What is the probability that it will not rain tomorrow?

Sol. P(not rain) = 1 – P(rain)

⇒ P(not rain) = 1 – 0.85 = 0.15

17. What is the arithmetic mean of first n natural numbers?

Sol. Arithmetic mean of first 'n' natural numbers = $\frac{1+2+\cdots+n}{n}$

1 + 2 + 3 +… is an AP such that

First term, a = 1

Common difference, d = 1

$\therefore S_n = \frac{n}{2}(a+a_n)$

$$= \frac{n}{2}(1 + n)$$

$$\Rightarrow \text{Arithemetic Mean} = \frac{\text{n(n + 1)}}{\text{2n}} = \frac{\text{n + 1}}{2}$$

18. Two right circular cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1, what is the ratio of their volumes?

Sol. Volume of cone $= \frac{1}{3}\pi r^2 h$

$$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{3}{1}\right)^2 \left(\frac{1}{3}\right) = \frac{3}{1}$$

∴ ratio is 3 : 1.

19. Using the empirical formula, find the mode of a distribution whose mean is 8.32 and the median is 8.05.

Sol. We know,

3 Median = 2 Mean + Mode

⇒ 3 × 8.05 = 2 × 8.32 + Mode

⇒ 24.15 = 16.64 + Mode

⇒ Mode = 24.15 – 16.64 = 7.51

20. Evaluate (sec A + tan A)·(1 – sin A) for A = 60°.

Sol. (sec 60° + tan 60°)(1 – sin 60°)

$$= \left(2 + \sqrt{3}\right)\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$= \frac{\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)}{2} = \frac{4 - 3}{2} = \frac{1}{2}$$

SECTION-B

Q. Nos. 21 to 26 carry 2 marks each.

21. Find the value of p, if the mean of the following distribution is 7.5.

Classes	2-4	4-6	6-8	8-10	10-12	12-14
Frequency (fi)	6	8	15	P	8	4

Sol.:

$18 + 40 + 105 + 9p + 88 + 52 = (41 + p)7.5$

$-4.5 = -1.5p$

$p = 3$

22. Read the following passage and answer the questions given at the end:

Students of Class XII presented a gift to their school in the form of an electric lamp in the shape of a glass hemispherical base surmounted by a metallic cylindrical top of same radius 21 com and weight 3.5 cm. the top was silver coated and the glass surface was painted red.

(i) What is the cost of silver coating the top at the rate of ₹ 5per 100 cm^2?

(ii) What is the surface area of glass to be painted red?

Sol.:

(i) $2\pi r^2 + (2\pi rh + \pi r^2)\ \pi r(2h + r) = 1848 \times \frac{5}{100} \times 92.4$

(ii) $2 \times \frac{22}{7} \times 21 \times 21 = 2772\ cm^2$

23. Find the 11^{th} term from the last term (towards the first term) of the AP 12,8,4, …., -84.

OR

Solve the equation:

1+5+9+13+… +x=1326

Sol.:

Reversing the AP, −84…, 4, 8, 12

$a = -84$

$d = 12 - 8 = 4$

$a_{11} = a + (11 - 1)d$

$a_{11} = -84 + 10 \times 4 = -44$

OR

$a = 1$

$d = 5 - 1 = 4$

Last term = $a + (n - 1)d$

$x = 1 + (n - 1)4$

$x - 1 = 4n - 4$

$4n = x + 3$

$n = \frac{x+3}{4}$

Some of n terms = 1326

$\frac{x+3}{4}(x + 1) = 1326$

$x^2 + 4x + 3 = 10608$

$x^2 + 4x - 10605 = 0$

$x^2 + 105x - 101x - 10605 = 0$

$x(x + 105) - 101(x + 105) = 0$

$(x - 101)(x + 105) = 0$

$x = 101, x = -105$

Neglecting $x = -105$ as AP is increasing

$\therefore x = 101$

24. If $\tan\theta = 3/4$ find the value of $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$

Sol.:

Given $\left(\frac{1-\cos^2\theta}{1+\cos^2\theta}\right)$

Dividing numerator and denominator by $\cos^2\theta$

$$\frac{\sec^2\theta - 1}{\sec^2\theta + 1} = \frac{\tan^2\theta}{2+\tan^2\theta} = \frac{\frac{9}{10}}{2+\frac{9}{16}} = \frac{9}{41}$$

OR

If $\tan\theta = \sqrt{3}$, find the value of $\left(\frac{2\sec\theta}{1+\tan^2\theta}\right)$

Sol.:

Given $\tan\theta = \sqrt{3}$

$\tan\theta = \tan 60$

$\Rightarrow \theta = 60°$

Now, $\left(\frac{2\sec\theta}{1+\tan^2\theta}\right)$

Put value of θ in above expression

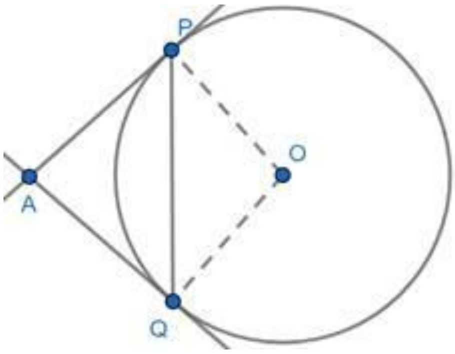
$\left(\frac{2\sec 60°}{1+\tan^2 60°}\right)$

$$\frac{2 \times 2}{1 + 3} = 1$$

25. Prove that the tangents at the extremities of any chord of a circle make equal angles with the chord.

Sol.:

Let the circle with centre O and chord PQ with tangents from point A as AP and AQ as shown:



We have to prove that ∠APQ = ∠AQP

Consider ΔOPQ

⇒ OP = OQ …radius

Hence ΔOPQ is an isosceles triangle

⇒ ∠OPQ = ∠OQP …base angles of isosceles triangle …(a)

As radius OP is perpendicular to tangent AP at point of contact P

⇒ ∠APO = 90°

From figure ∠APO = ∠APQ + ∠OPQ

⇒ 90° = ∠APQ + ∠OPQ

⇒ ∠APQ = 90° - ∠OPQ …(i)

As radius OQ is perpendicular to tangent AQ at point of contact Q

⇒ ∠AQO = 90°

From figure ∠AQO = ∠APQ + ∠OPQ

⇒ 90° = ∠AQP + ∠OQP

⇒ ∠AQP = 90° - ∠OQP

Using (a)

⇒ ∠AQP = 90° - ∠OPQ …(ii)

Using (i) and (ii), we can say that

⇒ ∠APQ = ∠AQP

Hence proved

Hence, the tangents at the extremities of any chord of a circle make equal angles with the chord

26. Two dice are thrown together once. Find the probability of getting a sum of more than 9.

Sol.:

When two dice are thrown, the possible outcomes are

{(1,1) (1,2) (1,3) (1,4) (1,5) (1,6), (2,1) (2,2) (2,3) (2,4) (2,5) (2,6), (3,1) (3,2) (3,3) (3,4) (3,5) (3,6), (4,1) (4,2) (4,3) (4,4) (4,5) (4,6), (5,1) (5,2) (5,3) (5,4) (5,5) (5,6), (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

The outcomes in which sum of no's is 9 are ={(3,6) (4,5) (5,4) (6,3)}

No of Total possible outcomes = 36

No of favourable outcomes = 4

and, Probability of an event $= \frac{\text{No of favourable outcomes}}{\text{No of Total outcomes}}$

Therefore,

P(Getting sum 9)$= \frac{4}{36} = \frac{1}{9}$

Section -C

Q. Nos, 27 to 34 carry 3 marks each.

27. 500 persons are taking dip into a cuboidal pond which is 80 m long and 50 m broad. What is the rise of water level in the pond, if the average displacement of the water by a person is 0.04m^3 ?

Sol.:

Raise in water level by 1 person = 0.04m^3

Raise in water level by 500 persons = 500 × 0.04 = 20 m^3

Now, let the raise in water level be 'h' m.

∴ Volume of raise in water = Length of pond × breadth of pond × raise in water

⇒ 20 = 80 × 50 × h

$$\Rightarrow \mathrm{h} = \frac{20}{80 \times 50} = \frac{20}{4000} = \frac{1}{200} \text{ m}$$

$$\Rightarrow \mathrm{h} = \frac{1}{200} \times 100 = \frac{1}{2} = 0.5 \text{ cm}$$

28. If sin θ +cos θ =p and sec θ +cosec θ =q, show that $q(p^2-1)=2p$

Sol.:

LHS

$q(p^2 - 1) = (\sec\theta + \operatorname{cosec}\theta)[(\sin\theta + \cos\theta)^2 - 1]$

$$= \left(\frac{1}{\sin\theta} + \frac{1}{\cos\theta}\right)(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1)$$

$$= \left(\frac{(\sin\theta+\cos\theta)}{\sin\theta\cos\theta}\right)(1 + 2\sin\theta\cos\theta - 1)$$

$$= \frac{(\sin\theta+\cos\theta)(2\sin\theta\cos\theta)}{\sin\theta\cos\theta}$$

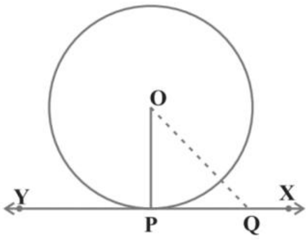
= 2p = RHS

Hence, proved!

29. Prove that, a tangent to a circle is perpendicular to the radius through the point of contact.

Sol.:

Let us consider a circle with center O, OP be a radius and XY be a tangent at point P.



To prove: OP ⊥ XY

Proof: Let's take a point Q on XY other than P.

Clearly, point Q lies outside the circle because if Q lies inside the circle then XY will be a secant (as it will intersect the circle at two points)

Point P is on circle and point Q is outside the circle

⇒ OP < OQ

This is true for all points on XY other than P

⇒ OP is the shortest distance between point P and line XY.

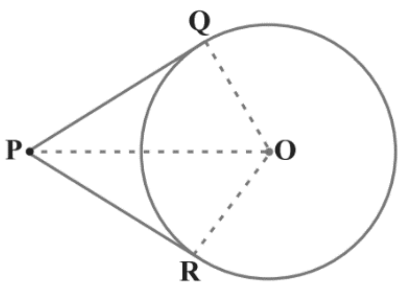
⇒ OP ⊥ XY [Shortest side is perpendicular]

Hence, proved!

OR

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Sol.:



Let us consider a circle with center O. PQ and PR be two tangents from an external point P to the circle.

To prove: ∠QPR + ∠QOR = 180°

Proof: In quadrilateral PROQ, we have

∠QPR + ∠PQO + ∠QOR + ∠ORP = 360°

Now, $\angle OQP = \angle ORP = 90°$ [Tangent at a point on a circle is perpendicular to the radius through point of contact]

$\Rightarrow \angle QPR + 90° + \angle QOR + 90° = 360°$

$\Rightarrow \angle QPR + \angle QOR = 180°$

Hence, proved!

30. On dividing x^3-3 x^2+x+2 by a polynomial a cos. the quotient and remainder were x-2 and -2 x+4 respectively. Find g(x).

Sol.:

We know that,

p(x) = g(x)q(x) + r(x)

Putting values, we get

$x^3 - 3x^2 + x + 2 = g(x)(x - 2) - 2x + 4$

$\Rightarrow g(x)(x - 2) = x^3 - 3x^2 + x + 2 + 2x - 4$

$\Rightarrow g(x)(x - 2) = x^3 - 3x^2 + 3x - 2$

$\Rightarrow g(x)(x - 2) = x^3 - 2x^2 - x^2 + 2x + x - 2$

$\Rightarrow g(x)(x - 2) = x^2(x - 2) - x(x - 2) + x - 2$

$\Rightarrow g(x)(x - 2) = (x - 2)(x^2 - x + 1)$

$\Rightarrow g(x) = x^2 - x + 1$

OR

If the sum of the squares of zeros of the quadratic polynomial $f(x)=x^2$-8 x + k is 40, find the value of k

Sol.:

Let zeroes of $x^2 - 8x + k = 40$ be α and β

$\Rightarrow \alpha + \beta = 8$ [-b/a]

$\Rightarrow \alpha\beta = k$ [c/a]

Given,

$\alpha^2 + \beta^2 = 40$

$\Rightarrow (\alpha + \beta)^2 - 2\alpha\beta = 40$

$\Rightarrow 8^2 - 2k = 40$

$\Rightarrow 64 - 2k = 40$

$\Rightarrow 2k = 24$

$\Rightarrow k = 12$

31. Find a, b and c if it is given that the numbers a, 7, b, 23, c are in A.P.]

First term of given AP is 'a', let common difference be 'd'.

Sol.

We have,

Second term, $a_2 = 7$

$\Rightarrow a + d = 7$ [1]

Fourth term, $a_4 = 23$

$\Rightarrow a + 3d = 23$ [2]

[2] – [1] gives,

$3d - d = 23 - 7$

$\Rightarrow 2d = 16$

$\Rightarrow d = 8$

$\Rightarrow a + 8 = 7$

$\Rightarrow a = -1$

$b = 7 + d = 7 + 8 = 15$

$c = 23 + d = 23 + 8 = 31$

OR

If m times the m^{th} term of an AP is equal to n times its n^{th} term, show that the $(m+n)^{th}$ term of the AP is zero.

Sol.:

We know that nth term of A.P. , $t_n = a + (n - 1) d$.

First, $t_n = a + (n - 1) d$

Then, $t_m = a + (m - 1) d$

Given, $mt_m = nt_n$

$\Rightarrow m [a + (m - 1) d] = n [a + (n - 1) d]$

$\Rightarrow ma + m^2d - md = na + n^2d - nd$

$\Rightarrow ma - na + m^2d - n^2d - md + nd = 0$

$\Rightarrow a (m - n) + d (m^2 - n^2) - d (m - n) = 0$

We know that $a^2 - b^2 = (a - b) (a + b)$

$\Rightarrow (m - n) [a + d (m + n) - d] = 0$

$\Rightarrow [a + d (m + n) - d] = 0$

$\Rightarrow a + (m + n - 1) d = 0$

$\therefore$ (m + n)th term, $t_{m+n} = 0$

Hence proved.

32. Find the values of k for which the points A(k+1,2 k), B(3 k, 2 k+3) and C(5 k-1,5 k) are collinear.

Sol.:

Three points are collinear if the area of triangle formed by them is zero.

Area of a triangle $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$0 = (k + 1)(2k + 3 - 5k) + 3k(5k - 2k) + (5k - 1)(2k - 2k - 3)$

$\Rightarrow 0 = (k + 1)(3 - 3k) + 3k(3k) + (5k - 1)(-3)$

$\Rightarrow 0 = -3k^2 + 3 + 9k^2 - 15k + 3$

$\Rightarrow 6k^2 - 15k + 6 = 0$

$\Rightarrow 6k^2 - 12k - 3k + 6 = 0$

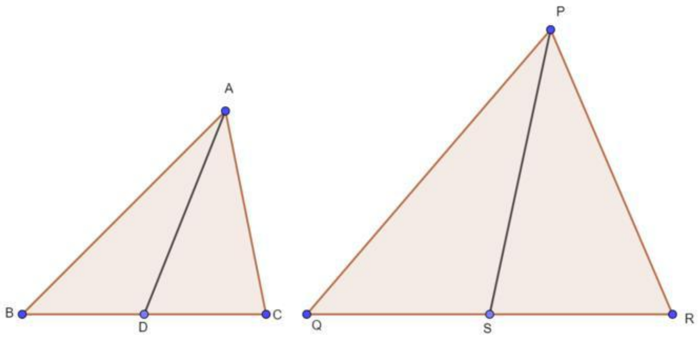
$\Rightarrow 6k(k - 2) - 3(k - 2) = 0$

$\Rightarrow (6k - 3)(k - 2) = 0$

$\Rightarrow k = 2$ or $k = ½$

33. Prove that the ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding medians.

Sol.:



Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$.

Let AD and PS be the medians of these triangles

Then, because $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(i)

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ ...(ii)

Since AD and PS are medians,

BD = DC = BC/2

And, QS = SR = QR/2

Equation (i) becomes,

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ ...(iii)

In ΔABD and ΔPQS,

$\angle B = \angle Q$ [From (ii)]

$\frac{AB}{PQ} = \frac{BD}{QS}$ [From (iii)]

$\Delta ABD \sim \Delta PQS$ (SAS similarity)

Therefore, it can be said that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

From (i) and (iv), we get

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

and hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Hence, proved!

34. Find the value of k for which the quadratic equation $kx^2+1-2(k-1)x+x^2=0$ has equal roots. Hence find the roots of the equation.

Sol.:

Equal roots i.e. determinant is 0

$\Rightarrow b^2 - 4ac = 0$

Equation given: $kx^2 + 1 - 2(k - 1)x + x^2 = 0$

$\Rightarrow x^2(k + 1) - 2(k - 1)x + 1 = 0$

$\Rightarrow (-2(k - 1))^2 - 4(k + 1)1 = 0$

$\Rightarrow 4(k - 1)^2 - 4k - 4 = 0$

$\Rightarrow 4k^2 - 8k + 4 - 4k - 4 = 0$

$\Rightarrow 4k^2 - 12k = 0$

$\Rightarrow 4k(k - 3) = 0$

$\Rightarrow k = 0$ or $k = 3$

If $k = 0$, equation is $x^2 + 2x + 1 = 0$

$\Rightarrow (x + 1)^2 = 0$

$\therefore$ roots will be -1 and -1

If $k = 3$, equation is $4x^2 - 4x + 1 = 0$

$\Rightarrow (2x - 1)^2 = 0$

$\therefore$ roots will be ½ and ½ .

Section -D

Q. Nos. 35 to 40 carry 4 marks each.

35. In an equilateral triangle A B C, D is a point on the side BC such that BD =1/3BC. Prove that $9\ AD^2=7\ AB^2$.

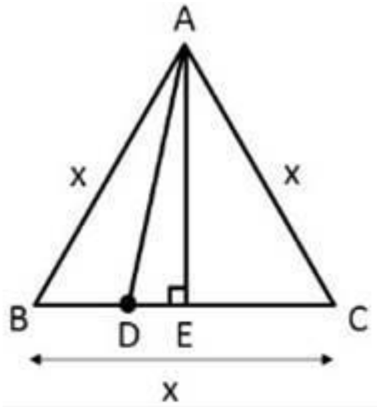
Sol.:

Given: AB = BC = CA = x (say)

$BD = \frac{1}{3} BC$

To prove: $9\ AD^2 = 7\ AB^2$

Proof: Construct AE perpendicular to BC.



As BC = x,

BD = $\frac{x}{3}$

In ΔABE and ΔACE,

AB = AC [∵ AB = AC = x]

AE = AE [common sides in both triangles]

∠AEB = ∠AEC [∵ ∠AEB = ∠AEC = 90°]

Thus, ΔABE ≅ ΔACE by RHS congruency, i.e., Right angle-Hypotenuse-Side congruency.

If ΔABE ≅ ΔACE,

BE = CE

[∵ corresponding parts of congruent triangles are congruent]

So, BE = CE = $\frac{1}{2}$ BC

⇒ BE = CE = $\frac{x}{2}$

We have BE = x/2, BD = x/3 and clearly

BD + DE = BE

$\Rightarrow \frac{x}{3} + DE = \frac{x}{2}$

$\Rightarrow DE = \frac{x}{2} - \frac{x}{3} = \frac{3x-2x}{6} = \frac{x}{6}$

In ΔABE using Pythagoras theorem,

$(\text{hypotenuse})^2 = (\text{perpendicular})^2 + (\text{base})^2$

$\Rightarrow x^2 = AE^2 + \left(\frac{x}{2}\right)^2$

$\Rightarrow AE^2 = x^2 - \frac{x^2}{4}$

$\Rightarrow AE^2 = \frac{3x^2}{4}$ ...(i)

Similarly, using pythagoras theorem in right ΔAED,

$AD^2 = AE^2 + ED^2$

$\Rightarrow AD^2 = \frac{3x^2}{4} + \frac{x^2}{36} \left[\because ED = \frac{x}{6}\right]$

$\Rightarrow AD^2 = \frac{27x^2+x^2}{36}$

$\Rightarrow AD^2 = \frac{28x^2}{36}$

$\Rightarrow AD^2 = \frac{7}{9}x^2$

$\Rightarrow 9\ AD^2 = 7\ x^2$

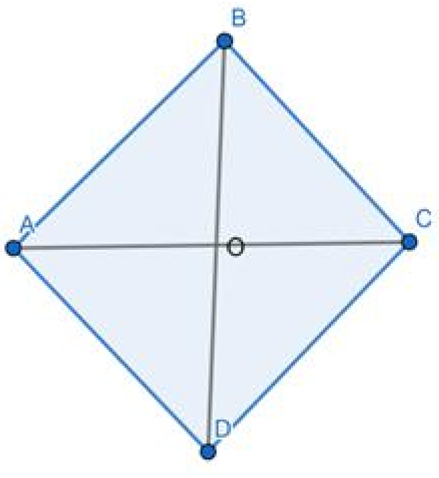
$\Rightarrow 9AD^2 = 7\ AB^2$ [$\because AB = x \Rightarrow AB^2 = x^2$]

OR

Prove that the sum of squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Sol.:

Need to prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals



ABCD is a rhombus in which diagonals AC and BD intersect at point O.

We need to prove $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + DB^2$

$\Rightarrow$ In Δ AOB; $AB^2 = AO^2 + BO^2$

$\Rightarrow$ In Δ BOC; $BC^2 = CO^2 + BO^2$

$\Rightarrow$ In Δ COD; $CD^2 = DO^2 + CO^2$

$\Rightarrow$ In Δ AOD; $AD^2 = DO^2 + AO^2$

$\Rightarrow$ Adding the above 4 equations we get

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2$

$= AO^2 + BO^2 + CO^2 + BO^2 + DO^2 + CO^2 + DO^2 + AO^2$

$\Rightarrow = 2(AO^2 + BO^2 + CO^2 + DO^2)$

Since, $AO^2 = CO^2$ and $BO^2 = DO^2 = 2(2\ AO^2 + 2\ BO^2)$

$= 4(AO^2 + BO^2)$ ......eq(1)

Now, let us take the sum of squares of diagonals

$\Rightarrow AC^2 + DB^2 = (AO + CO)^2 + (DO + BO)^2 = (2AO)^2 + (2DO)^2$

$= 4\ AO^2 + 4\ BO^2$ ......eq(2)

From eq(1) and eq(2) we get

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + DB^2$

36. If the angle of elevation of a cloud from a point 10 metres above a lake is 30° and the angle of depression of its reflection in the lake is 60°, find the height of the cloud from the surface of lake.

Sol.:

Here, BN $\rightarrow$ Lake

C $\rightarrow$ Position of level

Clearly CN = C′N

I-Δ CAM

$$\tan 30^\circ = \frac{h}{AM}$$

$$\tan 30^\circ = \frac{h}{AM}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{AM} \Rightarrow AM = h\sqrt{3} \qquad ...(1)$$

In Δ AMC′

$$\tan 60^\circ = \frac{MC'}{AM}$$

$$\sqrt{3} = \frac{10 + h + 10}{AM} \Rightarrow AM = \frac{20 + h}{\sqrt{3}} \qquad ...(2)$$

From eq. (1) and (2)

$$h\sqrt{3} = \frac{20 + h}{\sqrt{3}} \Rightarrow 3h = 20 + h \Rightarrow 2h = 20$$

h = 10 m

OR

A vertical tower of height 20 m stands on a horizontal plane and is surmounted by a vertical flag - staff of height h. At a point on the plane, the angle of elevation of the bottom and top of the flag staff are 45° and 60° respectively. Find the value of h.

Sol.:

AB → Flag Staff (1 m)

BC → Tower (20 m)

DC → x(8ay)

Diagram

In Δ ADC,

$$\tan 60^\circ = \frac{AC}{DC}$$

$$\sqrt{3} = \frac{h+20}{x}$$

$$h = \frac{h+20}{\sqrt{3}} \qquad ...(1)$$

In Δ BDC

$$\tan 45^\circ = \frac{BC}{DC}$$

$$h = \frac{20}{x} \Rightarrow h = 20 \quad (2)$$

Fro, eq. (1) & (2)

$$\frac{h+20}{\sqrt{3}} = 20$$

$$h + 20 = 20\sqrt{3}$$

$$h = 20(\sqrt{3} - 1)\text{ m}$$

37. Show that $(12)^n$ cannot end with digit 0 or 5 for any natural number n.

Sol.:

If the number 12n, for any natural number n, ends with the digit 0 or 5, then it is divisible by 5.

That is, the prime factorization of 12n contains the prime 5.

This is not possible because prime factorisation of

$12n = (2^2 \times 3)^n = 2^{2n} \times 3^n$

So, the only primes in the factorisation of 12^n are 2 and 3

And according to the fundamental theorem of arithmetic guarantees that there are no other primes in the factorization of 12^n. So, there is no natural number n for which 12^n ends with the digit zero.

OR

Prove that $(\sqrt{2} + \sqrt{5})$ is irrational.

Sol.:

Let $\sqrt{2}+\sqrt{5}$ be a rational number.

A rational number can be written in the form of p/q where p,q are integers.

$\sqrt{2}+\sqrt{5} = p/q$

Squaring on both sides,

$(\sqrt{2}+\sqrt{5})^2 = (p/q)^2$

$\sqrt{2}^2+\sqrt{5}^2+2(\sqrt{5})(\sqrt{2}) = p^2/q^2$

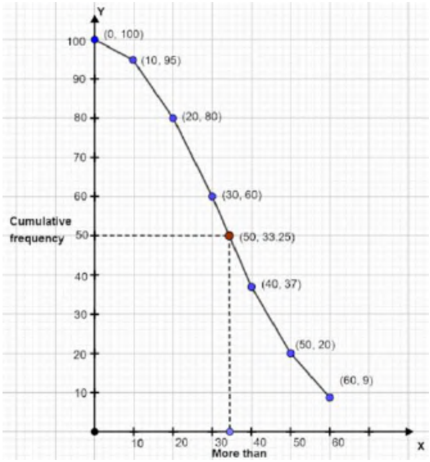
38. For the following frequency distribution, draw a cumulative frequency curve of 'more than' type and hence obtain the median value.

Classes	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

Sol.:

More than type	C.f
More than 0	100
10	100-5 = 95
20	95-15 = 80
30	80-20 = 60
40	60-23 = 37
50	37-17 = 20
60	20-11 = 9

Flor (0,100) (10,95) (20,80) (30,60) (40,37) (50,20) (60,9)



39. If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes 1/2 if we only add 1 to the denominator. What is the fraction?

Sol: Let the fraction be $\frac{x}{y}$

Case I

$\frac{x+1}{y-1}=1$

x + 1 = y − 1

x − y = -2 (1)

Case II

$\frac{x}{y+1}=\frac{1}{2}$

2x = y + 1

$\Rightarrow$ y = 2x -1 (2)

Substitute (2) in (1)

$x-(2x-1)=-2$

$x-2x+1=-2$

tx = 3

x = 3

$\therefore$ put x = 3 in q(1)

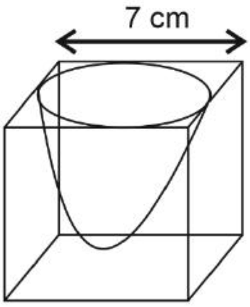
3 − y = -2

y = 5

$\therefore$ fraction = $\frac{x}{y}=\frac{3}{5}$

40. A hemispherical depression is cut out from one face of a cuboidal block of side 7cm such that the diameter of the hemisphere is equal to the edge of the cube. Find the surface area of the remaining solid.

Sol.



$x = 7cm$

$r = \frac{7}{2}cm$

Surface Area = TSA of cube + CSA of hemisphere - πr^2

$= 6x^2 + 2\pi r^2 - \pi r^2$

$= 6x^2 + \pi r^2$

$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

$= 294 + 38.5$

= 332.5 cm^2

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