UNIT-9

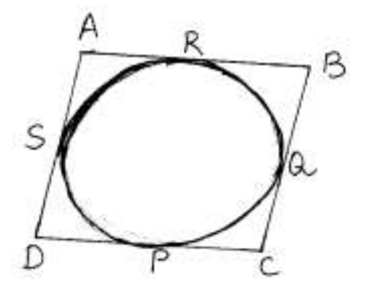
CIRCLES

1. Prove that the parallelogram circumscribing a circle is rhombus.

Ans Given : ABCD is a parallelogram circumscribing a circle.
To prove : - ABCD is a rhombus
or
AB=BC=CD=DA

Proof: Since the length of tangents from external are equal in length
$\therefore$ AS = AR …..(1)
BQ = BR …..(2)
QC = PC …..(3)
SD = DP …..(4)

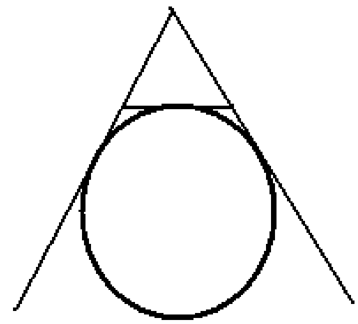
Adding (1), (2), (3) & (4).
AS + SD + BQ + QC = AR + BR + PC + DP
AD + BC = AB + DC
AD + AD = AB + AB
Since BC = AD & DC = AB (opposite sides of a parallelogram are equal)
2AD = 2AB
$\therefore$ AD = AB …..(5)
BC = AD } (opposite sides of a parallelogram)
DC = AB } …..(6)
From (5) and (6)
AB = BC = CD = DA
Hence proved



2. A circle touches the side BC of a triangle ABC at P and touches AB and AC when produced at Q and R respectively as shown in figure.

Show that AQ= $\frac{1}{2}$ (perimeter of triangle ABC)

A
P
B C
Q R



Ans: Since the length of tangents from external point to a circle are equal.

AQ = AR

BQ = BP

PC = CR

Since AQ = AR

AB + BQ = AC + CR

$\therefore$ AB + BP = AC + PC (Since BQ = BP & PC = CR)

Perimeter of Δ ABC = AB + AC + BC

= AB + BP + PC + AC

= AQ + PC + AC (Since AB + BP = AQ)

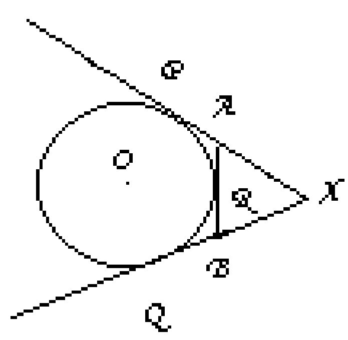
= AQ + AB + BP (Since PC + AC = AB + BP)

= AQ + AQ (Since AB + BP = AQ)

Perimeter of Δ ABC = 2AQ

$\therefore$ AQ = $\frac{1}{2}$ (perimeter of triangle ABC)

3. In figure, XP and XQ are tangents from X to the circle with centre O. R is a point on the circle. Prove that XA+AR=XB+BR



Ans: Since the length of tangents from external point to a circle are equal

XP = XQ

PA = RA

BQ = BR

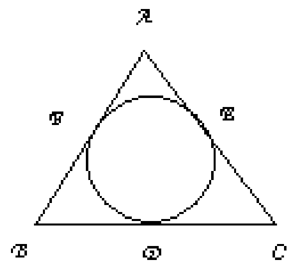
XP = XQ

$\Rightarrow$XA + PA = XB + BQ

$\Rightarrow$XA + AR = XB + BR $\left[\because PA = AR \;\&\; BQ = BR\right]$

Hence proved

4. In figure, the incircle of triangle ABC touches the sides BC, CA, and AB at D, E, and F respectively. Show that AF+BD+CE=AE+BF+CD= $\frac{1}{2}$ (perimeter of triangle ABC),



Ans: Since the length of tangents from an external point to are equal

$\therefore$ AF = AE
FB = BD
EC = CD

$$\text{Perimeter of } \Delta ABC = AB + BC + AC$$
$$= AF + FB + BD + DC + AE + EC$$
$$= AF + BD + BD + CE + AF + CE$$
$$(\because AF=AE,\ FB=BD,\ EC=CD)$$
$$= AF + AF + BD + BD + CE + CE$$

$$\text{Perimeter of } \Delta ABC = 2(AF + BD + CE)$$

$$\therefore AF + BD + CE = \frac{1}{2}(\text{perimeter of } \Delta ABC) \quad \ldots\ldots..(1)$$

$$\text{Perimeter of } \Delta ABC = AB + BC + AC$$
$$= AF + FB + BD + DC + AE + EC$$
$$= AE + BF + BF + CD + AE + CD$$
$$(\because AF = AE,\ FB = BD,\ EC = CD)$$
$$= AE + AE + BF + BF + CD + CD$$

$$\text{Perimeter of } \Delta ABC = 2(AE + BF + CD)$$

$$\therefore AE + BF + CD = \frac{1}{2}(\text{perimeter of } \Delta ABC) \quad \ldots\ldots..(2)$$

From (1) and (2)

$$AF + BD + CE = AE + BF + CD = \frac{1}{2}(\text{perimeter of } \Delta ABC)$$

5. A circle touches the sides of a quadrilateral ABCD at P, Q, R and S respectively. Show that the angles subtended at the centre by a pair of opposite sides are supplementary.

Ans: To prove :- $\angle AOB + \angle DOC = 180^o$
$\angle BOC + \angle AOD = 180^o$

Proof : - Since the two tangents drawn from an external point to a circle subtend equal angles at centre.

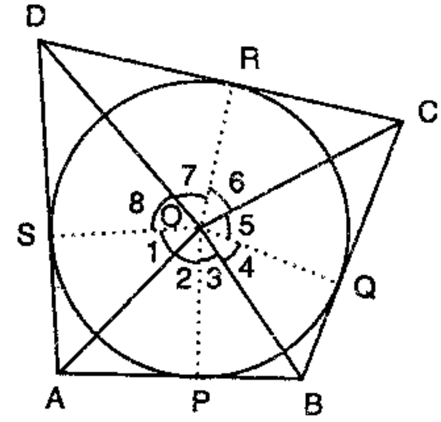
$\therefore \angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$

but $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360^o$

$\angle 2 + \angle 3 + \angle 6 + \angle 7 = 360^o$

$\therefore \angle AOB + \angle DOC = 180^o$



Similarly

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

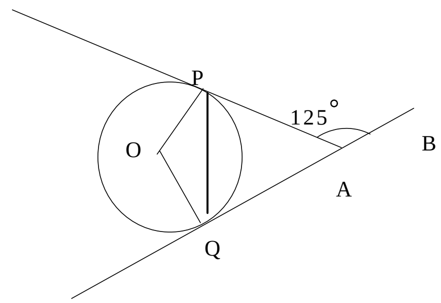
$2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360^o$

$\angle 1 + \angle 8 + \angle 5 = 180^o$

$\therefore \angle BOC + \angle AOD = 180^o$

Hence proved

6. In figure, O is the centre of the Circle .AP and AQ two tangents drawn to the circle. B is a point on the tangent QA and $\angle$ PAB = 125°, Find $\angle$ POQ.
(Ans: 125^o)



Ans: Given $\angle PAB = 125^o$

To find : - $\angle POQ = ?$

Construction : - Join PQ

Proof : - $\angle PAB + \angle PAQ = 180^o$ (Linear pair)

$\angle PAQ + 125^o = 180^o$

$\angle PAQ = 180^o - 125^o$

$\angle PAQ = 55^o$

Since the length of tangent from an external point to a circle are equal.

PA = QA

$\therefore$ From ΔPAQ

$\angle APQ = \angle AQP$

In ΔAPQ

$\angle APQ + \angle AQP + \angle PAQ = 180^o$ (angle sum property)

$\angle APQ + \angle AQP + 55^o = 180^o$

$2\angle APQ = 180^o - 55^o$ ($\because \angle APQ = \angle AQP$)

$\angle APQ = \frac{125^o}{2}$

$\therefore \angle APQ = \angle AQP = \frac{125^o}{2}$

OQ and OP are radii

QA and PA are tangents

$\therefore \angle OQA = 90^o$

& $\angle OPA = 90^o$

$\angle OPQ + \angle QPA = \angle OPA = 90^o$ (Linear Pair)

$\angle OPQ + \frac{125^o}{2} = 90^o$

$\angle OPQ = 90^o - \frac{125^o}{2}$

$= \frac{180^o - 125^o}{2}$

$\angle OPQ = \frac{55^o}{2}$

Similarly $\angle OQP + \angle PQA = \angle OQA$

$\angle OQP + \frac{125^o}{2} = 90^o$

$\angle OQP = 90^o - \frac{125^o}{2}$

$\angle OQP = \frac{55^o}{2}$

In ΔPOQ

$\angle OQP + \angle OPQ + \angle POQ = 180^o$ (angle sum property)

$\frac{55^o}{2} + \frac{55^o}{2} + \angle POQ = 180^o$

$\angle POQ + \frac{110}{2} = 180^o$

$\angle POQ = 180^o - \frac{110}{2}$

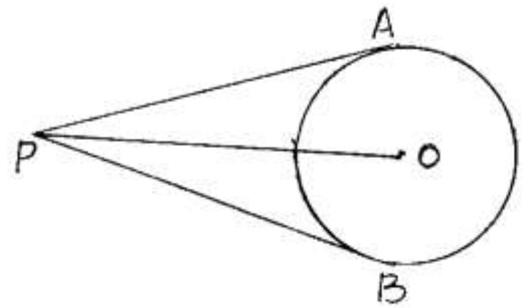
$\angle POQ = \frac{360^o - 110^o}{2}$

$\angle POQ = \frac{250^o}{2}$

$\angle$POQ =125^o
$\therefore \angle$POQ =125^o

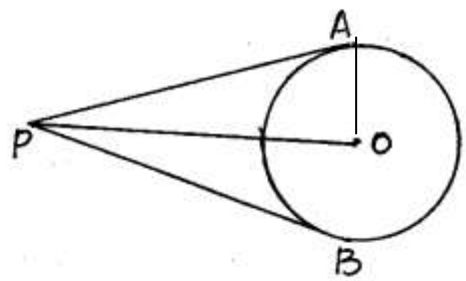
7. Two tangents PA and PB are drawn to the circle with center O, such that $\angle$APB=120^o. Prove that OP=2AP.

Ans: Given : - $\angle$APB = 120^o
Construction : -Join OP
To prove : -OP = 2AP
Proof :- $\angle$APB = 120^o
$\therefore \angle$APO = $\angle$OPB = 60^o
Cos 60^o = $\frac{AP}{OP}$
$\frac{1}{2} = \frac{AP}{OP}$
$\therefore$ OP = 2AP
Hence proved



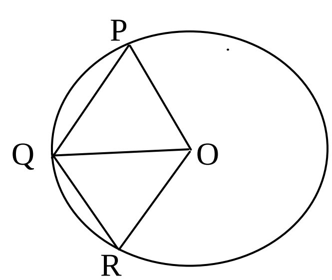
8. From a point P, two tangents PA are drawn to a circle with center O. If OP=diameter of the circle show that triangle APB is equilateral.

Ans: PA=PB (length of tangents from an external point
From ΔOAP,
sin$\angle$APO = $\frac{OA}{OP} = \frac{1}{2}$
Since OP = 2OA (Since OP=Diameter)
$\therefore \angle$APO = 30^o
since Δ APO ≅ ΔBPO
$\angle$APO = $\angle$BPO = 30^o
$\therefore \angle$APB = 60^o
ΔAPB is equilateral



9. In the given fig OPQR is a rhombus, three of its vertices lie on a circle with centre O If the area of the rhombus is 32√3 cm^2. Find the radius of the circle.

Ans: QP = OR
OP = OQ
$\therefore$ΔOPQ is a equilateral Δ.
area of rhombus = 2 (ar of ΔOPQ)
$32\sqrt{3} = 2\left(\frac{\sqrt{3}r^2}{4}\right)$
$32\sqrt{3} = \frac{\sqrt{3}r^2}{2}$

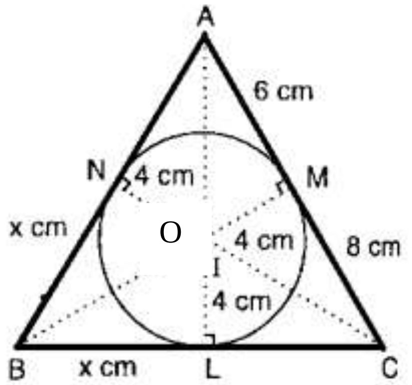


$r^2 = 32 \times 2 = 64$
$\Rightarrow r = 8cm$
$\therefore$ Radius = 8cm

10. If PA and PB are tangents to a circle from an outside point P, such that PA=10cm and $\angle APB = 60^o$. Find the length of chord AB.

Self Practice

11. The radius of the in circle of a triangle is 4cm and the segments into which one side is divided by the point of contact are 6cm and 8cm. Determine the other two sides of the triangle.



(Ans: 15, 13)

Ans: a = BC = $x + 8$
b = AC = 6 + 8 = 14cm
c = AB = $x + 6$

Semi – perimeter = $\frac{a+b+c}{2}$

$= \frac{BC + AC + AB}{2}$

$= \frac{x + 8 + 14 + x + 6}{2}$

$= \frac{2x + 28}{2}$

$= x + 14$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ on substituting we get

$= \sqrt{(x+14)(6)(x)(8)}$

$= \sqrt{(x+14)(48x)}$ ……………..(1)

Area of ΔABC = area ΔAOB + area BOC + area Δ AOC area $\Delta AOC = \left(\frac{1}{2}b.h\right) = \frac{1}{2}$ x 4 x 14

= 28

On substituting we get

$\therefore$ area ΔABC = area ΔAOC + area ΔBOC + area ΔAOB

= 4x + 56 ……………(2)

From (1) and (2)

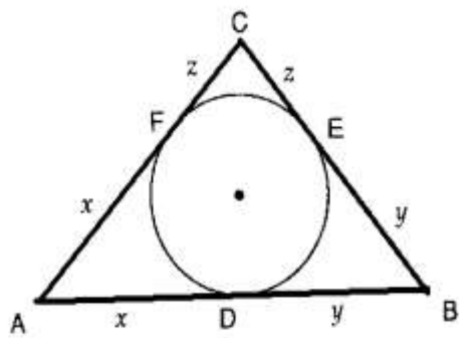
$4x + 56 = \sqrt{(x + 14)(48\ x)}$

Simplify we get $x = 7$

$\therefore$ AB = x + 6 = 7 + 6 = 13cm

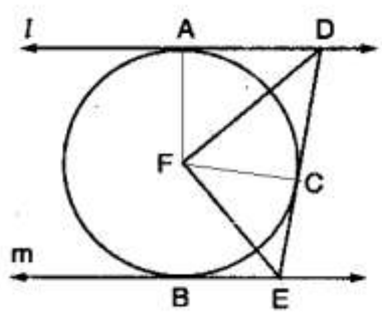
$\therefore$ BC = x + 8 = 7 + 8 = 15cm

12. A circle is inscribed in a triangle ABC having sides 8cm, 10cm and 12cm as shown in the figure. Find AD, BE and CF. (Ans :7cm ,5cm,3cm)



Self Practice

13. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.



Since Δ ADF $\cong$ Δ DFC

$\angle ADF = \angle CDF$

$\therefore \angle ADC = 2\angle CDF$

Similarly we can prove $\angle CEB = 2\angle CEF$

Since $l \parallel m$

$\angle ADC + \angle CEB = 180^o$

$\Rightarrow 2\angle CDF + 2\angle CEF = 180^o$

$\Rightarrow \angle CDF + \angle CEF = 90^o$

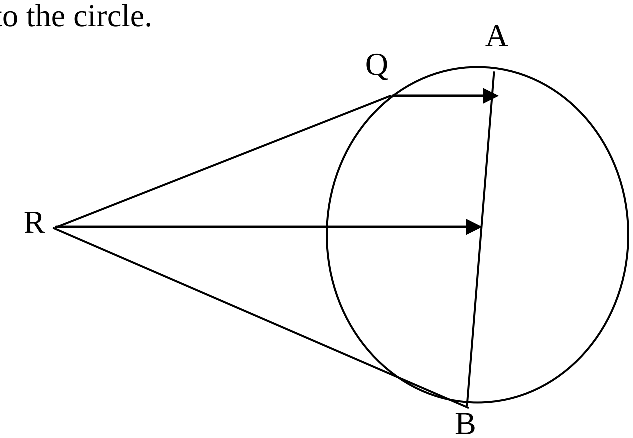
In Δ DFE

$\angle DFE = 90^o$

14. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Ans: Same as question No.5

15. QR is the tangent to the circle whose centre is P. If QA || RP and AB is the diameter, prove that RB is a tangent to the circle.



Self Practice