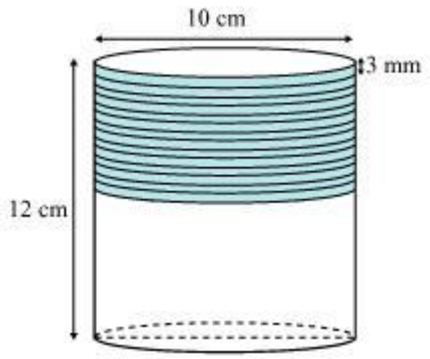
SOLVED- MENSURATION TEST PAPER

Q1. A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.



It can be observed that 1 round of wire will cover 3 mm height of cylinder.

$$\text{Number of rounds} = \frac{\text{Height of cylinder}}{\text{Diameter of wire}}$$
$$= \frac{12}{0.3} = 40 \text{ rounds}$$

Length of wire required in 1 round = Circumference of base of cylinder

$= 2\pi r = 2\pi \times 5 = 10\pi$

Length of wire in 40 rounds = $40 \times 10\pi$

$= \frac{400 \times 22}{7} = \frac{8800}{7}$ = 1257.14 cm = 12.57 m

Radius of wire

$= \frac{0.3}{2} = 0.15 \text{ cm}$

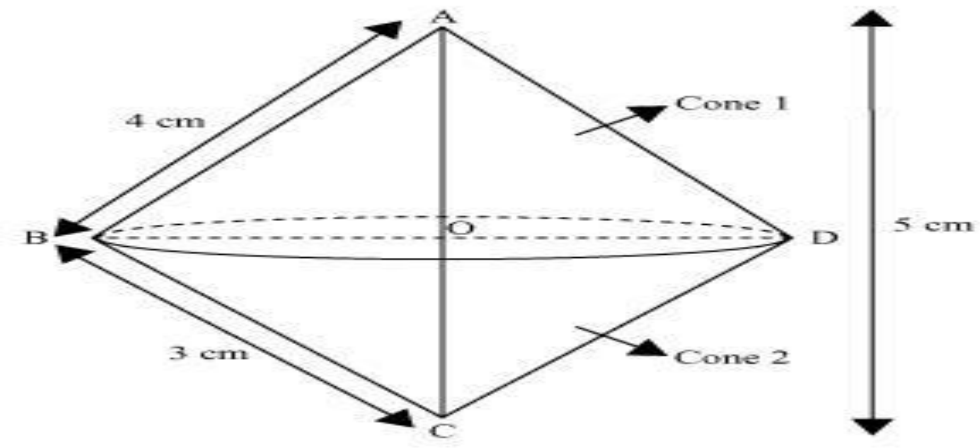
Volume of wire = Area of cross-section of wire × Length of wire

$= \pi(0.15)^2 \times 1257.14 = 88.898 \text{ cm}^3$

Mass = Volume × Density

= 88.898 × 8.88 = 789.41 gm

Q2. A right triangle whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate.



The double cone so formed by revolving this right-angled triangle ABC about its hypotenuse is shown in the figure.

Hypotenuse $AC = \sqrt{3^2 + 4^2} = \sqrt{25}$ = 5 cm

Area of ΔABC $= \frac{1}{2} \times AB \times AC$

$$\frac{1}{2} \times AC \times OB = \frac{1}{2} \times 4 \times 3$$
$$\frac{1}{2} \times 5 \times OB = 6$$
$$OB = \frac{12}{5} = 2.4 \text{ cm}$$

Volume of double cone = Volume of cone 1 + Volume of cone 2

$$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$$
$$= \frac{1}{3}\pi r^2 (h_1 + h_2) = \frac{1}{3}\pi r^2 (OA + OC)$$
$$= \frac{1}{3} \times 3.14 \times (2.4)^2 (5)$$

= 30.14 cm^3

Surface area of double cone = Surface area of cone 1 + Surface area of cone 2

= $\pi r l_1 + \pi r l_2$ $= \pi r [4 + 3] = 3.14 \times 2.4 \times 7$ = 52.75 cm^2

Q.03. A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being 22.5 cm × 7.5 cm × 6.5 cm?

Solution : Volume of cistern = 150 × 120 × 110 = 1980000 cm^3

Volume to be filled in cistern = 1980000 – 129600 = 1850400 cm^3

Let n numbers of porous bricks were placed in the cistern.

Volume of n bricks = n × 22.5 × 7.5 × 6.5 = 1096.875n

As each brick absorbs one-seventeenth of its volume, therefore, volume absorbed by these bricks

$$=\frac{n}{17}(1096.875)$$

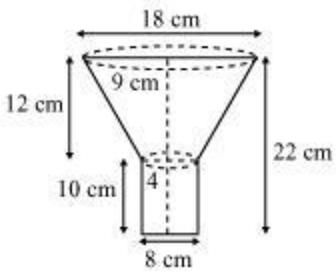
$$1850400+\frac{n}{17}(1096.875)=(1096.875)n$$

$$1850400=\frac{16n}{17}(1096.875)$$

n = 1792.41

Therefore, 1792 bricks were placed in the cistern.

Q.04. An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see the given figure).



Radius (r_1) of upper circular end of frustum part $=\frac{18}{2}=9 \text{ cm}$

Radius (r_2) of lower circular end of frustum part = Radius of circular end of cylindrical

part

$$=\frac{8}{2}=4 \text{ cm}$$

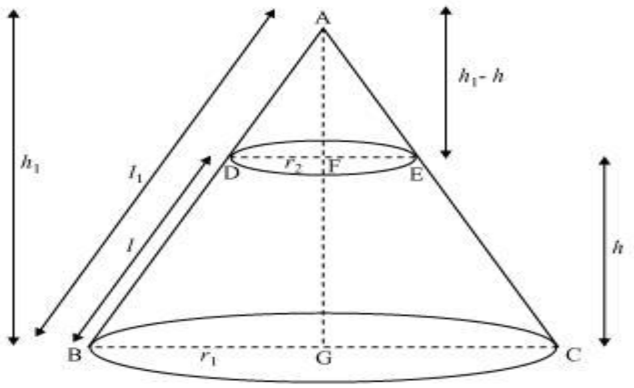
Height (h_1) of frustum part = 22 − 10 = 12 cm Height (h_2) of cylindrical part = 10 cm

Slant height (l) of frustum part $=\sqrt{(r_1-r_2)^2+h^2}=\sqrt{(9-4)^2+(12)^2}=13 \text{ cm}$

Area of tin sheet required = CSA of frustum part + CSA of cylindrical part

$$= \pi(r_1 + r_2)l + 2\pi r_2 h_2$$
$$= \frac{22}{7} \times (9+4) \times 13 + 2 \times \frac{22}{7} \times 4 \times 10$$
$$= \frac{22}{7}[169+80] = \frac{22 \times 249}{7}$$
$$= 782\frac{4}{7} \text{ cm}^2$$

Q.05 Derive the formula for the curved surface area and total surface area of the frustum of cone.



Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base. Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In ΔABG and ΔADF, DF||BG

ΔABG ≈ ΔADF

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$
$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$
$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$
$$1 - \frac{l}{l_1} = \frac{r_2}{r_1}$$
$$\frac{l}{l_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$
$$\frac{l_1}{l} = \frac{r_1}{r_1 - r_2}$$
$$l_1 = \frac{r_1 l}{r_1 - r_2}$$

CSA of frustum DECB = CSA of cone ABC − CSA cone ADE

$$= \pi r_1 l_1 - \pi r_2 (l_1 - l)$$
$$= \pi r_1 \left(\frac{l r_1}{r_1 - r_2}\right) - \pi r_2 \left[\frac{r_1 l}{r_1 - r_2} - l\right]$$
$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \pi r_2 \left(\frac{r_1 l - r_1 l + r_2 l}{r_1 - r_2}\right)$$

→

$$= \frac{\pi r_1^2 l}{r_1 - r_2} - \frac{\pi r_2^2 l}{r_1 - r_2}$$
$$= \pi l \left[\frac{r_1^2 - r_2^2}{r_1 - r_2}\right]$$

CSA of frustum = $\pi(r_1 + r_2) l$

Total surface area of frustum = CSA of frustum + Area of upper circular end + Area of lower circular end

$$= \pi(r_1 + r_2) l + \pi r_2^2 + \pi r_1^2$$
$$= \pi\left[(r_1 + r_2) l + r_1^2 + r_2^2\right]$$

Q.06: Derive the formula for the volume of the frustum of a cone.

Solution: Let ABC be a cone. A frustum DECB is cut by a plane parallel to its base.

Let r_1 and r_2 be the radii of the ends of the frustum of the cone and h be the height of the frustum of the cone.

In ΔABG and ΔADF, DF||BG

ΔABG ≈ ΔADF

$$\frac{DF}{BG} = \frac{AF}{AG} = \frac{AD}{AB}$$
$$\frac{r_2}{r_1} = \frac{h_1 - h}{h_1} = \frac{l_1 - l}{l_1}$$
$$\frac{r_2}{r_1} = 1 - \frac{h}{h_1} = 1 - \frac{l}{l_1}$$

→

$$1 - \frac{h}{h_1} = \frac{r_2}{r_1}$$
$$\frac{h}{h_1} = 1 - \frac{r_2}{r_1} = \frac{r_1 - r_2}{r_1}$$
$$\frac{h_1}{h} = \frac{r_1}{r_1 - r_2}$$
$$h_1 = \frac{r_1 h}{r_1 - r_2}$$

Volume of frustum of cone = Volume of cone ABC − Volume of cone ADE

$$=\frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 (h_1 - h)$$

$$=\frac{\pi}{3}\left[r_1^2 h_1 - r_2^2 (h_1 - h)\right]$$

$$=\frac{\pi}{3}\left[r_1^2\left(\frac{hr_1}{r_1 - r_2}\right) - r_2^2\left(\frac{hr_1}{r_1 - r_2} - h\right)\right]$$

$$=\frac{\pi}{3}\left[\left(\frac{hr_1^3}{r_1 - r_2}\right) - r_2^2\left(\frac{hr_1 - hr_1 + hr_2}{r_1 - r_2}\right)\right]$$

$$=\frac{\pi}{3}\left[\frac{hr_1^3}{r_1 - r_2} - \frac{hr_2^3}{r_1 - r_2}\right]$$

$$=\frac{\pi}{3}h\left[\frac{r_1^3 - r_2^3}{r_1 - r_2}\right]$$

$$=\frac{\pi}{3}h\left[\frac{(r_1 - r_2)\left(r_1^2 + r_2^2 + r_1 r_2\right)}{r_1 - r_2}\right]$$

$$=\frac{1}{3}\pi h\left[r_1^2 + r_2^2 + r_1 r_2\right]$$