CCE SAMPLE QUESTION PAPER

FIRST TERM (SA–I)

MATHEMATICS

(*With Solutions*)

CLASS X

Time Allowed : 3 to 3½ Hours] *[Maximum Marks : 80*

General Instructions :

(*i*) *All questions are compulsory.*

(*ii*) *The question paper consists of 34 questions divided into four sections A, B, C and D. Section A comprises of 10 questions of 1 mark each, Section B comprises of 8 questions of 2 marks each, Section C comprises of 10 questions of 3 marks each and Section D comprises of 6 questions of 4 marks each.*

(*iii*) *Question numbers 1 to 10 in Section A are multiple choice questions where you are to select one correct option out of the given four.*

(*iv*) *There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only one of the alternatives in all such questions.*

(*v*) *Use of calculators is not permitted.*

Section 'A'

Question numbers 1 to 10 are of one mark each.

1. If $\tan\theta = \frac{1}{\sqrt{7}}$, then the value of $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{4}$

(*c*) $\frac{4}{5}$ (*d*) $\frac{2}{\sqrt{7}}$

Solution. Choice (*b*) is correct.

$$\sec^2\theta = 1 + \tan^2\theta = 1 + \frac{1}{7} = \frac{8}{7}$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{1}{\tan^2\theta} = 1 + 7 = 8$$

$$\therefore \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{(56-8)/7}{(56+8)/7}$$

$$= \frac{48}{64}$$

$$= \frac{3}{4}$$

2. If $\sin \alpha = \frac{1}{2}$, then the value of $4 \cos^3 \alpha - 3 \cos \alpha$ is

(*a*) 0 (*b*) 1

(*c*) –1 (*d*) $\frac{1}{8}$

Solution. Choice (*a*) is correct.

$$\sin \alpha = \frac{1}{2}$$

$$\Rightarrow \quad \sin^2 \alpha = \frac{1}{4}$$

$$\Rightarrow \quad 1 - \sin^2 \alpha = 1 - \frac{1}{4}$$

$$\Rightarrow \quad \cos^2 \alpha = \frac{3}{4}$$

$$\Rightarrow \quad \cos \alpha = \frac{\sqrt{3}}{2}$$

Now, $4 \cos^3 \alpha - 3 \cos \alpha$

$$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{4(3\sqrt{3})}{8} - \frac{3\sqrt{3}}{2}$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$

3. If $\cos 2\theta = \sin (\theta - 12°)$, where (2θ) and $(\theta - 12°)$ are both acute angles, then the value of θ is

(*a*) 24° (*b*) 28°

(*c*) 32° (*d*) 34°

Solution. Choice (*d*) is correct.

$$\cos 2\theta = \sin (\theta - 12°)$$

$$\Rightarrow \quad \sin (90° - 2\theta) = \sin (\theta - 12°) \qquad [\because \cos \theta = \sin (90° - \theta)]$$

$$\Rightarrow \quad 90° - 2\theta = \theta - 12°$$

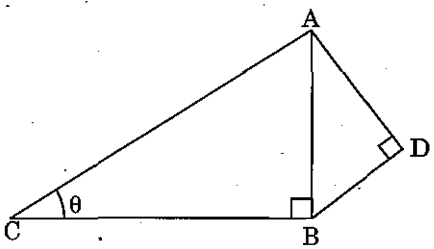
$$\Rightarrow \quad 2\theta + \theta = 90° + 12°$$

$$\Rightarrow \quad 3\theta = 102°$$

$$\Rightarrow \quad \theta = 102° \div 3$$

$$\Rightarrow \quad \boldsymbol{\theta = 34°}$$

4. In figure, $AD = 3\sqrt{3}$ cm, $BD = 3$ cm and $CB = 8$ cm, then the value of cosec θ is



(a) $\frac{2}{3}$ (b) $\frac{4}{3}$

(c) $\frac{5}{3}$ (d) $\frac{7}{3}$

Solution. Choice (c) is correct.

In right ΔADB, $AB^2 = AD^2 + BD^2$

$\Rightarrow \quad AB^2 = (3\sqrt{3})^2 + (3)^2$

$\Rightarrow \quad AB^2 = 27 + 9 = 36 = (6)^2$

$\Rightarrow \quad AB = 6$ cm

In right ΔABC, $AC^2 = CB^2 + AB^2$

$\Rightarrow \quad AC^2 = (8)^2 + (6)^2$

$\Rightarrow \quad AC^2 = 64 + 36 = 100 = (10)^2$

$\Rightarrow \quad AC = 10$ cm

In ΔACB, cosec $\theta = \frac{AC}{AB} = \frac{10}{6} = \frac{5}{3}$.

5. For a given data with 100 observations the 'less than ogive and the more than ogive' intersect at (525, 50). The median of the data is

(a) 20 (b) 30

(c) 50 (d) 525

Solution. Choice (d) is correct.

The x-coordinate of the intersection point (525, 50) of 'less than ogive and more than ogive' is 525. Therefore, **525** is the median of the given data.

6. Which of the following is not a rational number ?

(a) $\sqrt{3}$ (b) $\sqrt{9}$

(c) $\sqrt{16}$ (d) $\sqrt{25}$

Solution. Choice (a) is correct.

Since 3 is a prime number, $\sqrt{3}$ is an irrational number.

7. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, then the other number is

(a) 415 (b) 425

(c) 435 (d) 445

Solution. Choice (c) is correct.

LCM × HCF = Product of two numbers a and b.

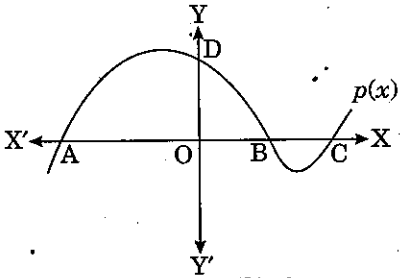
$\Rightarrow \quad 145 \times 2175 = 725 \times b$, where $a = 725$

$$\Rightarrow \quad b = \frac{145 \times 2175}{725}$$

$$\Rightarrow \quad b = 145 \times 3$$

$$\Rightarrow \quad b = \mathbf{435}$$

8. In figure, the graph of a polynomial $p(x)$ is shown. The number of zeroes of $p(x)$ is



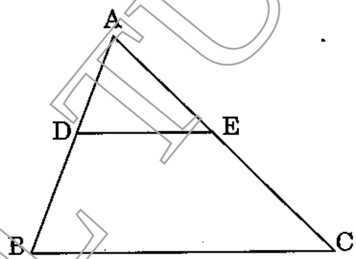
(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Solution. Choice (*c*) is correct.

The number of zeroes of $p(x)$ is **3** as the graph intersects the x-axis at three points A, B and C in figure.

9. In ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $\frac{AD}{DB} = \frac{2}{3}$ and $AC = 18$ cm, then AE is equal to



(*a*) 5.2 cm (*b*) 6.2 cm

(*c*) 7.2 cm (*d*) 8.2 cm

Solution. Choice (*c*) is correct.

In figure, since $DE \parallel BC$, then by BPT, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{AE}{AC - AE}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{AE}{18 - AE}$$

$$\Rightarrow \quad 36 - 2AE = 3AE$$

$$\Rightarrow \quad 3AE + 2AE = 36$$

$$\Rightarrow \quad 5AE = 36$$

$\Rightarrow \quad AE = 36 \div 5$

$\Rightarrow \quad AE = \mathbf{7.2\ cm}$

10. If the pair of linear equations $2x + 3y = 7$ and $2\alpha x + (\alpha + \beta)y = 28$ has infinitely many solutions, then the values of α and β are

(a) 3 and 5 (b) 4 and 5

(c) 4 and 7 (d) 4 and 8

Solution. Choice (d) is correct.

The given pair of linear equations will have infinitely many solution, if

$$\frac{2}{2\alpha} = \frac{3}{\alpha+\beta} = \frac{-7}{-28}$$

$$\Rightarrow \quad \frac{1}{\alpha} = \frac{3}{\alpha+\beta} = \frac{1}{4}$$

$$\Rightarrow \quad \alpha = 4 \quad \text{and} \quad \alpha + \beta = 12$$

$$\Rightarrow \quad \boldsymbol{\alpha = 4} \quad \text{and} \quad \boldsymbol{\beta = 8}$$

Section 'B'

Question numbers 11 to 18 carry 2 marks each.

11. Find the LCM and HCF of 510 and 92 by the prime factorisation method.

Solution. The prime factorisation of 510 and 92 gives :

$$510 = 2^1 \times 3^1 \times 5^1 \times 17^1 \text{ and } 92 = 2 \times 2 \times 23 = 2^2 \times 23^1$$

Here, 2^1 is the smallest power of the common factor 2.

So, HCF (510, 92) = 2^1 = **2** = Product of the smallest power of each common prime factor in the numbers.

LCM (510, 92) = $2^2 \times 3^1 \times 5^1 \times 17^1 \times 23^1$ = **23460**

= Product of the greatest power of each prime factor, involved in the numbers.

12. If the product of zeroes of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a'.

Solution. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

Let α and β be the zeroes of the polynomial $ax^2 - 6x - 6$.

Then, product of the zeroes = $\alpha\beta = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-6}{a}$.

But the product of zeroes of the polynomial $ax^2 - 6x - 6$ is 4.

$$\therefore \quad \frac{-6}{a} = 4$$

$$\Rightarrow \quad a = -\frac{6}{4}$$

$$\Rightarrow \quad a = -\frac{3}{2}$$

Thus, the value of a is $-\mathbf{\dfrac{3}{2}}$.

13. 2 tables and 3 chairs together cost ₹ 3500 whereas 3 tables and 2 chairs together cost ₹ 4000. Find the cost of a table and a chair.

Solution. Let the cost of a table be ₹ x and the cost of a chair be ₹ y.
Then, according to the given condition, we have

$$2x + 3y = 3500 \quad \ldots(1)$$
$$3x + 2y = 4000 \quad \ldots(2)$$

Adding (1) and (2), we get

$$5x + 5y = 7500$$
$$\Rightarrow \quad x + y = 1500 \quad \ldots(3)$$

Subtracting (1) from (2), we get

$$x - y = 500 \quad \ldots(4)$$

Adding (3) and (4), we get

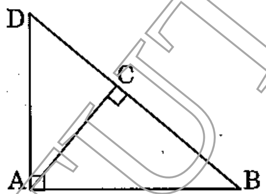
$$2x = 2000$$
$$\Rightarrow \quad x = 1000$$

Substituting $x = 1000$ in (3), we get

$$1000 + y = 1500$$
$$\Rightarrow \quad y = 1500 - 1000$$
$$\Rightarrow \quad y = 500$$

Hence, the cost of a table = ₹ **1000** and the cost of a chair = ₹ **500.**

14. In figure, ΔABD is a right triangle, right-angled at A and $AC \perp BD$. Prove that $AB^2 = BC.BD$.



Solution. Given : ΔABD is a right triangle, right-angled at A and $AC \perp BD$.
To prove : $AB^2 = BC.\,BD$.
Proof : In ΔABD and ΔCAB, we have

$$\angle BAD = \angle ACB \quad \text{[Each = 90°]}$$
$$\angle B = \angle B \quad \text{[Common]}$$

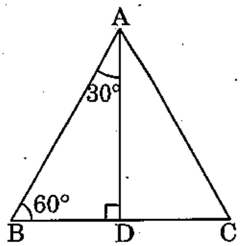
So, by AA-criterion of similarity of triangles, we have

$$\Delta ABD \sim \Delta CAB$$
$$\Rightarrow \quad \frac{AB}{BD} = \frac{BC}{AB}$$

Hence, $AB^2 = BC.\,BD$.

15. Find the value of tan 60°, geometrically.

Solution. Consider an equilateral triangle ABC. Let $2a$ be the length of each side of the traingle ABC such that



$$AB = BC = CA = 2a$$

Since each angle in an equilateral triangle is 60°, therefore,

$$\angle A = \angle B = \angle C = 60°$$

Draw the perpendicular AD from A to the side BC.

Clearly, $\Delta ABD \cong \Delta ACD$

Therefore, $BD = DC$ and $\angle BAD = \angle CAD$ [CPCT]

ΔABD is a right triangle, right angled at D with $\angle ABD = 60°$

Also, $\quad BD = \frac{1}{2}BC = a$

In ΔABD, we have

$$AD^2 = AB^2 - BD^2 = (2a)^2 - (a)^2 = 3a^2 \Rightarrow AD = \sqrt{3}\,a$$

Now, $\quad \tan 60° = \frac{AD}{BD} = \frac{\sqrt{3}a}{a} = \sqrt{3}$.

Or

Without using the trigonometric tables, evaluate the following :

$$\mathbf{\frac{11}{7}\cdot\frac{\sin 70°}{\cos 20°} - \frac{4}{7}\cdot\frac{\cos 53° \operatorname{cosec} 37°}{\tan 15° \tan 35° \tan 55° \tan 75°}}$$

Solution. We have

$$\frac{11}{7}\cdot\frac{\sin 70°}{\cos 20°} - \frac{4}{7}\cdot\frac{\cos 53° \operatorname{cosec} 37°}{\tan 15° \tan 35° \tan 55° \tan 75°}$$

$$= \frac{11}{7}\cdot\frac{\sin (90° - 20°)}{\cos 20°} - \frac{4}{7}\cdot\frac{\cos (90° - 37°)\cdot \operatorname{cosec} 37°}{\tan 15° \tan 35°\cdot\tan (90° - 35°)\cdot\tan (90° - 15°)}$$

$$= \frac{11}{7}\cdot\frac{\cos 20°}{\cos 20°} - \frac{4}{7}\cdot\frac{\sin 37°\cdot \operatorname{cosec} 37°}{\tan 15° \tan 35°\cdot\cot 35°\cdot\cot 15°}$$

[$\because \sin (90° - \theta) = \cos \theta, \cos (90° - \theta) = \sin \theta, \tan (90° - \theta) = \cot \theta$]

$$= \frac{11}{7}\cdot(1) - \frac{4}{7}\cdot\frac{(\sin 37°\cdot \operatorname{cosec} 37°)}{(\tan 15°\cdot\cot 15°)(\tan 35°\cdot\cot 35°)}$$

$$= \frac{11}{7} - \frac{4}{7}\cdot\frac{1}{(1)(1)} \qquad [\because \sin\theta\cdot\operatorname{cosec}\theta = 1, \tan\theta\cdot\cot\theta = 1]$$

$$= \frac{11}{7} - \frac{4}{7}$$

$$= \frac{7}{7} = \mathbf{1}.$$

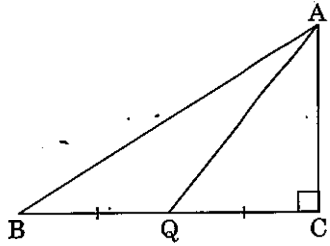
16. In a ΔABC, $\angle BCA$ is a right angle. If Q is the mid point of the side BC, $AC = 4$ cm, and $AQ = 5$ cm, find $(AB)^2$.

Solution. Since ΔACB is a right angle, right-angled at C, therefore

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow \quad AB^2 = AC^2 + (2QC)^2 \qquad \left[\because Q \text{ is the mid-point of } BC, BQ = QC = \frac{1}{2}BC\right]$$

$$\Rightarrow \quad AB^2 = AC^2 + 4QC^2 \qquad ...(1)$$



Again, ΔACQ is right triangle, right-angled at C, therefore

$$AQ^2 = AC^2 + QC^2$$

$$\Rightarrow \quad QC^2 = AQ^2 - AC^2$$

$$= (5)^2 - (4)^2 \qquad [\because AQ = 5 \text{ cm and } AC = 4 \text{ cm}]$$

$$\Rightarrow \quad QC^2 = 25 - 16 = 9 \qquad ...(2)$$

From (1) and (2), we have

$$AB^2 = (4)^2 + 4 \times 9 \qquad [\because AC = 4 \text{ cm}]$$

$$\Rightarrow \quad AB^2 = 16 + 36 = 52$$

Hence, $\quad (AB)^2 = \mathbf{52\ cm^2}$.

17. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality.

Monthly consumption (in units)	**65 – 85**	**85 – 105**	**105 – 125**	**125 – 145**	**145 – 165**	**165 – 185**	**185 – 205**
Number of consumers	**4**	**5**	**13**	**20**	**14**	**8**	**4**

Write the above distribution as less than type cumulative frequency distribution.

Solution. Cumulative Frequency Table as less than type is given below :

Monthly consumption (in units)	*Number of consumers [Frequency (f)]*	*Monthly consumption less than*	*Cumulative frequency (cf)*
65 – 85	4	85	4
85 – 105	5	105	9 (5 + 4)
105 – 125	13	125	22 (13 + 9)
125 – 145	20	145	42 (22 + 20)
145 – 165	14	165	56 (42 + 14)
165 – 185	8	185	64 (56 + 8)
185 – 205	4	205	68 (64 + 4)

18. The length of 42 leaves of a plant are measured correct up to the nearest millimetre and the data is as under :

Length (in mm)	**118 – 126**	**126 – 134**	**134 – 142**	**142 – 150**	**150 – 158**	**158 – 166**
Number of leaves	**4**	**5**	**10**	**14**	**4**	**5**

Find the mode length of the leaves.

Solution. Since the maximum number of leaves is 14, therefore, the modal class is 142 – 150.

$\therefore \quad l = 142, h = 8, f_1 = 14, f_0 = 10, f_2 = 4$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 142 + \frac{14 - 10}{2 \times 14 - 10 - 4} \times 8$$

$$= 142 + \frac{4}{28 - 14} \times 8$$

$$= 142 + \frac{4}{14} \times 8$$

$$= 142 + \frac{16}{7}$$

$$= 142 + 2.29$$

$$= \mathbf{144.29 \text{ mm}}$$

Section 'C'

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $3 + \sqrt{2}$ is an irrational number.

Solution. Let us assume to contrary, that $3 + \sqrt{2}$ is rational.
That is, we can find coprime a and b ($b \neq 0$) such that

$$3 + \sqrt{2} = \frac{a}{b}$$

Rearranging, we get

$$\sqrt{2} = \frac{a}{b} - 3$$

$$\Rightarrow \quad \sqrt{2} = \frac{a - 3b}{b}$$

Since a and b are integers, we get $\frac{a-3b}{b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $3 + \sqrt{2}$ is rational.

So, we conclude that $3 + \sqrt{2}$ is **irrational.**

Or

Prove that $5\sqrt{2}$ is irrational number.

Solution. Let us assume to the contrary, that $5\sqrt{2}$ is rational.
Then, there exist co-prime positive integers p and q such that

$$5\sqrt{2} = \frac{p}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{p}{5q}$$

$$\Rightarrow \quad \sqrt{2} \text{ is rational} \qquad \left[\begin{array}{l} \because 5, p \text{ and } q \text{ are integers.} \\ \therefore \frac{p}{5q} \text{ is a rational number} \end{array}\right]$$

But this contradicts the fact that $\sqrt{2}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5\sqrt{2}$ is rational.

So, we conclude that $5\sqrt{2}$ is **irrational.**

20. For any positive integer n, $n^3 - n$ is divisible by 6.

Solution. We know that any positive integer is of the form $6m$, $6m + 1$, $6m + 2$, $6m + 3$, $6m + 4$, $6m + 5$, for some positive integer n.

When $n = 6m$, then

$$n^3 - n = (6m)^3 - (6m)$$

$= 216m^3 - 6m$
$= 6m(36m^2 - 1)$
$= 6p$, where $p = m(36m^2 - 1)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

When $n = 6m + 1$, then

$n^3 - n = (n-1)n(n+1)$
$= (n-1)(n^2 + n)$
$= (6m + 1 - 1)[(6m + 1)^2 + 6m + 1]$
$= 6m[36m^2 + 12m + 1 + 6m + 1]$
$= 6m(36m^2 + 18m + 2)$
$= 6q$, where $q = m(36m^2 + 18m + 2)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

When $n = 6m + 2$, then

$n^3 - n = (n-1)(n)(n+1)$
$= (6m + 2 - 1)(6m + 2)(6m + 2 + 1)$
$= (6m + 1)(6m + 2)(6m + 3)$
$= (6m + 1)[36m^2 + 30m + 6]$
$= 6m(36m^2 + 30m + 6) + 36m^2 + 30m + 6$
$= 6m(36m^2 + 30m + 6) + 6(6m^2 + 5m + 1)$
$= 6p + 6q$, where $p = m(36m^2 + 30m + 6)$ and $q = 6m^2 + 5m + 1$
$= 6(p + q)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

When $n = 6m + 3$, then

$n^3 - n = (6m + 3)^3 - (6m + 3)$
$= (6m + 3)[(6m + 3)^2 - 1]$
$= 6m[(6m + 3)^2 - 1] + 3[(6m + 3)^2 - 1]$
$= 6m[(6m + 3)^2 - 1] + 3[36m^2 + 36m + 9 - 1]$
$= 6m[(6m + 3)^2 - 1] + 3[36m^2 + 36m + 8]$
$= 6m[(6m + 3)^2 - 1] + 6(18m^2 + 18m + 4)$
$= 6p + 6q$, where $p = m[(6m + 3)^2 - 1]$ and $q = 18m^2 + 18m + 4$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

When $n = 6m + 4$, then

$n^3 - n = (6m + 4)^3 - (6m + 4)$
$= (6m + 4)[(6m + 4)^2 - 1]$
$= 6m[(6m + 4)^2 - 1] + 4[(6m + 4)^2 - 1]$
$= 6m[(6m + 4)^2 - 1] + 4[(36m^2 + 48m + 16 - 1]$
$= 6m[(6m + 4)^2 - 1] + 12[12m^2 + 16m + 5]$
$= 6p + 6q$, where $p = m[(6m + 4)^2 - 1]$ and $q = 2(12m^2 + 16m + 5)$
$= 6(p + q)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

When $n = 6m + 5$, then

$n^3 - n = (6m + 5)^3 - (6m + 5)$
$= (6m + 5)[(6m + 5)^2 - 1]$
$= 6m[(6m + 5)^2 - 1] + 5[(6m + 5)^2 - 1]$
$= 6m[(6m + 5)^2 - 1] + 5[(36m^2 + 60m + 25 - 1]$
$= 6m[(6m + 5)^2 - 1] + 30[6m^2 + 10m + 4]$

$= 6p + 30q$, where $p = m[(6m + 5)^2 - 1]$ and $q = 6m^2 + 10m + 4$

$= 6(p + 5q)$

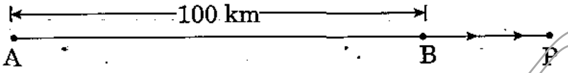
$\Rightarrow$ $n^3 - n$ is divisible by 6.

Hence, $\boldsymbol{n^3 - n}$ **is divisible by 6 for any positive integer $\boldsymbol{n}$.**

21. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars ?

Solution. Let X and Y be the two cars starting from places A and B respectively. Let x km/h and y km/h be the speeds of the cars X and Y respectively.

Case 1 : When two cars move in the same direction :



Suppose two cars meet at a point P, then

Distance travelled by the car X in 5 hours is AP

$= \text{speed} \times \text{time}$

$= (x \text{ km/h}) \times (5 \text{ h})$

$= 5x$ km ...(1)

Distance travelled by the car Y in 5 hours is BP

$= (y \text{ km/h}) \times (5 \text{ h})$

$= 5y$ km ...(2)

Distance between the two places A and B ($= AB$)

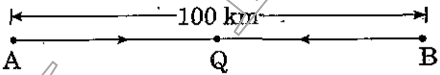
= Distance travelled by the car X – Distance travelled by the car Y

$\Rightarrow$ $AB = AP - BP$

$\Rightarrow$ $100 = 5x - 5y$ [using (1), (2) and AB = 100 km]

$\Rightarrow$ $x - y = 20$ ...(3) [Dividing both sides by 5]

Case 2 : When two cars move in the opposite directions (towards each other) :



Suppose two cars meet at a point Q, then

Distance travelled by the car X in 1 hour is AQ

$= (x \text{ km/h}) \times (1 \text{ h})$

$= x$ km ...(4)

Distance travelled by the car Y in 1 hour is BQ

$= (y \text{ km/h}) \times (1 \text{ h})$

$= y$ km ...(5)

Distance between two places A and B ($= AB$)

= Distance travelled by the car X + Distance travelled by the car Y

$\Rightarrow$ $AB = AQ + BQ$

$\Rightarrow$ $100 = x + y$ ...(6) [using (4), (5) and AB = 100 km]

Adding and subtracting (3) and (6), we get

$2x = 120$ and $2y = 80$

$\Rightarrow$ $x = 60$ and $y = 40$

Hence, the speed of the two cars are **60 km/h** and **40 km/h** respectively.

Or

Solve the following pair of equations :

$$\frac{10}{x+y}+\frac{2}{x-y}=4$$

$$\frac{10}{x+y}+\frac{2}{x-y}=-2$$

Solution. We have :

$$\frac{10}{x+y}+\frac{2}{x-y}=4 \qquad \text{...(1)}$$

and $$\frac{15}{x+y}-\frac{5}{x-y}=-2 \qquad \text{...(2)}$$

Multiplying (1) by 5 and (2) by 2, we get

$$\frac{50}{x+y}+\frac{10}{x-y}=20 \qquad \text{...(1}a\text{)}$$

and $$\frac{30}{x+y}-\frac{10}{x-y}=-4 \qquad \text{...(2}a\text{)}$$

Adding (1a) and (2a), we get

$$\left(\frac{50}{x+y}+\frac{10}{x-y}\right)+\left(\frac{30}{x+y}-\frac{10}{x-y}\right)=20-4$$

$$\Rightarrow \quad \frac{80}{x+y}=16$$

$$\Rightarrow \quad x+y=80\div 16$$

$$\Rightarrow \quad x+y=5 \qquad \text{...(3)}$$

Substituting $x + y = 5$ in (1), we obtain

$$\frac{10}{5}+\frac{2}{x-y}=4$$

$$\Rightarrow \quad \frac{2}{x-y}=4-2$$

$$\Rightarrow \quad \frac{2}{x-y}=2$$

$$\Rightarrow \quad x-y=1 \qquad \text{...(4)}$$

Now, adding (3) and (4), we get : $2x = 6 \Rightarrow x = 3$

Subtracting (4) from (3), we get : $2y = 4 \Rightarrow y = 2$

Hence, $x = 3, y = 2$ is the required solution of the given pair of equations.

22. Find all the zeroes of the polynomial $2x^3 + x^2 - 6x - 3$, if two of its zeroes are $-\sqrt{3}$ and $\sqrt{3}$.

Solution. Since two zeroes are $-\sqrt{3}$ and $\sqrt{3}$, therefore $(x+\sqrt{3})(x-\sqrt{3}) = x^2 - 3$ is a factor of the given polynomial.

Now, we divide the given polynomial by $x^2 - 3$.

$$\begin{array}{r|l} x^2-3 & 2x^3+x^2-6x-3 \quad (2x+1) \\ & \underline{2x^3 \quad\;\; -6x} \\ & \quad\; x^2 \quad\;\; -3 \\ & \quad\; \underline{x^2 \quad\;\; -3} \\ & \qquad\quad 0 \end{array}$$

$\left[\text{First term of quotient is } \frac{2x^3}{x^2} = 2x\right]$

$\left[\text{Second term of quotient is } \frac{x^2}{x^2} = 1\right]$

$\therefore \; 2x^3 + x^2 - 6x - 3 = (x^2 - 3)(2x + 1)$

$= (x + \sqrt{3})(x - \sqrt{3})(2x + 1)$

So, the zero of the polynomial $(2x + 1)$ is given by $x = \frac{-1}{2}$.

Hence, all zeroes of the given polynomial are $-\sqrt{3}$, $\sqrt{3}$ and $\frac{-1}{2}$.

23. Prove that :

$$\sec^4\theta(1 - \sin^4\theta) - 2\tan^2\theta = 1$$

Solution. We have

L.H.S. $= \sec^4\theta(1 - \sin^4\theta) - 2\tan^2\theta$

$= \sec^4\theta - \sec^4\theta . \sin^4\theta - 2\tan^2\theta$

$= \sec^4\theta - \frac{\sin^4\theta}{\cos^4\theta} - 2\tan^2\theta$

$= \sec^4\theta - \tan^4\theta - 2\tan^2\theta$

$= (\sec^4\theta - \tan^4\theta) - 2\tan^2\theta$

$= [(\sec^2\theta)^2 - (\tan^2\theta)^2] - 2\tan^2\theta$

$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta) - 2\tan^2\theta$ $\quad [\because a^2 - b^2 = (a - b)(a + b)]$

$= (1 + \tan^2\theta - \tan^2\theta)(1 + \tan^2\theta + \tan^2\theta) - 2\tan^2\theta$

$= (1)(1 + 2\tan^2\theta) - 2\tan^2\theta$

$= 1 + 2\tan^2\theta - 2\tan^2\theta$

$= 1$

$=$ R.H.S.

24. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$.

Solution. We have

L.H.S. $= (m^2 + n^2)\cos^2\beta$

$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)\cos^2\beta$ $\quad \left[\text{using } m = \frac{\cos\alpha}{\cos\beta} \text{ and } n = \frac{\cos\alpha}{\sin\beta}\right]$

$= \cos^2\alpha\left(\frac{1}{\cos^2\beta} + \frac{1}{\sin^2\beta}\right)\cdot\cos^2\beta$

$= \cos^2\alpha\cdot\left[\frac{\cos^2\beta}{\cos^2\beta} + \frac{\cos^2\beta}{\sin^2\beta}\right]$

$= \cos^2 \alpha \, [1 + \cot^2 \beta]$

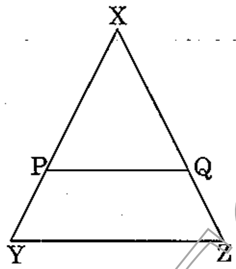
$= \cos^2 \alpha \, . \, \text{cosec}^2 \beta$ [$\because$ $\text{cosec}^2 \theta = 1 + \cot^2 \theta$]

$= \dfrac{\cos^2 \alpha}{\sin^2 \beta}$

$= n^2$

= R.H.S.

25. In figure, $\dfrac{XP}{PY} = \dfrac{XQ}{QZ} = 3$, if the area of ΔXYZ is 32 cm^2, then find the area of the quadrilateral $PYZQ$.



Solution. We have

$$\frac{XP}{PY} = \frac{XQ}{QZ}$$

$\therefore$ $PQ \parallel YZ$ [By converse of BPT]

$\therefore$ $\angle XPQ = \angle XYZ$ [Corresponding angles]

$\angle X = \angle X$ [Common]

By AA-criterion of similarity, we have

$$\Delta XPQ \sim \Delta XYZ$$

$$\Rightarrow \quad \frac{\text{ar}(\Delta XPQ)}{\text{ar}(\Delta XYZ)} = \frac{(XP)^2}{(XY)^2} = \frac{(XQ)^2}{(XZ)^2} = \frac{(PQ)^2}{(YZ)^2} \quad \text{...(1)}$$

[$\because$ The ratio of the areas of two similar triangles is equal to the ratio of squares of their corresponding sides]

Now, $\dfrac{XP}{PY} = \dfrac{XQ}{QZ} = \dfrac{3}{1}$ (given)

$$\Rightarrow \quad \frac{PY}{XP} = \frac{QZ}{XQ} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{PY}{XP} + 1 = \frac{QZ}{XQ} + 1 = \frac{1}{3} + 1$$

$$\Rightarrow \quad \frac{PY + XP}{XP} = \frac{QZ + XQ}{XQ} = \frac{1+3}{3}$$

$$\Rightarrow \quad \frac{XY}{XP} = \frac{XZ}{XQ} = \frac{4}{3}$$

$\Rightarrow \quad \dfrac{XP}{XY} = \dfrac{XQ}{XZ} = \dfrac{3}{4}$ ...(2)

From (1) and (2), we have

$$\frac{\text{ar}(\Delta XPQ)}{\text{ar}(\Delta XYZ)} = \left(\frac{XP}{XY}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$\Rightarrow \quad \text{ar}(\Delta XPQ) = \dfrac{9}{16} \times \text{ar}(\Delta XYZ)$

$\Rightarrow \quad \text{ar}(\Delta XPQ) = \dfrac{9}{16} \times 32$ [$\because$ ar (ΔXYZ) = 32 cm^2 (given)]

$\Rightarrow \quad \text{ar}(\Delta XPQ) = 18 \text{ cm}^2$

$\Rightarrow$ ar (quad $PYZQ$) = ar (ΔXYZ) – ar (ΔXPQ)

$= (32 - 18) \text{ cm}^2$

$= \mathbf{14 \text{ cm}^2}$

26. Find the length of an altitude of an equilateral triangle of side 2 cm.

Solution. Let ABC be an equilateral triangle of side 2 cm in which $AD \perp BC$, *i.e.*, AD is the altitude of ΔABC.

In ΔABD and ΔACD

$AB = AC$ [given]

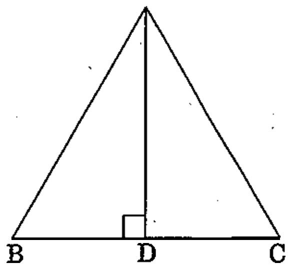
$AD = AD$ [Common]

and $\angle ADB = \angle ADC$ [Each = 90°]

$\therefore \quad \Delta ABD \cong \Delta ACD$ [R.H.S. criterion of congruence]

$\therefore \quad BD = DC$ [CPCT]

$\Rightarrow \quad BD = DC = \dfrac{1}{2}BC = \dfrac{1}{2}AB$ ... (1) [$\because AB = BC$]



In right ΔABD, we have

$AB^2 = BD^2 + AD^2$

$\Rightarrow \quad AD^2 = AB^2 - BD^2$

$\Rightarrow \quad AD^2 = AB^2 - \left(\dfrac{1}{2}AB\right)^2$ [using (1)]

$\Rightarrow \quad AD^2 = AB^2 - \dfrac{1}{4}AB^2$

$\Rightarrow \quad AD^2 = \dfrac{3}{4}AB^2$

$\Rightarrow \quad AD^2 = \dfrac{3}{4} \times (2)^2$ [$\because AB$ = 2 cm (side of an equilateral Δ)]

$\Rightarrow \quad AD^2 = 3$

$\Rightarrow \quad AD = \sqrt{3}$ cm

Hence, the length of an altitude of an equilateral triangle of side 2 cm is $\mathbf{\sqrt{3}}$ **cm.**

27. The table below gives the percentage distribution of female teachers in primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers by using step-deviation method.

Percentage of female teachers	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75	75 – 85
No. of States/U.T.	6	11	7	4	4	2	1

Solution. Let the assumed mean $a = 50$ and $h = 10$

Calculation of Mean

Percentage of female teachers	*No. of States / U.T.* (f_i)	*Class-mark* (x_i)	$u_i = \frac{x_i - 50}{10}$	f_iu_i
15 – 25	6	20	– 3	– 18
25 – 35	11	30	– 2	– 22
35 – 45	7	40	– 1	– 7
45 – 55	4	50 = a	0	0
55 – 65	4	60	1	4
65 – 75	2	70	2	4
75 – 85	1	80	3	3
Total	$n = \Sigma f_i = 35$			$\Sigma f_iu_i = -36$

Using the formula :

$$\text{Mean} = a + \frac{\Sigma f_iu_i}{\Sigma f_i} \times h$$

$$= 50 + \frac{(-36)}{35} \times 10$$

$$= 50 - \frac{72}{7}$$

$$= 50 - 10.29$$

$$= 39.71$$

Hence the mean percentage of female teachers = **39.71.**

Or

The mean of the following distribution is 8.1. Find the value of *p*.

Classes	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Frequency	1	2	1	p	6	2	3

Solution. **Calculation of Mean**

Classes	*Frequency* (f_i)	*Class-mark* (x_i)	f_ix_i
0 – 2	1	1	1
2 – 4	2	3	6
4 – 6	1	5	5
6 – 8	p	7	$7p$
8 – 10	6	9	54
10 – 12	2	11	22
12 – 14	3	13	39
Total	$n = \Sigma f_i = 15 + p$		$\Sigma f_ix_i = 127 + 7p$

Using the formula :

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$\Rightarrow$ (given) $8.1 = \dfrac{127 + 7p}{15 + p}$

$\Rightarrow$ $121.5 + 8.1p = 127 + 7p$

$\Rightarrow$ $8.1p - 7p = 127 - 121.5$

$\Rightarrow$ $1.1p = 5.5$

$\Rightarrow$ $p = 5.5 \div 1.1$

$\Rightarrow$ $\mathbf{p = 5}$

28. The following distribution shows the number of runs scored by some top batsman of the world in one-day cricket matches :

Runs-scored	***Number of batsman***
3000 – 4000	**4**
4000 – 5000	**18**
5000 – 6000	**9**
6000 – 7000	**7**
7000 – 8000	**6**
8000 – 9000	**3**
9000 – 10000	**1**
10000 – 11000	**1**

Find the mode.

Solution. Since the maximum frequency of batsman is 18, therefore, the modal class is 4000 – 5000. Thus, the lower limit (l) of the modal class = 4000.

$\therefore$ $f_1 = 18, f_0 = 4, f_2 = 9, h = 1000$

Using the formula :

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 4000 + \frac{18 - 4}{2 \times 18 - 4 - 9} \times 1000$$

$$= 4000 + \frac{14}{36 - 13} \times 1000$$

$$= 4000 + \frac{14000}{23}$$

$$= 4000 + 608.70$$

$$= 4608.70$$

So, the maximum number of batsman scored **4608.70 runs.**

Section 'D'

Question numbers 29 to 34 carry 4 marks each.

29. If two zeroes of the polynomial $x^4 + 3x^3 - 20x^2 - 6x + 36$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeroes of the polynomial.

Solution. Since two zeroes are $\sqrt{2}$ and $-\sqrt{2}$, therefore $(x - \sqrt{2})(x + \sqrt{2}) = x^2 - 2$ is a factor of the given polynomial.

Now, we divide the given polynomial by $x^2 - 2$.

$$\begin{array}{r|l} & x^2 + 3x - 18 \\ \hline x^2 - 2 & x^4 + 3x^3 - 20x^2 - 6x + 36 \\ & \underset{-}{x^4} \quad\quad \underset{+}{-}\, 2x^2 \\ \hline & \quad 3x^3 - 18x^2 - 6x + 36 \\ & \quad \underset{-}{3x^3} \quad\quad \underset{+}{-}\, 6x \\ \hline & \quad\quad -18x^2 + 36 \\ & \quad\quad \underset{+}{-}\,18x^2 \underset{-}{+}\, 36 \\ \hline & \quad\quad\quad 0 \end{array}$$

$\left[\text{First term of quotient is } \dfrac{x^4}{x^2} = x^2\right]$

$\left[\text{Second term of quotient is } \dfrac{3x^3}{x^2} = 3x\right]$

$\left[\text{Third term of quotient is } \dfrac{-18x^2}{x^2} = -18\right]$

So, $x^4 + 3x^3 - 20x^2 - 6x + 36 = (x^2 - 2)(x^2 + 3x - 18)$

$= (x - \sqrt{2})(x + \sqrt{2})[x^2 + 6x - 3x - 18]$

$= (x - \sqrt{2})(x + \sqrt{2})[x(x + 6) - 3(x + 6)]$

$= (x - \sqrt{2})(x + \sqrt{2})(x + 6)(x - 3)$

So, the zeroes of $x^2 + 3x - 18 = (x + 6)(x - 3)$ are given by $x = -6$ and $x = 3$.

Hence, the other zeroes of the given polynomial are **– 6** and **3**.

30. Solve the following system of linear equations graphically :

$$\mathbf{3x + y - 12 = 0}$$
$$\mathbf{x - 3y + 6 = 0}$$

Shade the region bounded by these lines and the x-axis. Also, find the ratio of areas of triangles formed by given lines with the x-axis and the y-axis.

Solution. The given system of linear equations are

$3x + y - 12 = 0 \Rightarrow y = 12 - 3x$ ...(1)

and $x - 3y + 6 = 0 \Rightarrow y = \dfrac{x + 6}{3}$ ...(2)

Let us draw the graphs of the equations (1) and (2) by finding two solutions for each of these equations.

They are given in tables :

$y = 12 - 3x$

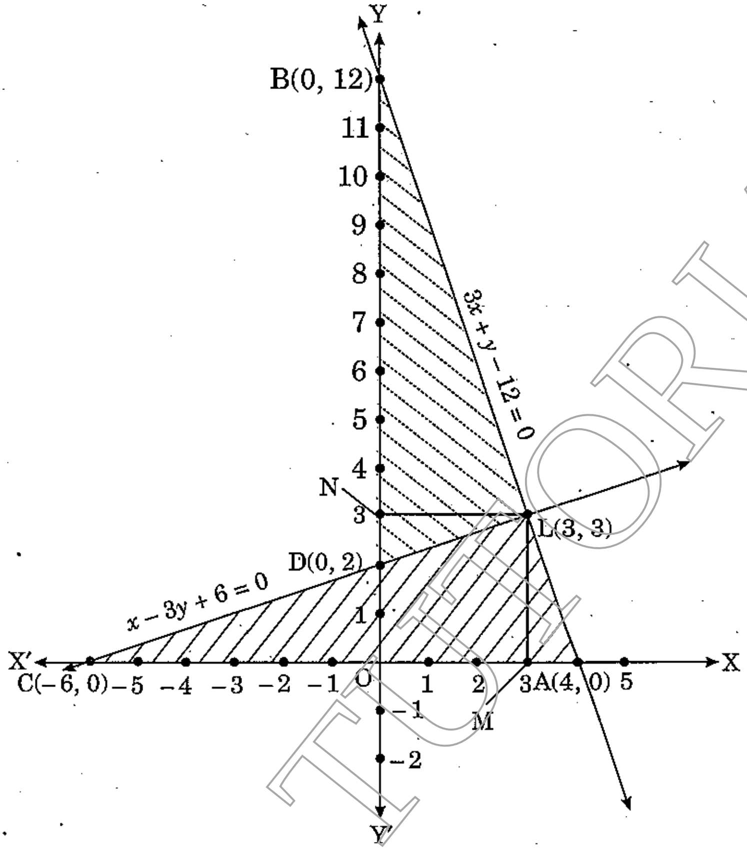
x	4	0
y	0	12
	A	B

$y = \dfrac{x + 6}{3}$

x	–6	0
y	0	2
	C	D

Plot the points $A(4, 0)$, $B(0, 12)$, $C(-6, 0)$, $D(0, 2)$ on graph paper and draw the lines AB and CD passing through them to represent the equations, as shown in figure.

The two lines intersect at the point $L(3, 3)$. So, $\boldsymbol{x = 3, y = 3}$ is the required solution of the system of linear equations.



Area of triangle formed by lines with x-axis

$= \text{Area of } \Delta ALC$

$= \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 10 \times 3$ [$\because$ Base $= CA = 10$ units and Height $LM = 3$ units]

$= 15$ sq. units.

Area of triangle formed by lines with y-axis

$= \text{Area of } \Delta BLD$

$= \frac{1}{2} \times \text{Base} \times \text{Height}$

$= \frac{1}{2} \times 10 \times 3$ [$\because$ Base $= BD = 10$ units and Height $LN = 3$ units]

$= 15$ sq. units.

Thus, the ratio of areas of the triangles formed by given lines with x-axis and the y-axis

$$= \frac{\text{Area of } \Delta ALC}{\text{Area of } \Delta BLD}$$

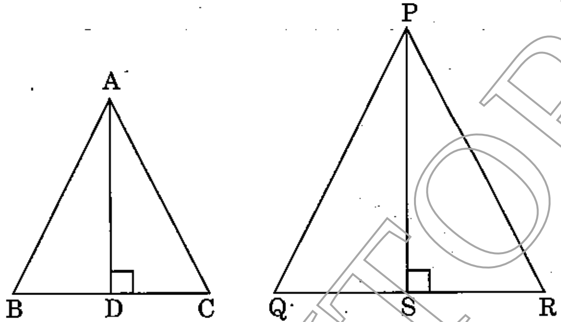
$$= \frac{15 \text{ sq. units}}{15 \text{ sq. units}} = \frac{1}{1}$$
$$= 1 : 1.$$

31. Prove that the ratio of areas of two similar triangles is equal to the square of their corresponding sides.

Solution. Given : ΔABC and ΔPQR such that $\Delta ABC \sim \Delta PQR$.

To prove : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$

Construction : Draw $AD \perp BC$ and $PS \perp QR$.



Proof : $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times QR \times PS}$ [Area of $\Delta = \frac{1}{2}$(base) × height]

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC \times AD}{QR \times PS}$ ...(1)

Now, in ΔADB and ΔPSQ, we have

$\angle B = \angle Q$ [As $\Delta ABC \sim \Delta PQR$]

$\angle ADB = \angle PSQ$ [Each = 90°]

3rd $\angle BAD$ = 3rd $\angle QPS$

Thus, ΔADB and ΔPSQ are equiangular and hence, they are similar.

Consequently $\frac{AD}{PS} = \frac{AB}{PQ}$ ...(2)

[If Δs are similar, the ratio of their corresponding sides is same]

But $\frac{AB}{PQ} = \frac{BC}{QR}$ [∵ $\Delta ABC \sim \Delta PQR$]

$\Rightarrow \frac{AD}{PS} = \frac{BC}{QR}$ ...(3) [using (2)]

Now, from (1) and (3), we get

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC}{QR} \times \frac{BC}{QR}$$

$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \frac{BC^2}{QR^2}$ ...(4)

As $\Delta ABC \sim \Delta PQR$, therefore

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad \ldots(5)$$

Hence, $\dfrac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \dfrac{AB^2}{PQ^2} = \dfrac{BC^2}{QR^2} = \dfrac{CA^2}{RP^2}$ [From (4) and (5)]

Or

Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

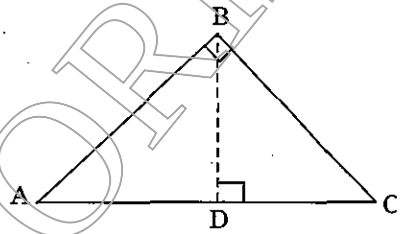
Solution. Given : A right triangle ABC, right angled at B.

To prove : $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction : Draw $BD \perp AC$

Proof : $\Delta ADB \sim \Delta ABC$.



[If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.]

So, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$ [Sides are proportional]

$\Rightarrow$ $AD.AC = AB^2$ ...(1)

Also, $\Delta BDC \sim \Delta ABC$ [Same reasoning as above]

So, $\dfrac{CD}{BC} = \dfrac{BC}{AC}$ [Sides are proportional]

$\Rightarrow$ $CD.AC = BC^2$ ...(2)

Adding (1) and (2), we have

$AD.AC + CD.AC = AB^2 + BC^2$

$\Rightarrow (AD + CD).AC = AB^2 + BC^2$

$\Rightarrow$ $AC.AC = AB^2 + BC^2$

Hence, $\mathbf{AC^2 = AB^2 + BC^2}$

32. The median of the following data is 20.75. Find the missing frequencies x and y, if the total frequency is 100.

Class Interval	*Frequency*
0 – 5	7
5 – 10	10
10 – 15	x
15 – 20	13
20 – 25	y
25 – 30	10
30 – 35	14
35 – 40	9

Solution. Here, the missing frequencies are x and y.

Class Interval	*Frequency*	*Cumulative Frequency*
0 – 5	7	7
5 – 10	10	17
10 – 15	x	$17 + x$
15 – 20	13	$30 + x$
20 – 25	y	$30 + x + y$
25 – 30	10	$40 + x + y$
30 – 35	14	$54 + x + y$
35 – 40	9	$63 + x + y$
Total	100	

It is given that $n = 100$ = Total frequency

$\therefore \quad 63 + x + y = 100$

$\Rightarrow \quad x + y = 100 - 63$

$\Rightarrow \quad x + y = 37$

$\Rightarrow \quad y = 37 - x$ ...(1)

$$\frac{n}{2} = \frac{100}{2} = 50 \text{ lies in the class-interval } 20 - 25$$

The median is 20.75 (given), which lies in the class 20 – 25.

So, **l = lower limit of median class = 20**

f = frequency of median class = y

cf = cumulative frequency of class preceding the median class = $30 + x$

h = class size = 5

Using the formula :

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 20.75 = 20 + \left(\frac{\frac{100}{2} - (30 + x)}{y}\right) \times 5$$

$$\Rightarrow \quad 0.75 = \left(\frac{50 - 30 - x}{y}\right) \times 5$$

$$\Rightarrow \quad \frac{3}{4} = \frac{(20 - x) \times 5}{y}$$

$\Rightarrow \quad 3y = 400 - 20x$

$\Rightarrow \quad 3(37 - x) = 400 - 20x$ [using (1)]

$\Rightarrow \quad 111 - 3x = 400 - 20x$

$\Rightarrow \quad 17x = 289$

$\Rightarrow \quad x = 17$

Substituting $x = 17$ in (1), we get

$$y = 37 - 17 = 20$$

Hence, the missing frequencies are $x = 17$ and $y = 20$.

33. Prove that :

$$\frac{\cot^3\theta}{1+\cot^2\theta} + \frac{\tan^3\theta}{1+\tan^2\theta} = \sec\theta\,.\,\text{cosec}\,\theta - 2\sin\theta\cos\theta$$

Solution. We have

$$\text{L.H.S.} = \frac{\cot^3\theta}{1+\cot^2\theta} + \frac{\tan^3\theta}{1+\tan^2\theta}$$

$$= \frac{\cot^3\theta}{\text{cosec}^2\theta} + \frac{\tan^3\theta}{\sec^2\theta} \qquad \left[\because \begin{array}{l} 1+\cot^2 A = \text{cosec}^2 A \\ 1+\tan^2 A = \sec^2 A \end{array}\right]$$

$$= \sin^2\theta \times \frac{\cos^3\theta}{\sin^3\theta} + \cos^2\theta \times \frac{\sin^3\theta}{\cos^3\theta}$$

$$= \frac{\cos^3\theta}{\sin\theta} + \frac{\sin^3\theta}{\cos\theta}$$

$$= \frac{\cos^4\theta + \sin^4\theta}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{\sin\theta\cos\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta} \qquad [\because a^2 + b^2 = (a+b)^2 - 2ab]$$

$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} - \frac{2\sin^2\theta\cos^2\theta}{\sin\theta\cos\theta}$$

$$= \sec\theta.\text{cosec}\,\theta - 2\sin\theta\cos\theta$$

$$= \text{R.H.S.}$$

Or

Without using trigonometrical tables, evaluate :

$$\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ\ \text{cosec}\ 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$$

Solution. We have

$$\frac{\cos 58^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos 68^\circ} - \frac{\cos 38^\circ\ \text{cosec}\ 52^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan 72^\circ \tan 55^\circ}$$

$$= \frac{\cos(90^\circ - 32^\circ)}{\sin 32^\circ} + \frac{\sin 22^\circ}{\cos(90^\circ - 22^\circ)} - \frac{\cos 38^\circ\ \text{cosec}\,(90^\circ - 38^\circ)}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \tan(90^\circ - 18^\circ)\tan(90^\circ - 35^\circ)}$$

$$= \frac{\sin 32^\circ}{\sin 32^\circ} + \frac{\sin 22^\circ}{\sin 22^\circ} - \frac{\cos 38^\circ \sec 38^\circ}{\tan 18^\circ \tan 35^\circ \tan 60^\circ \cot 18^\circ \cot 35^\circ}$$

[$\because \cos(90^\circ - \theta) = \sin\theta$, $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$, $\tan(90^\circ - \theta) = \cot\theta$]

$$= 1 + 1 - \frac{\cos 38^\circ \sec 38^\circ}{(\tan 18^\circ \cdot \cot 18^\circ) \tan 60^\circ (\tan 35^\circ \cdot \cot 35^\circ)}$$

$$= 2 - \frac{1}{(1)(\sqrt{3})(1)}$$

$\left[\begin{array}{l} \because \cos\theta \cdot \sec\theta = 1 \\ \tan\theta \cdot \cot\theta = 1 \text{ and } \tan 60^\circ = \sqrt{3} \end{array}\right]$

$$= \mathbf{2 - \frac{1}{\sqrt{3}}}.$$

34. Prove that :

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}.$$

Solution. We have

$$\text{L.H.S.} = (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$$

[$\because \sin^2 A + \cos^2 A = 1$]

$$= \cos A \sin A.$$

Now, $\text{R.H.S.} = \dfrac{1}{\tan A + \cot A}$

$$= \frac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$$

$$= \frac{1}{\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \sin A \cos A$$

[$\because \sin^2 A + \cos^2 A = 1$]

Hence, L.H.S. = R.H.S.