# ARITHMETIC PROGRESSION

**if we are given the first term $a$ and the common difference $d$, we can completely identify the sequence as,**

$$a,\ a + d,\ a + 2d,\ a + 3d, ...,\ a + (n - 1)d, ...$$

**If the A.P. is finite and terminates at the $n$th step, then $T_n = a + (n - 1)d$ is also called the last term.**

**An important result :**

$d = \frac{T_m - T_n}{m - n}$, $m \neq n$.

In fact $T_m - T_n = a + (m - 1)d - (a + (n - 1)d)$

$= (m - n)d$

$\therefore \quad d = \frac{T_m - T_n}{m - n}$ **$(m - n \neq 0)$**

| | |
|---|---|
| Q.1 | Find the 101st term of A.P. 5, 11, 17,... |
| | **Solution :** $a = 5,\ d = 6$<br>$T_n = a + (n - 1)d$<br>$\therefore \quad T_{101} = 5 + (101 - 1)6 = 5 + 600 = 605$<br>$\therefore$ The 101st term is 605. |
| Q,2 | For a given A.P. 5, 10, 15, 20,..., 200, what is the number of terms ? |
| | **Solution :** Suppose 200, the last term is the $n$th term.<br>$T_n = 200,\ a = 5,\ d = 5$<br>$\therefore \quad a + (n - 1)d = 200$<br>$\therefore \quad 5 + (n - 1)5 = 200$<br>$\therefore \quad 1 + n - 1 = 40$<br>$\therefore \quad n = 40$<br>$\therefore$ The number of terms is 40. |
| Q.3 | Is 0 a term of A.P. 200, 196, 192,..., −200 ? If yes, what is its order ? |
| | **Solution :** $a = 200,\ d = 196 - 200 = -4$ Let $T_n = 0$, if possible. $T_n = a + (n - 1)d$<br>$\therefore \quad 200 - 4(n - 1) = 0 \quad \therefore \quad n = 51 \quad 50 - n + 1 = 0$ Yes, 51st term of the A.P. is zero. |
| Q.4 | If in an A.P., 7th term is 108 and 11th term is 212, find its $n$th term. |
| | **Solution :** Here $T_7 = 108,\ T_{11} = 212$<br>$\therefore \quad a + 6d = 108$ and $a + 10d = 212$ **$(T_n = a + (n - 1)d)$**<br>Subtraction of the equations gives $4d = 104$ |

$\therefore \quad d = 26$

$\therefore \quad a = 108 - 6d = 108 - 156 = -48$

$\therefore \quad T_n = a + (n - 1)d = -48 + 26(n - 1)$

$\therefore \quad T_n = 26n - 74$

Here $d = \frac{T_{11} - T_7}{11 - 7} = \frac{212 - 108}{4} = \frac{104}{4} = 26$

**Q.5** How many three digit multiples of 7 are there ?

**Solution :** Multiples of 7 having 3 digits are 105, 112,..., 994.

$\therefore \quad a = 105,\ d = 7,\ \ T_n = 994$

$\therefore \quad 994 = 105 + 7(n - 1)$

$\therefore \quad \frac{889}{7} = n - 1$

$\therefore \quad n = 128$

$\therefore$ There are 128 three digit multiples of 7.

**Q.5** Which is the first negative term of A.P. 112, 107, 102,... ?

**Solution :** Let the $n$th term of the sequence be its first negative term.

$\therefore \quad T_n < 0$

$\therefore \quad 112 + (n - 1)(-5) < 0$

$\therefore \quad 112 < 5(n - 1)$

$\therefore \quad n > \frac{112}{5} + 1$

$\therefore \quad n > 23.4$

$\therefore$ The smallest $n \in N$ greater than 23.4 is 24.

$\therefore$ 24th term is the first negative term of A.P. 112, 107, 102,...

**Tips** Infact $T_{23} = 112 + (23 - 1)(-5) = 2$,

$T_{24} = 112 + (24 - 1)(-5) = -3$

**Q.6** Determine the A.P. whose 4th term is 17 and the 10th term exceeds the 7th term by 12.

**Solution :** We know $d = \frac{T_m - T_n}{m - n}$

Here $T_{10} = T_7 + 12$. So $T_{10} - T_7 = 12$

$\therefore \quad d = \frac{T_{10} - T_7}{10 - 7} = \frac{12}{3} = 4$

Now, $T_4 = a + 3d = 17$

$\therefore \quad a + 12 = 17$

$\therefore \quad a = 5$

$\therefore$ The A.P. is 5, 9, 13, 17, 21, 25,... $T_n = 5 + 4(n - 1) = 4n + 1$

**Q.7** For which $n$, some terms of 231, 228, 225,... and 3, 6, 9,... are equal ?

**Solution :** For the sequence 231, 228, 225,...

$T_n = 231 + (n - 1)(-3) = -3n + 234$

For the sequence 3, 6, 9,... $T_n' = 3n$

We want $T_n = T_n' \quad \therefore \quad 3n = -3n + 234 \qquad n = 39$

The 39th terms of both A.P. s are same, namely 117.